REAL TIME ACOUSTIC BASED MODAL FILTERING APPROACH TO SENSING SYSTEM DESIGN

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INTRODUCTION

In designing an active control system for a given structural acoustic radiation problem, two important ideals come into conflict: to be able to measure, and so attenuate, an error criterion that is directly related to a global quantity such as acoustic power; and to minimize the number of input signals that must be handled by the controller. The first ideal may lead to the greatest levels of global disturbance attenuation [1]. However, it is the latter which governs the complexity and speed of the control system [2]. Considering the case of noise radiated from electrical transformers, the earliest researchers [3] found that, when minimizing acoustic pressure amplitude at a limited number of sensors locations, attenuation was confined to the immediate vicinity of the sensor(s). A result which has been reported in more recent research [4, 5]. To increase the area over which attenuation is achieved the number of sources and sensors needs to be increased [6–9], with significant noise reductions being reported over angles of 35-40 degrees around the circumference of the transformer. However, simply scaling up smaller systems in an attempt to improve global performance around, say, a transformer with edge dimensions of the order of 3-5 meters is not practical. The sheer number of sensor signals required as controller inputs would be too difficult for the controller CPU and would consequently reduce the system’s performance [2].

To seek a balance for the two ideals above, many active control researchers have turned to modal filtering. The aim of modal filtering [10–13] is to resolve global quantities, traditionally the amplitude of structural modes, from a large number of point sensor measurements. Any control strategy would then target attenuation of the resolved quantities rather than the point measurements. A large number of sensor signals are then condensed into a relatively small number of measurements of global quantities (modes) for use by the control system. This balances the two ideals of (global) error criteria measurement and minimising the number of input signals. While free space sound fields do not have any true “modes”, it is possible to mathematically express the acoustic power radiated from a vibrating structure as the sum of contributions from orthogonal combinations of structural modes [1, 10, 14–16]. To do this, the acoustic power is expressed as a quadratic function of structural modes. The eigenvectors of this expression effectively take on the role of modes in the modal filtering exercise; hence the term "radiation modes" [15]. The eigenvalues provide a measure of each radiation mode’s
efficiency, quantify the importance of each mode to overall power. The sensing system design problem is then to efficiently measure the target modes, through the use of point [14] or shaped piezoelectric film sensors [14, 16–19].

While the above approach to sensing system design works well in the laboratory on simple structures, it is difficult to apply to practical problems for a number of reasons, including:

- At the heart of the design methodology is the requirement of structural vibration characteristics, commonly structural mode shapes, to construct the (quadratic) acoustic power expression. For many applications of interest, such as an electrical transformer, obtaining this information is not practical. This same shortcoming also applies to other sensing system design strategies (for example, [16–19]) developed for active control.

- The techniques are explicitly vibration-based, to be used only with vibration sensors and actuators. This greatly limits the scope of application of the techniques, as well as increasing the number of sensors that must be used when the structural wavelength is smaller than the acoustic wavelength, as is often the case.

It is the aim of this work to overcome these problems through an alternative sensing system design strategy, formulated entirely in the acoustic space. This will simplify implementation in practice, broaden the scope of application, remove the need for knowledge of structural mode shapes, and provide the flexibility needed to use either vibration or acoustic actuators in the control solution. Some of the detailed rationale and development of the theory has been presented elsewhere [20], and so will not be repeated here. Of chief interest are a number of practical issues associated with implementation of the sensing system scheme in the time domain, as will be required in a practical setting. The associated sensing system simulations will use a novel time-domain approach.

It is important to state at the outset that no control simulations will be presented. The aim of this exercise is to develop an approach to sensing system design for active noise control that can be used in a variety of control system design techniques: adaptive feedforward, LQG, etc. By starting the development with a standard-form quadratic performance metric, it is straightforward to apply the technique in a range of control settings.

**THEORY**

**QUADRATIC PERFORMANCE MEASURE DEVELOPMENT**

For the application of interest here, the global performance measure, $J$, is expressed as a quadratic function,

$$J(t) = q^T(t)A(\omega)q(t)$$

(1)

where $q$ is a state vector of measured quantities and $A$ is a weighting matrix, often frequency dependent. The aim here is derivation of a quadratic expression of the form in Eq. (1) for radiated acoustic power, in terms of some fundamental, “mode-like” quantity, which provides the basis for a modal filtering problem.

Consider a planar structure in an infinite baffle subject to harmonic excitation and radiating into air, as shown in Fig. 1. To calculate acoustic power $W$ radiated by this source the acoustic
intensity is integrated over a hemisphere enclosing the source. Using the geometry of Fig. 1, acoustic power can be written as

\[
W = \int_0^{2\pi} \int_0^{\pi/2} \frac{|p(\mathbf{r})|^2}{2 \rho_0 c_0} |\mathbf{r}|^2 \sin \theta \, d\theta \, d\varphi
\]  

(2)

In Eq. (2), \(p(\mathbf{r})\) is the acoustic pressure \(p\) at some location \(\mathbf{r}\) in space, with the location defined by the spherical coordinates \(\mathbf{r} = (r, \theta, \phi)\). The terms \(\rho_0\) and \(c_0\) are the density of air and speed of sound in air respectively.

Letting the acoustic pressure be decomposed in terms of acoustic multipole radiation patterns, analogous to the decomposition of a structural velocity distribution in terms of \textit{in vacuo} mode shape functions, pressure is written as

\[
p(\mathbf{r}) = \sum_{i=1}^{\infty} a_i \psi_i(\mathbf{r})
\]  

(3)

where \(a_i\) is the amplitude of the \(i\)th multipole, and \(\psi_i(\mathbf{r})\) is the radiation transfer function for the \(i\)th multipole between the origin and the location \(\mathbf{r}\) (ie, the value of the radiation pattern generated by the multipole at location \(\mathbf{r}\) in space). The multipole radiation patterns used in the decomposition will be derived from in-phase/out-of-phase combinations of monopoles situated in an array on the baffle surface, as depicted in Fig. 2.

Figure 1: System Geometry

Figure 2: Monopoles situated on a baffle

By using an array of monopoles to generate the multipole patterns one can simply increase the number of multipoles used in the problem by increasing the size of the array. The precise extent of the array will be problem-specific, influenced by factors such as a structure size and frequency range.

The infinite sum in Eq. (3) can be truncated at \(n\) acoustic multipoles, enabling the pressure to be written as the matrix expression

\[
p(\mathbf{r}) = \mathbf{\psi}(\mathbf{r}) \mathbf{a}
\]  

(4)

where \(\mathbf{\psi}(\mathbf{r})\) is a row vector with elements corresponding to the values of the radiation transfer functions for the \(n\) multipoles included in the calculation,

\[
\mathbf{\psi}(\mathbf{r}) = [\psi_1(\mathbf{r}) \, \psi_2(\mathbf{r}) \cdots \psi_n(\mathbf{r})]
\]  

(5)
where
\[ \psi_{\text{multipoles}}(r) = \frac{j \omega \rho_0}{2 \pi r_1} e^{-j k r_1} + \frac{j \omega \rho_0}{2 \pi r_2} e^{-j k r_2} + \cdots + \frac{j \omega \rho_0}{2 \pi r_m} e^{-j k r_m} \]  
(6)
The parameter \( \mathbf{a} \) is a column vector with the complex amplitudes of the \( n \) multipoles under consideration.

Equation (4) can be used to expand the pressure term in Eq. (2) producing
\[ W \approx \int_0^{2\pi} \int_0^{\pi/2} \frac{\mathbf{a}^H \psi^H(r) \psi(r) \mathbf{a}}{2 \rho_0 c_0} |r|^2 \sin \theta \, d\theta \, d\varphi \]  
(7)
or
\[ W \approx \mathbf{a}^H \mathbf{A}_a \mathbf{a} \]  
(8)
where \( \mathbf{A}_a \) is a square weighting matrix, the \((i, j)\) term of which is defined by
\[ \mathbf{A}_a(i, j) = \int_0^{2\pi} \int_0^{\pi/2} \frac{\psi_i^H(r) \psi_j(r)}{2 \rho_0 c_0} |r|^2 \sin \theta \, d\theta \, d\varphi \]  
(9)
The expression in Eq. (8) has the desired form given in Eq. (1), where the weighting matrix will indeed be frequency dependent. Ideally the weighting matrix \( \mathbf{A}_a \) will be diagonal, meaning that the multipoles are independent contributors to the performance measure, and so a reduction in the amplitude of any measured multipole will guarantee a reduction in the performance measure.

**EVALUATION OF MULTIPOLe AMPLITUDES**

Practical evaluation acoustic power, as defined in Eq. (8), requires measurement of the amplitude of the acoustic multipoles used to describe the sound field. The usual modal filtering approach is to express the measured quantity, in this case acoustic pressure at \( m \) measurement points, in the form
\[ \mathbf{p} = \mathbf{\Psi} \mathbf{a} \]  
(10)
Here \( \mathbf{p} \) is the vector of complex pressure at the \( m \) points under consideration, \( \mathbf{a} \) is the vector of amplitudes of the acoustic multipoles, and \( \mathbf{\Psi} \) is the matrix of transfer functions between the \( n \) multipoles and the \( m \) measurement points under consideration, and is given by
\[ \mathbf{\Psi} = [\psi_1 \psi_2 \cdots \psi_n] = \begin{bmatrix} \psi_1(\mathbf{r}_1) & \psi_2(\mathbf{r}_1) & \cdots & \psi_n(\mathbf{r}_1) \\ \psi_1(\mathbf{r}_2) & \psi_2(\mathbf{r}_2) & \cdots & \psi_n(\mathbf{r}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_1(\mathbf{r}_m) & \psi_2(\mathbf{r}_m) & \cdots & \psi_n(\mathbf{r}_m) \end{bmatrix} \]  
(11)
It is then possible to resolve the amplitudes of the acoustic multipoles using a matrix inversion or pseudo-inversion,
\[ \mathbf{a} = \mathbf{\Psi}^{-1} \mathbf{p} \]  
(12)
This expression takes the standard form of a modal filtering problem \([2, 10, 13]\), where a number of sensor measurements, in this case acoustic pressure, are decomposed into a number of modal quantities.
To generate a given multipole pattern, the amplitudes of the sources are fixed to be equal. For example, the phasing of the sources from 8 multipoles is governed by the relationship

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2 \\
q_3 \\
q_4 \\
q_5 \\
q_6 \\
q_7 \\
q_8 \\
\end{bmatrix}
\]

(13)

Referring to Eq. (13), the first multipole pattern is generated by having all sources in phase, the second by having sources 1-4 in phase and 5-8 out of phase, the third by having sources 1,2,7,8 in phase and 3-6 out of phase, etc. The matrix in Eq. (13) contains a set of orthogonal functions composed of +/-1, referred to as a Hadamard matrix [21, 22].

A set of 8 multipole radiation transfer functions to a point \( r \) in space that constitute the elements in a given row in \( \Psi \) in Eq. (11), are then calculated by

\[
\begin{bmatrix}
\psi_1(r) \\
\psi_2(r) \\
\vdots \\
\psi_8(r) \\
\end{bmatrix}^T = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\
\end{bmatrix}
\begin{bmatrix}
j\omega \alpha_1 e^{-jkr_1} \\
j\omega \alpha_1 e^{-jkr_2} \\
j\omega \alpha_1 e^{-jkr_3} \\
j\omega \alpha_1 e^{-jkr_4} \\
\vdots \\
j\omega \alpha_1 e^{-jkr_8} \\
\end{bmatrix}
\]

(14)

where \( r_1, r_2, \) etc, are the distances from the individual monopoles to the point of interest.

**SENSING SYSTEM DEVELOPMENT**

The overall aim of the work here is to develop a sensing system of the type shown in Fig. 3, where a relatively large number of sensor signals are decomposed into a much smaller group of signals, which are subsequently used to evaluate the global performance measure, \( W \) and sent to a controller. The underlying mathematics is derived by substituting Eq. (12) into Eq. (8):

\[
W \approx p^H \left\{ \Psi^{-1} \right\}^H A_a \Psi^{-1} p
\]

(15)

The vector of measured pressure measurements are then decomposed into multipole components by multiplication by \( \Psi^{-1} \) (modal filtering) and weighted by \( A_a \) to give a global performance measure. The relatively small number of resulting signals, (multipole amplitudes) would then be passed through filters which have frequency characteristics defined by \( A_a \), weighting the importance of the signals [1, 16].

In many control settings, design and implementation are simplified if \( A_a \) is diagonal, decoupling the measured quantities. The weighting matrix \( A_a \) is not, in general, diagonal. However, since it is symmetric it can be diagonalised using an orthonormal transformation:
where the columns of $Q$ are the eigenvectors of $A_a$ and $\Lambda$ is a diagonal matrix of the associated eigenvalues. Substituting this into Eq. (15) yields:

$$W \approx p^H \left\{ \Psi^{-1} \right\}^H \Lambda \Lambda^{-1} \Psi^{-1} p$$  \hspace{1cm} (17)

With this decoupling the modal filtering weights are now defined by $Q^{-1} \Psi^{-1}$.

Another important consideration is the frequency independence of the terms in $\Psi^{-1}$ in Eq. (12). Frequency independence is necessary for practical implementation of the modal filtering procedure, where sensor signals are multiplied by a real weighting value and then added to produce the measurement quantities of interest. If the terms in $\Psi^{-1}$ are frequency dependent, then direct implementation would require the single-number multiplication to be replaced with digital filtering, and the computational requirements for large numbers of sensors render the approach impractical. However, the modal filter matrix $\Psi^{-1}$ is typically frequency dependent, and so $Q^{-1} \Psi^{-1}$ in Eq. (17) can be re-expressed as

$$Q^{-1} \Psi^{-1} = N \Psi^{-1}_{n}$$  \hspace{1cm} (18)

where $N$ is a diagonal matrix of frequency-dependent terms, and $\Psi^{-1}_{n}$ is a normalised version of $Q^{-1} \Psi^{-1}$ with the maximum coefficient amplitude equal to 1.

With these two changes the frequency independent modal filter weights are defined by $\Psi^{-1}_{n}$ and the frequency dependent performance weighting matrix is defined by $N^H \Lambda N$. Such quantities would be used in the practical sensing system implement of the type shown in Fig. 3, letting Eq. (17) take on the final form

$$W \approx p^H \left\{ \Psi^{-1}_{n} \right\}^H \Lambda \Psi^{-1}_{n} p$$  \hspace{1cm} (19)

where

$$\Lambda = N^H \Lambda N$$  \hspace{1cm} (20)
SENSING SYSTEM ENVIRONMENT

The described technique relies on obtaining complex (number) pressure measurements for the decomposition process. However measured pressure will be a real number. If this approach was to use vibration measurements, then provided that the structure was lightly damped, this requirement could be simplified to positive or negative real values [23]. However, in the acoustic space, the phase difference between sensing points is not due to combinations of modes vibrating in or out of phase, but rather propagation delays between source(s) and sensor(s). Since practical implementation is in the time domain, where all pressure measurements are real, simulation in the frequency domain is not enough to guide system development. In this section, a novel simulation environment is developed to assess the sensing system performance in the time-domain. Shown in Fig. 4 is an overview of the simulation procedure.

EVALUATING SOUND PRESSURE IN THE TIME DOMAIN

There are two steps to simulate the acoustic radiation from the structure: simulation of the structural vibration in response to the force input, followed by simulation of radiation into free space. The method of simulation of radiation in the frequency domain is well known [10], but is surprisingly unpublished for the time domain. In the time domain, vibration of a single structural mode is governed by the state equation

\[ \dot{x} = Ax + Bu \quad y = Cx + Du \]  

(21)

where \( u \) is the input force, \( x_n \) are the system states and \( y_n \) is the system output vector, given by

\[
\begin{align*}
  u &= \begin{bmatrix} 0 \\ f(t) \end{bmatrix} \quad x_n = \begin{bmatrix} x_n \\ \dot{x}_n \end{bmatrix} \quad n(=1,2,\ldots) \\
  y_n &= \begin{bmatrix} y_n \\ \dot{y}_n \end{bmatrix} \quad n(=1,2,\ldots)
\end{align*}
\]

and matrices \( A_n, B_n, C_n \) and \( D_n \) defined as:

\[
\begin{align*}
  A_n &= \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta \omega_n \end{bmatrix} \quad n(=1,2,\ldots) \\
  B_n &= \begin{bmatrix} 0 \\ \frac{1}{m} \Psi_n \end{bmatrix} \quad n(=1,2,\ldots)
\end{align*}
\]

\[
\begin{align*}
  C_{d,n} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \\
  C_{v,n} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \quad n(=1,2,\ldots) \\
  D_n &= 0
\end{align*}
\]

(22)

where \( n \) is the modal index and \( C_{v,n} \) denotes the \( C \) matrix for velocity as an output and \( C_{d,n} \) for displacement, \( m \) is the modal mass of the structure, \( \omega_n \) the natural frequency of the \( n \)th mode, \( \zeta \) the damping ratio and \( \Psi_n \) the mode shape function value for the \( n \)th mode at the input locations (\( B \) matrix) and measurement locations (\( C \) matrix). The total structural vibration model is built up from multiple modes in parallel.

Radiation from a planar, baffled structure with the geometry shown in Fig. 3 is governed by the Raleigh integral. If a far field measurement point is considered and the structure simplified to a simply supported rectangular panel for discussion here, the radiation transfer function between pressure and velocity can be simplified as, [1, 24]

\[
Z_{rad}(i, r) = \frac{j \omega \rho_0}{2\pi r} e^{-jkr} \frac{L_x L_y}{M_i N_i \pi^2} \left[ \frac{(-1)^{M_i} e^{-j\alpha} - 1}{\left( \frac{\alpha}{M_i \pi} \right)^2 - 1} \right] \left[ \frac{(-1)^{N_i} e^{-j\beta} - 1}{\left( \frac{\beta}{N_i \pi} \right)^2 - 1} \right]
\]

(23)
where \( r \) is the distance, \( \rho_0 \) is the density of air, \( L_x \) and \( L_y \) are the dimensions of the panel, \( M_i \) and \( N_i \) are the \( i \)th modal indices in the \( x \) and \( y \) directions respectively and

\[
\alpha = k L_x \sin \theta \cos \phi \quad \beta = k L_x \sin \theta \sin \phi
\]  

\( k \) being the acoustic wave number with \( \theta \) and \( \phi \) defined as in Fig. 1. Note that the radiation transfer function is a frequency-domain quantity, complex, time invariant and purely dependent on system geometry and the excitation frequency.

To use the radiation transfer function in the time domain simulations, the complex numbers in each transfer function are subjected to the Euler substitution,

\[
s = j \omega
\]

The resulting entirely real \( s \)-domain transfer function describing pressure/vibration is multiplied by the transfer function evaluated in Eq. (22) to produce the overall transfer function of pressure/force. Further details on the transformation is contained in Hill et al [25, 26].

**SENSING SYSTEM SIMULATION**

To assess the quality of the approach described previously, the sound power radiated by a simple structure will be compared to the predictions of this global quantity obtained using the modal filtering arrangement in a numerical simulation. These comparisons will provide insight into where the sensing system design approach will be applicable, and how the range of operation can be extended.

The simple structure used in the simulation is a rectangular steel panel, dimensions 0.612m x 1.212m x 0.004m. The panel is lightly damped, with a damping coefficient, \( \zeta = 0.005 \), and excited by a disturbance point force at the location \( L_x/3, L_y/4 \). The frequency range of interest is up to 500 Hz where there are 24 modal resonances, as listed in Table 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Resonance Frequency (Hz)</th>
<th>Mode</th>
<th>Resonance Frequency (Hz)</th>
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<tr>
<td>1 , 1</td>
<td>32.8</td>
<td>2 , 5</td>
<td>271.3</td>
</tr>
<tr>
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<td>52.8</td>
<td>3 , 3</td>
<td>295.4</td>
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<td>86.2</td>
<td>3 , 4</td>
<td>342.1</td>
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<td>111.3</td>
<td>2 , 6</td>
<td>344.7</td>
</tr>
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<td>2 , 2</td>
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<td>2 , 7</td>
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<td>266.2</td>
<td>4 , 3</td>
<td>478.4</td>
</tr>
</tbody>
</table>

Table 1: Structural resonances below 500 Hz for the test panel
The sensing system contains 128 microphones evenly spread over the range \((2\pi)\) of \(\phi\) at each of \(\theta = 10^\circ, 20^\circ, 30^\circ\) and \(40^\circ\), as illustrated in Fig. 3. The sensing grid location is chosen to be in the far-field, at a radius of 10 metres, with the sensors on a constant radius from the center of the structure to minimise phase differences outside of \(0^\circ\) and \(180^\circ\) between the modal filtering weights assigned to the signals carried from the microphone. While this is not a perfect assumption, it does minimise phase differences in a simplistic manner. If the modal filter weight phases are all \(0^\circ\) or \(180^\circ\) different, then the system is purely real and the complex number requirement can be dropped. The modal filtering parameters for the simulation problem are derived on the basis of radiation patterns from multipole combinations of eight monopole sources, arranged as shown in Fig. 5. The measured sound pressure level at each microphone location will be filtered into the eight multipoles, with the sound power calculated using Eq. (17).

![Monopole arrangement](image1)

**Figure 5: Monopole arrangement**

![Eigenvalues of the weighting matrix](image2)

**Figure 6: Eigenvalues of the weighting matrix \(A_n\) plotted as a function of frequency**

An important question is, how many multipoles are required to adequately model a given sound field? As mentioned, the radiation efficiency of the various multipoles is related to the eigenvalues in \(\Lambda\) in Eq. (17). The eigenvalues for the simulated system are plotted in Fig. 6. At low frequencies (up to approximately 250 Hz) the radiation efficiencies of five of the modal filter quantities (the multipoles) greatly exceed the other three. A sensing system that derives only five multipole quantities would then provide a reasonably accurate estimate of radiated acoustic power at low frequencies. At higher frequencies, above 400 Hz, the radiation efficiencies of the eight multipole quantities converge, which, suggests that resolving eight multipole quantities may not be enough to provide an accurate prediction of radiated acoustic power over the wider frequency range.

Illustrated in Fig. 7 is a plot of radiated sound power estimated by resolving all 8 multipoles, in Eq. (15), averaged over time. For reference, the actual acoustic power radiated from the plate, calculated using the frequency domain representation [10] is also shown. The plots compare well, especially below 220Hz. However at higher frequencies the sensing systems performance drops off, as would be expected considering the number of monopoles used in the decomposition. It should be emphasised though that the estimate of acoustic power from the
modal filtering arrangement comes from only 8 resolved quantities, a number of signals that many controllers could easily work with.

Illustrated in Fig. 7 (b) is a plot of the component of sound power estimated to be in each (independent) multipole, calculated by using the resolved modal quantities in Eq. (19), averaged over time. The modal filter weights in $\Psi^{-1}$ were fixed to be the exact values at 35Hz. This plot can provide some insight into the quality of the sensing system design procedure. However it must be emphasised that the modal filter weights where fixed (as in any practical situation) so the acoustic power contained in each multipole does not converge as suggested by Fig. 6. At low frequencies, a few resolved multipoles will provide a good estimate of the radiated power. However, as expected from examination of the eigenvalues, the accuracy of the estimate will fall away at higher frequencies, unless more multipoles are included in the calculation.

These results suggest the following:

- The number of resolved quantities from the modal filtering process is valid at "low" frequencies. Importantly, this methodology allows the designer to obtain a measurement of a global error criterion, in this case radiated acoustic power, with a minimum number of inputs to the control law and tuning algorithm.

- At higher frequencies, the number of quantities resolved by the sensing system must increase for an accurate estimate of the global error criterion to be obtained.

- By placing the acoustic sensors on a constant radius, it is possible to ignore small phase differences outside of 0° and 180° between the modal filter weights, and so overcome the need for complex (number) acoustic pressure measurements. The resulting sensing system can predict spectral peaks accurately, but not the depth of the troughs.
CONCLUSIONS

A time domain modal filtering method based on radiation patterns produced by acoustic multipoles has been presented. The techniques uses the fundamental acoustic radiation patterns produced by monopoles as the basis functions, and so requires no knowledge of a structure’s mode shapes. This generalised technique then has application to more complex structural radiation problems, something not possible with previous techniques. Simulations of the sensing system in the time domain illustrate that a minimum number of signals (resolved multipoles) are required to provide a good approximation of radiated power. This result implies that a large number of sensors signals can be condensed into only a few inputs when the control system is implemented, simplifying the controller.

REFERENCES


