Vibration Analysis of Waffle Floors

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Abstract

Two way grillage or waffle floors are used extensively in semiconductor factories as they provide high impedance mounts for manufacturing equipment that is extremely vibration sensitive. This paper presents a mathematical model for the analysis of vibration at the center of a bay and the transmission of vibration along a waffle floor. The mathematical model is compared with finite element models and experimental results from several manufacturing buildings, and shows good agreement. Trends are shown for the displacement and resonance frequency of the floor as the thickness of the floor, size of the bays and the stiffness of the columns are varied.

Key words: vibration sensitive equipment, waffle floors, grillage, semi-conductor, plate

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1 Introduction

Advanced integrated circuits, such as central processing units (CPUs) currently have feature sizes around 0.13 microns and are made using a photolithography process. Extreme precision is required to manufacture these cir-

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cuits and hence the photo-lithography machines are extremely vibration sen-
sitive. Modern photo-lithography machines, called scanners, generate inertial
reaction forces when their internal mechanisms move. These forces are trans-
mitted into the feet of the scanner. The feet, in turn, transmit forces into a
stiff pedestal, or platform, which is mounted on the waffle floor. The com-
ponents involved in the transmission of the inertial forces are the internal
mechanisms of the scanner, the mounting feet, the supporting pedestal and
the floor. The combined impedance of these components determines whether
or not the machine can operate successfully. Manufacturers of scanners place
specifications on the impedance that are necessary for the successful operation
of their equipment. The specifications are in terms of some type of dynamic
impedance, such as accelerance (or inertiance), mobility or dynamic stiffness,
depending on the manufacturer of the equipment. There is no consensus among
the manufacturers as to how to specify this requirement. The important point
is that manufacturers provide amplitude and frequency specifications for the
impedance, which encompass both the dynamic stiffness and damping prop-
erties of the combined systems that support their equipment.

There are several reasons why the floors in semiconductor manufacturing
buildings require sufficiently high impedance:

- The floor must resist the inertial forces generated by the photo-lithography
  machines,
- the transmission of energy from vibration sources (such as pumps, compres-
sors, air handling units, people walking on the floor) to vibration sensitive
  machines should be minimized,
- the vibration generated from one scanner should not affect the operation of
  adjacent scanners.

Hence, the transmission of vibration throughout the building, which includes
the transmission of vibration along the waffle floor is of concern to designers
of these buildings.

These manufacturing buildings contain many vibrating sources such as pumps, compressors, air handling units and other equipment that all contribute to the excitation of the floors. The locations of these items are distributed throughout the manufacturing buildings and adjacent central utility buildings. These items generate tonal and broad-band vibration associated with the rotational frequency of shafts, and from the friction and turbulence of liquids and gases in piping. Vibration propagates through soil into the building, from the ground level up columns, from the fan deck down columns, and as base excitation shaking the foundations of the building. Due to the complexity of this vibration problem, it has not been solved previously with a precise mathematical model. Instead, designers use an empirical relationship that has been developed based on vibration measurements in many buildings. The empirical relationship for the velocity response of the floor due to broad-band mechanical excitation is given by $V_{\text{mech}} = C/k$, where $C$ is a constant and $k$ is the stiffness at the midbay [1].

The mathematical model presented in this paper covers one concern faced by designers of these special purpose buildings, namely the vibration transmission along the waffle floor due to tonal or random vibration.

A typical design for waffle floors (also known as two-way grillages) in semiconductor manufacturing buildings, is shown in Figure 1. It consists of thick concrete beams approximately 60cm thick and 15cm wide, with a concrete layer (topping) of 5cm thick on top of the beams. Often the topping layer is punctured with holes about 15cm in diameter, so that air can flow from the fabrication (fab) level to the (sub-fab) level beneath, and the holes allow pipes to pass from the lower level to the process level.

There are few papers that discuss the vibration characteristics of waffle floors, which is a reflection of the competition between semiconductor manufactur-
ers. The civil engineering codes for the design of buildings are mainly related to the strength of the building to withstand static and seismic loads, and there are no codes for the design of building with waffle floors for vibration sensitive equipment. These special purpose buildings are designed by a few consultants that make their best engineering judgements for their designs.

Petyt et. al. [2] used finite element modelling to calculate the mode shapes of floor slabs on four column supports. Several configurations of bays were considered and their results showed that the lowest resonance frequency occurs when adjacent bays are vibrating out-of-phase. Their work showed that the response of one bay is influenced by the vibration of adjacent bays.

Amick et. al. [3] described a model for the vibration attenuation with distance along the floor. Several models were proposed, such as exponential, linear, and power decaying with distance from the source. However none of the models take into account the modal behavior of the floor, and this is the impetus for the derivation of the mathematical model described here.

The concept design for semi-conductor factories is frequently done in group
meetings with architects, the building owner and structural engineers. Designers are often required to quickly assess several concept designs of waffle floors. The mathematical model presented here is intended to aid designers for this purpose.

2 Description of the Model

Figure 1 shows that the cross section of a typical waffle floor is a composite section of rectangular beams and a topping layer. This composite section can be transformed into an equivalent plate section of constant thickness, by equating the cross sectional moments of inertia of the composite section and a plate section of unit width. Note that the steel reinforcing bars that are within concrete sections are ignored, as they do not significantly alter the flexural rigidity of the floor. This reduction of the complex cross section geometry into an equivalent flat plate can only be made when the moments of inertia along the $x$ and $y$ axes are the same. If the waffle has different cross-sectional moments of inertia along each axis, then the transformation is invalid.

The waffle floor can be modelled as a simply supported flat plate, which is supported by columns and driven by harmonic forces at points on the floor. Note that the mathematical model does not require a uniform grid spacing for the columns. The simply supported edge condition around the perimeter of the plate is a reasonable assumption as the perimeter of the waffle floor is usually supported by shear walls. The columns are modelled as linear elastic springs, which apply a restoring force along the vertical axis and restoring moments about two rotational axes, as shown in Figure 2.

One assumption used in this model is that vibration transmitted into the columns from the waffle floor is absorbed by the soil that supports the column, and is not re-radiated into adjacent columns and transmitted back into
the waffle floor. However, it is possible to include the cross-coupling between columns by altering the stiffness matrix for the columns. Another assumption is that each column is attached to the floor at a single point, which is a reasonable assumption for the low frequency range considered here.

3 Equations of Motion

The plate is driven by sinusoidal forces on the floor at $J = 1, \cdots, L_0$ locations at a concentrated point $\mathbf{\sigma}_J = \mathbf{\sigma}(x_J, y_J)$, which could originate from a scanner or a reciprocating machine. The Dirac delta function, $\delta$, can be used to describe the point application of the force per unit area to the plate. The plate is supported by a grid of $K = 1, \cdots, L_c$ columns that apply point forces and moments at $\mathbf{\sigma}_K = \mathbf{\sigma}(x_K, y_K)$, and the simply supported edge condition is assumed to exist along each of the four sides. The displacement of the plate $w(\mathbf{\sigma}, \omega)$ at frequency $\omega$ can be described by the following partial differential
equations [4,5]:

\[
\frac{Eh^3}{12(1-\nu^2)} \nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = \\
\sum_{K=1}^{L_0} \left\{ F_{zK}^c \delta(\sigma - \sigma_K) - M_{xK}^c \frac{\partial \delta(\sigma - \sigma_K)}{\partial y} \right. \\
+ M_{yK}^c \frac{\partial \delta(\sigma - \sigma_K)}{\partial x} \left. \right\} + \sum_{J=1}^{L_0} F_0^J \delta(\sigma - \sigma_J)
\]

(1)

where \( \rho, \nu, E, h \) are the density per unit volume, Poisson’s ratio, Young’s modulus, thickness, respectively of the plate. The units on the right hand side of Eq. (1) are force per unit area. The forces that are applied to the plate along the vertical axis are from the columns \( F_{zK}^c \) and the harmonic driving forces \( F_0^J \). Rotational moments \( M_{xK}^c, M_{yK}^c \), are applied by the columns, which are modelled as rotational springs that apply restoring moments around the \( x \) and \( y \) axes respectively. A moment can be converted into a force couple, as shown in Appendix A. The Dirac delta function can be expanded to \( \delta(\sigma - \sigma_J) = \delta(x - x_J) \delta(y - y_J) \). The gradient operator is defined as

\[
\nabla^4 = \nabla^2 \nabla^2 = \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right]^2
\]

(2)

For a simply supported plate, the following harmonic solution can be employed to describe the out-of-plane displacement of the plate,

\[
w(\sigma, \omega; t) = w(\sigma, \omega) e^{j\omega t}
\]

(3)

where \( e \) is the exponential function, \( \omega \) is the driving frequency, \( j = \sqrt{-1} \), \( t \) is the time variable, and

\[
w(\sigma, \omega) = \sum_{m,n=1}^{\infty} \eta_{mn} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y}
\]

(4)

where \( \eta_{mn} \) is the modal participation factor, and \( L_x, L_y \) are the dimensions of the plate along the \( x, y \) axes, respectively. The rotational displacements of
the plate are given by [4],

\[ \theta_x = -\frac{\partial w}{\partial y} \]  \hspace{1cm} (5)

\[ \theta_y = \frac{\partial w}{\partial x} \]  \hspace{1cm} (6)

The resonance frequencies \( \omega(m, n) \) of a plate are given by [4]

\[ \omega_{m,n} = \pi^2 \sqrt{\frac{Eh^2}{12\rho(1-\nu^2)}} \left[ \left( \frac{m}{L_x} \right)^2 + \left( \frac{n}{L_y} \right)^2 \right] \]  \hspace{1cm} (7)

in units of radians/s. This equation is based on thin shell theory, where the effect rotational inertia is not important. For plates that are thick or at high frequencies, these effects need to be included in the analysis. The waffle floor systems under investigation here typically have velocity criterion of 6.4 microns/s RMS (250 micro-inches/s RMS) for a photo-lithography area and the fundamental resonance frequency is about 10Hz. The equivalent floor thickness is about 0.5m hence the ratio of displacement of the floor to the thickness is around \( 1.8 \times 10^{-5} \), and hence the effects of rotary inertia can be ignored [5]. The modal combinations \( (m, n) \) can be re-ordered into increasing resonance frequencies and denoted by the subscript \( I \). If only \( P \) modes are used to model the dynamics of the system then \( I = 1 \cdots P \), and Eqs. (4) to (6) can be re-written as

\[
\begin{bmatrix}
w \\
\theta_x \\
\theta_y
\end{bmatrix} =
\begin{bmatrix}
\psi(\sigma) \eta \\
-\frac{\partial \psi(\sigma)}{\partial y} \eta \\
\frac{\partial \psi(\sigma)}{\partial x} \eta
\end{bmatrix} = R^b \eta
\]  \hspace{1cm} (8)

where the mode shape functions and modal participation factors can be grouped
into vectors as

\[ \psi(\sigma) = [\psi_1(\sigma), \psi_2(\sigma), \cdots, \psi_P(\sigma)] \]  

(9)

\[ \eta = [\eta_1, \eta_2, \cdots, \eta_P]^T \]  

(10)

where the superscript \( T \) denotes the matrix transpose operator. The mode shape functions are,

\[ \psi_I(\sigma) = \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \]  

(11)

\[ \frac{\partial \psi_I(\sigma)}{\partial y} = \frac{n\pi}{L_y} \sin \frac{m\pi x}{L_x} \cos \frac{n\pi y}{L_y} \]  

(12)

\[ \frac{\partial \psi_I(\sigma)}{\partial x} = \frac{m\pi}{L_x} \cos \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \]  

(13)

The driving forces and moments can be grouped into a vector as

\[ F^0_j = [F^0_{z,j} M^0_{x,j} M^0_{y,j}]^T \]  

(14)

The columns supporting the floor are assumed to be elastic springs that exert forces and moments proportional to the vertical or rotational displacement of the floor at \( \sigma_K \). The forces are given by

\[ F^c_K = \begin{bmatrix} -Q_c^{z,K} w(\sigma_K, \omega) \\ -Q_c^{\theta_x,K} \theta_x(\sigma_K, \omega) \\ -Q_c^{\theta_y,K} \theta_y(\sigma_K, \omega) \end{bmatrix} \]  

(15)

\[ = -[K_K] [R^K_b] \eta \]  

(16)

where \([K_K]\) is a diagonal stiffness matrix that has entries along the diagonal of \(Q^z_{z,K}, Q^{\theta_x,K}_{\theta_x,K}, Q^{\theta_y,K}_{\theta_y,K}\) which are the stiffnesses of the \( K \)th column along the vertical \( z \) and around the rotational \( \theta_x \) and \( \theta_y \) axes, respectively.

Substitution of Eqs. (11) to (13) into Eq. (2), pre-multiplying by the transpose
of the mode shape function vector $[R^b]^T$, integrating over the surface area of
the floor and making use of the properties of the Dirac delta function [6]
\[
\int_\alpha F(\alpha) \frac{\partial}{\partial \alpha} [\delta(\alpha - \alpha^*)] \, d\alpha = -\frac{\partial F(\alpha^*)}{\partial \alpha}
\] (17)
and the orthogonality property of the mode shape function [7]
\[
\int_0^L \sin \lambda_1 s \sin \lambda_2 s \, ds = \frac{L}{2}
\] (18)
when $\lambda_1 = \lambda_2$ and 0 when $\lambda_1 \neq \lambda_2$, the response of the floor can be written as
\[
i\ddot{\eta}_I + 2\zeta_I\omega_I \dot{\eta}_I + \omega_I^2 \eta_I = F_I
\] (19)
where $\eta_I$ is the $I^{th}$ modal participation factor, and the dots represent differ-
entiation with respect to time, $\zeta_I$ is the viscous damping of the $I^{th}$ mode of
the plate, $\omega_I$ is the resonance frequency of the $I^{th}$ mode and $F_I$ is the $I^{th}$
modal force which is applied to the plate for the $I^{th}$ mode and is defined as
\[
F_I = \frac{1}{m_I} \left[ \sum_{J=1}^{L_f} [R^b_J]^T F^0_J + \sum_{K=1}^{L_c} [R^b_K]^T F^c_K \right]
\] (20)
where the modal mass of the plate $m_I$ is given by
\[
m_I = \frac{1}{4} \rho L_x L_y h
\] (21)
The effects of concentrated masses on the floor can be added to the formulation
as shown in Ref [8]. Eq. (19) can be expressed in matrix form as
\[
Z_p \eta = \sum_{J=1}^{L_f} [R^b_J]^T F^0_J + \sum_{K=1}^{L_c} [R^b_K]^T F^c_K
\] (22)
where the impedance matrix of the uncoupled plate $Z_p$ is defined as a diagonal

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matrix,

\[ Z_p = \begin{bmatrix} \Omega_1 \\ \vdots \\ \Omega_P \end{bmatrix} \tag{23} \]

where \( \Omega_I \) \((I = 1, \cdots, P)\) is defined as

\[ \Omega_I = m_I(-\omega_I^2 + 2\zeta_I \omega_I j\omega + \omega_I^2) \tag{24} \]

Eq. (16) can be substituted into Eq. (22) and re-arranged into

\[ [Z_p + Z_c] \eta = \sum_{j=1}^{L_f} [R^b_j]^T F^0_j \tag{25} \]

where \( Z_c \) is the impedance matrix that accounts for the stiffness matrix of the columns that support the waffle and is given by

\[ Z_c = \sum_{K=1}^{L_c} [R_K^b]^T [K_K] [R_K^b] \tag{26} \]

Interaction between adjacent columns can be taken into account by modification of Eq. (26).

The modal participation vector \( \eta \) can calculated by solving Eq. (25), and hence the displacement at any location on the plate can be calculated.

This mathematical model was formulated in MATLAB with parameters from newly constructed buildings. The results from the MATLAB model are compared here with experimentally measured data and finite element models.

4 Finite Element Model

A finite element model was created in ANSYS of a waffle floor that has 9 × 9 bays, where a bay is considered to be the square area that has vertices of four
column supports, as shown in Figure 3. The edges of the plate are simply supported. Each bay was modelled using 36 square shaped shell elements (shell63) and linear spring and dashpot elements (combin14) at the corner of each bay for the column supports. The nodes on the perimeter of model were set to simply supported boundary conditions. The parameters used in the ANSYS are identical to those used in the MATLAB model, and are listed in Table 1.

![Fig. 3. Finite element model of the waffle floor with 9 × 9 bays.](image)

5 Experimental Measurements

The point and transfer impedances were measured on several waffle floors in newly constructed semi-conductor manufacturing buildings. The impedance measurements were made using an instrumented sledgehammer to strike the floor, and the vibration response of the floor was recorded with a spectrum analyzer. Another instrument that is commonly used to excite the floor in a building is an electrodynamic shaker [9]. The author of this paper has found that the main advantages of using a sledgehammer are that measurements at many locations in a building can be gathered much more quickly than is possible using a shaker system, and a properly executed swing of a sledgehammer can provide greater excitation than a portable shaker. Figure 4 shows the equipment that was used to perform the measurements. The sledgehammer has a large steel mass (7.8kg) that was used to strike 25mm thick rubber pads.
that rested on the waffle floor. Attached to the back of the mass was a Brüel and Kjær accelerometer (model 4373). A Brüel and Kjær accelerometer (model W8318c) rested on the floor and was used to measure the vibration response of the floor when it was struck with the instrumented hammer. The signals from the accelerometer on the hammer and the response accelerometer on the floor were conditioned by Brüel and Kjær charge amplifiers (model 2635) and processed by a Data Physics ACE portable digital spectrum analyzer. The force exerted by the hammer on the floor is calculated by multiplying the acceleration of the hammer by its mass. It is preferable to measure the impact force directly with a force transducer attached to the hammer head. However many comparisons with the instrumented sledgehammer and an impact hammer with a calibrated force transducer (PCB piezotronics model 086C20) showed identical results, except at low frequencies where the sledgehammer was able to provide greater excitation. The point impedance is measured by striking the hammer against the floor and measuring the vibration response adjacent to where the hammer strikes the floor. The transfer impedance is measured by striking the hammer on the floor at one location and measuring the vibration response at another location on the floor. Point impedance and transfer impedance measurements were conducted on several waffle floors, to quantify the transmission of vibration along the floors.
Floors in vibration sensitive buildings are designed to meet vibration criteria in terms of the RMS velocity in one-third octave bands, as specified in the ISO 2631 standard [10]. The two types of floor considered here have vibration criteria of 6.4 and 50.8 microns/s RMS (250 and 2000 micro-inches/s RMS), which correspond to the vibration criterion VC-D and VC-A, respectively.

Presented in this paper are four sets of data that were obtained from measurements in three buildings. The parameters of the buildings are listed in Table 1.

6 Comparison of Results

The following sections show the predictions of the displacement of waffle floors for the MATLAB model, the finite element model and the experimentally measured vibration response in semi-conductor manufacturing buildings.

6.1 Resonant Behaviour of the Floor

Figure 5 shows the comparison of the displacement at the center of the bay (midbay) where a harmonic 1N driving force was applied, for the MATLAB model described above, the ANSYS finite element model, and measurements at two locations in a building. The curve labelled Experiment (1) was derived from measurements using a impulse hammer (PCB piezotronics model 086C20) that has a head that weighs 2.5kg. The curve labelled Experiment (2) was derived from experiments using a sledgehammer that has a head that weighs 7.8kg. The experimental results for the impulse hammers at frequencies below 5 Hz suffer from poor coherence (less than 0.8), as the impulse hammer cannot provide sufficient energy to excite the structure and should be disregarded. The results obtained using the sledgehammer had slightly
higher coherence at low frequencies than the experiments using the lighter-weight hammer, as the sledgehammer was better able to excite the structure.

![Graph showing displacement versus frequency for Matlab, Ansys, and experiments (1) and (2).](image)

**Fig. 5.** Comparison of the displacement at the midbay where the driving force was applied.

It is possible to calculate the resonance frequencies of the floor by calculating the eigenvalue solutions to Eq. (25), using standard techniques such as those found in MATLAB. The results of this calculation show that there are several modes that are clustered in a narrow frequency range, which is associated with the resonance behaviour of the floor. Figure 6 shows the distribution of the number of modes in 1 Hz bins (modal density) centered at the number shown on the abscissa. The results show that the resonant peak at 12.5 Hz is due to the summation of the modal responses of several modes.

The dynamic stiffness of the floor can be calculated by inverting the displacement shown in Figure 5, and the results are shown in Figure 7. The characteristics of the dynamic stiffness curve are what would be expected from a single degree of freedom oscillator. At low frequencies, from 0 to about 5 Hz, the experimentally measured dynamic stiffness increases in magnitude, which is a measurement artifact caused by the low coherence problems described previously. The dynamic stiffness curve is then constant in magnitude for a small
frequency range, which is the stiffness controlled region, and this value is the approximate static stiffness. The curve then dips into a trough, which is the resonance controlled region, and increases again, which is the mass controlled region.

The results presented so far should provide some confidence that the MATLAB model can provide a reasonable prediction of the vibration response of a waffle floor. This model is now used to investigate the trends in the vibration response for variations in the bay size, floor thickness, and stiffness of the columns. Further comparisons between the MATLAB model and experimental results are presented in the discussion on the attenuation of vibration with increasing distance from a vibration source.
6.2 Influence of Bay Size on Resonance Frequency

The physical dimension that has the greatest influence on the lowest resonance frequency of the floor is the bay size. Figure 8 shows the displacement at the midbay of the floor for a 1N harmonic excitation force, when the bay size is varied between 3.66m (12ft) to 8.53m (28ft) in steps of 1.22m (4ft), for a floor thickness of 0.347m. The results show that as the bay size is increased, the resonance frequency decreases and the displacement at resonance increases.

![Fig. 8. Displacement at the midbay of the floor for several bay sizes, for a floor thickness of 0.347m.](image)
6.3 Influence of Equivalent Floor Thickness on Resonance Frequency

Figure 9 shows the displacement of the floor at the midbay when the floor thickness is varied between 0.2m and 0.6m in steps of 0.02m, for a bay size of 7.32m × 7.32m (24ft × 24ft) and column stiffness of 1 × 10^9 N/m. The results show that as the equivalent thickness of the floor increases, which corresponds to a greater cross-sectional moment of inertia, the first resonance frequency increases and the amplitude at the resonance frequency decreases.

![Figure 9](image)

Fig. 9. Displacement of the floor for floor thicknesses varying between 0.2m and 0.6m in steps of 0.02m, for a bay size of 7.32m × 7.32m.

Similar trends are also observed for a bay size that is 3.66m × 3.66m (12ft × 12ft), as shown in Figure 10. However, as the floor thickness increases to 0.4m, the resonance frequency no longer increases with stiffness and instead begins to decrease with increasing thickness, as shown in Figure 11.
Fig. 10. Displacement of the floor for floor thicknesses varying between 0.2m and 0.6m in steps of 0.02m, for a bay size of 3.66m × 3.66m (12ft × 12ft).

Fig. 11. Change in the resonance frequency of the floor when the floor thickness is varied between 0.2m and 0.6m in steps of 0.02m, for bay sizes of 3.66m × 3.66m (12ft × 12ft) and 7.32m × 7.32m (24ft × 24ft).
This is because the bending stiffness of the floor becomes comparable to the vertical stiffness of the supporting columns, and so the displacement of the floor is controlled by the stiffness of the columns. When the bending stiffness of the floor, $K_b$, is less than the stiffness of the columns, $K_c$, one can show by re-arranging Eq. (7), that the resonance frequency of the floor will vary proportionally to the thickness. When the bending stiffness of the floor is greater than the stiffness of the columns, then the waffle floor system tends to act as a rigid body supported by springs, hence

$$\omega \propto \sqrt[3]{\frac{K_c}{\rho L_x L_y h}} \propto \frac{1}{\sqrt{h}}$$

(27)

where the resonance frequency will vary inversely proportional to the square root of the thickness.
6.4 Influence of Column Stiffness on Resonance Frequency

Figure 12 shows the displacement of waffle floors for several values of column stiffness. The floors have a bay size of 3.66m × 3.66m (12ft × 12ft) and floor thickness of 0.507m. The results show that as the column stiffness increases, the displacement at the resonance frequency increases and the displacement at the midbay decreases.

![Graph showing displacement vs. frequency for different column stiffness values.](image)

Fig. 12. Change in the resonance frequency of the floor when the stiffness of the columns is varied between $5 \times 10^7$ N/m to $5 \times 10^9$ N/m, for a bay size of 3.66m × 3.66m (12ft × 12ft) and a floor thickness of 0.507m.

6.5 Vibration Transmission along the Floor

The vibration transmission along a waffle floor was measured in three buildings. The geometries of the buildings are listed in Table 1. Figures 13 and 14 show the vibration attenuation with distance and response at the midbay of a waffle floor that was designed for photolithography tools, and has a vibration criterion of VC-D. Figure 13 shows that at a distance of about 10m, the displacement of the floor slightly increases in amplitude with increasing distance. This is due to the modal response of the floor.
Fig. 13. Attenuation with distance along the floor at 41Hz, for Case A (photolithography).

Fig. 14. Displacement response of the floor at the drive point for Case A (photolithography).
Similar trends can be seen for another waffle floor designed for photolithography tools, as shown in Figures 15 and 16.

Fig. 15. Attenuation with distance along the floor at 18Hz for Case B (photolithography).

Fig. 16. Displacement response of the floor at the drive point for Case B (photolithography).
The modal response of the floor is clearly seen in the results of displacement of a waffle floor for non-photolithography tools, as shown in Figures 17 and 18.

Fig. 17. Attenuation with distance along the floor at 12.5Hz for Case C (non-photolithography).

Fig. 18. Displacement response of the floor at the drive point for Case C (non-photolithography).
A finite element model was created of a fictitious waffle floor, which has $9 \times 9$ bays with the properties listed in Table 1. Figure 19 shows the contour plot of the displacement magnitude at 11.5Hz. The color bar on the right hand side of the figure indicates the displacement of the floor in units of decibels re 1m, where the red shading indicates the highest vibration amplitude, and the blue shading indicates the lowest vibration amplitude. The dashed lines show the boundaries of a bay and at the intersection of four dashed lines is a column support. The figure shows that there is a pattern of high and low amplitude vibration in alternating bays. Figures 20 and 21 show the vertical displacement of the floor at several frequencies, along paths labelled Path A and Path B in Figure 19, respectively. Both paths originate at the excitation point and end at the perimeter of the floor.

Figures 20 and 21 have dashed vertical reference lines to indicate the locations of the midbays. Figure 20 shows that at 11.5Hz there is an alternating pattern of high and low amplitude vibration at each midbay location, whereas Figure 21 shows that the maximum amplitude occurs at the midbay location. Both these figures show that at frequencies above 11.5Hz, the maximum displacement within a bay does not occur at the midbay, and occurs slightly offset from the midbay. Figure 22 shows the vertical displacement of the floor.
Fig. 20. Vertical displacement of the floor at several frequencies, along Path A.

Fig. 21. Vertical displacement of the floor at several frequencies, along Path B. at 20Hz, and illustrates this characteristic.
Fig. 22. Contour plot of the displacement magnitude of the floor at 20.0Hz. The color scale is in decibels re 1m.
6.6 Influence of Proximity to Perimeter

To examine the vibration response of the floor at locations close to the perimeter, another model was constructed of an existing fab floor. A finite element model was constructed of $5 \times 18$ bays for the VC-A floor and Figure 23 shows the contour plot of the displacement of the plate for a 1N harmonic excitation force, applied at the midbay location that is 1.5 bays along the x-axis and 2.5 bays along the y-axis from the lower left corner in the figure. Figure 24 shows the vertical displacement at the midbays where the 1N harmonic driving force was applied for the $5 \times 18$ and $9 \times 9$ models. Figure 22 shows that the response of the floor extends to the perimeter of the model, which is 4 bays from the central bay. Figure 23 shows that when the vibration source is placed only one bay from the edge of the floor, the vibration response at the midbay of the driving force is affected.

![Contour plot of displacement for $5 \times 18$ bays.](image1)

![Displacement at midbay for $5 \times 18$ and $9 \times 9$ bays.](image2)

Fig. 23. Contour plot of the displacement for $5 \times 18$ bays.

Fig. 24. Displacement at the midbay of the driving force for $5 \times 18$ bays and $9 \times 9$ bays.
Inspection of the contour plot shown in Figure 19 and the vibration amplitude in Figure 20 shows that the vibration level at the midbay, which is next to the bay with the driving load along Path A, has significantly lower amplitude than two bays from the driving load. One might suspect that it would be advantageous to place vibration sensitive tools one bay from the vibration source, instead of two bays away. However using the displacement at the midbay is not a good way to compare the vibration response. A better metric to compare is the kinetic energy within each bay.

Figures 25 and 26 show the approximate kinetic energy within bays along Path A and Path B, respectively. An accurate calculation of the kinetic energy involves the integration of the discretized mass multiplied by the squared velocity of the discrete mass, over the area of the bay. The approximate kinetic energy of a bay calculated here, was derived by summing the squared velocity at 49 nodal locations within and along the perimeter of a bay.

![Fig. 25. Approximate kinetic energy of bays along path A.](image)

Figure 25 shows that there is less vibration at the midpoint of the adjacent bay (curve labelled 1 Bay), than two bays away (curve labelled 2 Bay), within
Fig. 26. Approximate kinetic energy of bays along path B.

the frequency range from 10Hz to 12Hz. However, the difference in kinetic energy levels within this frequency range is only a few decibels and it cannot be said that the overall vibration level is lower in the bay adjacent to the driving bay than two bays away. Hence, it is not advisable to place vibration sensitive tools close to the driving source of vibration to exploit the modal behaviour of the floor.

7 Conclusions

A mathematical model of a waffle floor was described that can be used as an aid to designers of semi-conductor manufacturing buildings to predict the vibration transmission along a waffle floor and to optimize the floor thickness and column stiffness and spacing. The mathematical model was compared with experimental and finite element results and showed good agreement for the displacement response at the midbay locations and the attenuation of vibration with increasing distance from the source. The results showed that the vibration response of the floor is due to the summation of a number of closely spaced vibration modes.
Table 1. Parameters used in the analysis.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Units</th>
<th>9x9 Bays</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of plate along x-axis</td>
<td>$L_x$</td>
<td>m</td>
<td>36.58</td>
<td>124.36</td>
<td>67.06</td>
<td>124.36</td>
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<tr>
<td>Length of plate along y-axis</td>
<td>$L_y$</td>
<td>m</td>
<td>36.58</td>
<td>29.26</td>
<td>45.72</td>
<td>29.26</td>
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<tr>
<td>Bay size along x-axis</td>
<td>$b_x$</td>
<td>m</td>
<td>7.32m (24ft)</td>
<td>3.66m (12ft)</td>
<td>6.10m (20ft)</td>
<td>7.32m (24ft)</td>
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<tr>
<td>Bay size along y-axis</td>
<td>$b_y$</td>
<td>m</td>
<td>7.32m (24ft)</td>
<td>3.66m (12ft)</td>
<td>6.10m (20ft)</td>
<td>7.32m (24ft)</td>
</tr>
<tr>
<td>Column axial stiffness</td>
<td>$Q_{c_z}^K$</td>
<td>N/m</td>
<td>$1 \times 10^{20}$</td>
<td>$1.05 \times 10^9$</td>
<td>$8.0 \times 10^8$</td>
<td>$1.05 \times 10^9$</td>
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<tr>
<td>Column rotational stiffness</td>
<td>$Q_{c_\theta}^K$</td>
<td>Nm / rad</td>
<td>$1.0 \times 10^6$</td>
<td>$1.0 \times 10^7$</td>
<td>$1.0 \times 10^7$</td>
<td>$1.0 \times 10^7$</td>
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<td>Location of driving force</td>
<td>$\sigma_J$</td>
<td>m</td>
<td>(18.29,18.29)</td>
<td>(56.69,16.46)</td>
<td>(15.24,27.43)</td>
<td>(76.81,10.97)</td>
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<tr>
<td>Magnitude of driving force</td>
<td>$F_J^0$</td>
<td>N</td>
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<tr>
<td>Number of modes to analyze</td>
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<tr>
<td>Young’s modulus of the plate</td>
<td>$E$</td>
<td>Pa</td>
<td>$25.931 \times 10^9$</td>
<td>$25.931 \times 10^9$</td>
<td>$25.931 \times 10^9$</td>
<td>$25.931 \times 10^9$</td>
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<tr>
<td>Vibration Criterion</td>
<td>VC</td>
<td>-</td>
<td>VC-A</td>
<td>VC-D</td>
<td>VC-D</td>
<td>VC-A</td>
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<tr>
<td>Plate thickness for VC-A floor</td>
<td>$h$</td>
<td>m</td>
<td>0.347</td>
<td>0.507</td>
<td>0.6675</td>
<td>0.347</td>
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<td>Density of the plate</td>
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<td>kg/m$^3$</td>
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<td>Poisson’s ratio of the plate</td>
<td>$\nu$</td>
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</tbody>
</table>
A Dirac Delta Function Properties

(1) Figure A.1 shows a point force $F_z$ acting in the $Z$ direction on point $\sigma_J(x_J, y_J)$ on a support structure which is equivalent to a distributed load $F_z \delta(x - x_J, y - y_J)$.

![Fig. A.1. Point force $F_z$](image)

(2) Figure A.2 shows a point moment $M_y$, around the $Y$ axis, which is equivalent to a pair of point forces in the $Z$ direction of $\frac{M_y}{2\epsilon} \delta(x - x_J + \epsilon, y - y_J)$ and $-\frac{M_y}{2\epsilon} \delta(x - x_J - \epsilon, y - y_J)$ when $\epsilon \to 0$, they correspond to a distributed load

$$\lim_{\epsilon \to 0} \frac{M_y}{2\epsilon} \delta(x - x_J + \epsilon, y - y_J) - \frac{M_y}{2\epsilon} \delta(x - x_J - \epsilon, y - y_J)$$

$$\to M_y \frac{\partial \delta(x - x_J, y - y_J)}{\partial x}$$

(A.1)

![Fig. A.2. Point moment $M_y$](image)

(3) Similarly, a point moment around the $X$ axis, $M_x$, is equivalent to a distributed load in the $Z$ direction

$$-M_x \frac{\partial \delta(x - x_J, y - y_J)}{\partial y}$$

(A.2)
(4) Integral of Dirac delta functions

\[ \int_{\sigma} \Gamma_k(\sigma) \delta(\sigma - \sigma_J) d\sigma = \Gamma_k(\sigma_J) \]  \hspace{2cm} (A.3)

(5) Integral of the partial derivatives of Dirac delta functions

\[
\begin{align*}
\int_{\sigma} \Gamma_k(\sigma) \frac{\partial \delta(\sigma - \sigma_J)}{\partial x} d\sigma & = \lim_{\epsilon \to 0} \int_{\sigma} \frac{1}{2\epsilon} \Gamma_k(\sigma) \left\{ \delta(x - x_J + \epsilon, y - y_J) - \delta(x - x_J - \epsilon, y - y_J) \right\} d\sigma \\
& = \lim_{\epsilon \to 0} \frac{1}{2\epsilon} \{ \Gamma_k(x_J - \epsilon, y_J) - \Gamma_k(x_J + \epsilon, y_J) \} \\
& = -\frac{\partial \Gamma_k(\sigma_J)}{\partial x} \\
& = -\Gamma_{kx}(\sigma_J) \hspace{2cm} (A.4)
\end{align*}
\]

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References


