Optimisation of vibro-acoustic absorbers inside the payload bay of a launch vehicle using a parallel genetic algorithm and a distributed computing network

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Abstract

An asynchronous parallel genetic algorithm (GA) was used in conjunction with a distributed computing network to optimise the locations and parameters for combined Helmholtz resonators and tuned mass dampers, called passive vibro-acoustic devices (PVADs), placed on the walls of a composite cylinder model of the payload bay in a rocket used to launch satellites. The vibro-acoustic response was calculated using modal coupling theory with the modal response calculated using commercial finite element software. Binary and integer representations of the chromosomes in the genetic algorithm were used in the optimisation, and it was found that the integer representation converged twice as fast as the binary representation. Optimisations were conducted with a varying number of PVADs. The use of the distributed computing environment reduced the time taken to conduct the optimisation to 3 days compared to 75 days on a single desktop computer.

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1 List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{HR}^J$</td>
<td>nodal area at the attachment of the $J^{th}$ HR to the cavity</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of sound in the fluid</td>
</tr>
<tr>
<td>$C_{np}$</td>
<td>dimensionless coupling coefficient between the structural and acoustic modes</td>
</tr>
<tr>
<td>$E_p$</td>
<td>acoustic potential energy</td>
</tr>
<tr>
<td>$F_p$</td>
<td>modal force applied to the structure</td>
</tr>
<tr>
<td>$j$</td>
<td>complex number</td>
</tr>
<tr>
<td>$k_{HR}^J$</td>
<td>stiffness of the $J^{th}$ HR</td>
</tr>
<tr>
<td>$k_{TMD}^J$</td>
<td>stiffness of the $J^{th}$ TMD</td>
</tr>
<tr>
<td>$m_{HR}^J$</td>
<td>mass of the $J^{th}$ HR</td>
</tr>
<tr>
<td>$m_{TMD}^J$</td>
<td>mass of the $J^{th}$ TMD</td>
</tr>
<tr>
<td>$N_a$</td>
<td>number of acoustic modes included in the analysis</td>
</tr>
<tr>
<td>$N_{HR}$</td>
<td>number of Helmholtz resonators</td>
</tr>
<tr>
<td>$N_{nodes}$</td>
<td>number of structural nodes</td>
</tr>
<tr>
<td>$N_s$</td>
<td>number of structural modes included in the analysis</td>
</tr>
<tr>
<td>$N_{TMD}$</td>
<td>number of tuned mass dampers</td>
</tr>
<tr>
<td>$p$</td>
<td>acoustic pressure</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$p_n$</td>
<td>modal participation factor for the acoustic pressure of the $n^{th}$ mode</td>
</tr>
<tr>
<td>$\dot{Q}_n$</td>
<td>volume acceleration of the $n^{th}$ acoustic mode</td>
</tr>
<tr>
<td>$Q_{HR}^J$</td>
<td>acoustic volume velocity from the $J^{th}$ HR</td>
</tr>
<tr>
<td>$Q_{K}^{\text{int}}$</td>
<td>acoustic volume velocity from the $K^{th}$ acoustic source</td>
</tr>
<tr>
<td>$Q_T$</td>
<td>total acoustic volume velocity</td>
</tr>
<tr>
<td>$r$</td>
<td>arbitrary location in the fluid</td>
</tr>
<tr>
<td>$r_s$</td>
<td>arbitrary location on the structure</td>
</tr>
<tr>
<td>$S$</td>
<td>surface area of the structure in contact with the fluid</td>
</tr>
<tr>
<td>$V_n$</td>
<td>volume of an acoustic finite element</td>
</tr>
<tr>
<td>$w$</td>
<td>Structural displacement</td>
</tr>
<tr>
<td>$w_p$</td>
<td>modal participation factor for the structural displacement of the $p^{th}$ mode</td>
</tr>
<tr>
<td>$x_{HR}^J$</td>
<td>displacement of the $J^{th}$ HR</td>
</tr>
<tr>
<td>$x_{TMD}^J$</td>
<td>displacement of the $J^{th}$ TMD</td>
</tr>
<tr>
<td>$\eta$</td>
<td>structural loss factor</td>
</tr>
<tr>
<td>$\Lambda_n$</td>
<td>modal volume of the $n^{th}$ acoustic mode</td>
</tr>
<tr>
<td>$\Lambda_p$</td>
<td>modal mass of the $p^{th}$ structural mode</td>
</tr>
<tr>
<td>$\omega$</td>
<td>driving frequency in radians / sec</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>resonance frequency of the $n^{th}$ cavity mode</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>resonance frequency of the $p^{th}$ structural mode</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>structural mode shape of the $p^{th}$ mode</td>
</tr>
<tr>
<td>$\psi_n$</td>
<td>acoustic mode shape of the $n^{th}$ mode</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>density of the fluid</td>
</tr>
</tbody>
</table>

Table 1: List of Symbols.

2 Introduction

2.1 Motivation

Many companies are interested in optimising the designs of their products to reduce noise, vibration, weight, cost, etc. Computer simulation tools such as finite element analysis, boundary element analysis, statistical energy analysis, and rigid body kinematic analysis are used to predict the performance of a particular design. Engineers vary the design parameters and re-calculate the performance, or the cost function, for each new design. Almost every optimisation technique requires numerous calculations of the cost function for various input parameters, until the best result can be obtained. Unfortunately, most computational methods for complex vibro-acoustic systems require significant computational resources to evaluate each cost function. Optimisation of the design parameters is often not achievable within a reasonable time frame due to the complexity of the problem. The work presented here shows how a distributed computing network can be established relatively easily to conduct an optimisation of a complex vibro-acoustic system in a reasonable time.

Most companies have invested significant money into purchasing numerous
desktop computers that are used heavily during working hours, but remain idle after the working day has ended. Idle computers present an opportunity as an untapped computational resource that can be used for conducting calculations for optimisation routines.

In the work conducted here, an optimisation of the parameters for tuned mass dampers (TMDs) and Helmholtz resonators (HRs) attached to the walls of the payload bay of a rocket used to launch satellites was conducted to reduce the noise levels inside the payload bay. The cost function was the acoustic potential energy within the cavity and each evaluation took approximately 6 minutes on a 3.0GHz Pentium desktop computer. A Genetic Algorithm was used to optimise the parameters over 18,000 cost function evaluations. If the optimisation had been conducted on a single 3.0GHz Pentium it would have taken 75 days. By using a distributed computing network of about 150 computers of varying processor speeds from 1.8GHz to 2.4GHz, the time taken to conduct the optimisation was less than 3 days.

2.2 Background

The work presented here is one aspect of a multi-phase project with the Air Force Office of Scientific Research [1–3]. Previously published papers [4, 5] described modal coupling theory, which is based on the work of Cazzolato [6], to calculate the vibro-acoustic response, and was modified to include the effects of tuned mass dampers and Helmholtz resonators using a point impedance approach. The work described here takes a different approach, where instead of using a point impedance method, the coupling of the absorbers to the vibro-acoustic system is accomplished by combining the modal responses using a method similar to that described in Refs [7–9]. The advantage of this formulation over the previous work is that it can be easily extended to multi-degree of freedom absorbers. The previous optimisation using a genetic algorithm
took over one month using a single computer, even though the number of modes included in the analysis was reduced by removing modes that had low vibro-acoustic coupling coefficients [2, 5]. Unfortunately it was found that removing the poorly coupled modes from the search space eliminated possible good solutions, whereby the vibration energy could be shifted into modes that are poor radiators of sound. In the current work, the poorly coupled modes are retained in the search space. The conclusion from Refs [2, 5] was that a faster optimisation technique was required to reduce the calculation time to the order of a few days, rather than a month.

The contribution of the work presented here shows how a parallel genetic algorithm and a distributed computing network can be implemented relatively easily to solve vibro-acoustic optimisation problems. The vibro-acoustic equations presented here are an extension of the well known work of Fahy [10].

3 Modelling

3.1 Coupled Structural Acoustic Equations

Fahy [10, p249] describes equations for the coupled structural-acoustic response of a system in terms of the summation of structural and acoustic mode shapes. The structural displacement is described in terms of a summation over the \textit{in vacuo} normal modes as

\[ w(r_s) = \sum_{p=1}^{\infty} w_p \phi_p(r_s) \]  

where \( \phi_p \) is the mode shape of the \( p^{th} \) structural mode, \( r_s \) is an arbitrary location on the surface of the structure, and \( w_p \) is the modal participation factor of the \( p^{th} \) mode.
The acoustic pressure is described in terms of a summation of the acoustic modes of the fluid volume with rigid boundaries as

\[ p(r) = \sum_{n=0}^{\infty} p_n(t) \psi_n(r) \]  \hspace{1cm} (2)

where \( \psi_n \) is the acoustic mode shape of the \( n \)th mode, \( r \) is an arbitrary location within the volume of fluid, and \( p_n \) is the modal participation factor of the \( n \)th mode. Note that the \( n = 0 \) mode is the acoustic bulk compression mode of the cavity that must be included in the summation.

The equation for the coupled response of the structure is given by [10, Eq. (6.27)]

\[ \ddot{w}_p + \omega_p^2 w_p = \frac{S}{\Lambda_p} \sum_n p_n C_{np} + \frac{F_p}{\Lambda_p} \]  \hspace{1cm} (3)

where \( \omega_p \) are the structural resonance frequencies, \( \Lambda_p \) are the modal masses, \( F_p \) are the modal forces applied to the structure, \( S \) is the surface area of the structure, and \( C_{np} \) is the dimensionless coupling coefficient given by the integral of the product of the structural (\( \phi_p \)) and acoustic (\( \psi_n \)) mode shape functions over the surface of the structure, given by

\[ C_{np} = \frac{1}{S} \int_S \psi_n(r_s) \phi_p(r_s) \, dS \]  \hspace{1cm} (4)

The equation for the coupled response of the fluid is given by [10, Eq. (6.28)]

\[ \ddot{p}_n + \omega_n^2 p_n = -\left( \frac{\rho_0 c^2 S}{\Lambda_n} \right) \sum_p \ddot{w}_p C_{np} + \left( \frac{\rho_0 c^2}{\Lambda_n} \right) \dot{Q}_n \]  \hspace{1cm} (5)

where \( \omega_n \) are the resonance frequencies of the cavity, \( \rho_0 \) is the density of the fluid, \( c \) is the speed of sound in the fluid, \( \Lambda_n \) is the modal volume, and \( Q_n \) is the source strength with units of volume velocity (hence \( \dot{Q}_n \) has units of volume acceleration).
A Tuned Mass Damper (TMD) is a device consisting of a spring and mass, the resonance frequency of which is adjusted to coincide with the frequency of the forces driving the structure, or with a particular resonance frequency of structural vibration. The purpose of a TMD is to reduce the vibration levels on the structural boundary enclosing the cavity with the intention of reducing the sound transmitted into the cavity. However, alternative uses for such a device include tuning it to an acoustic resonance frequency of a cavity, or even attempting to shift the energy of one vibration mode into another vibration mode that has a poor acoustic radiation efficiency (radiation ratio or index are alternative preferences), as is used in Active Structural Acoustic Control (ASAC) [11, Ch. 9].

The method used to model the coupling of the TMD to the structure is similar to the method described in Refs [8, 9]. This formulation can be easily extended to multiple degree of freedom TMDs to include 3 translational and 3 rotational degrees of freedom, or multi-modal TMDs, and multi-modal acoustic Helmholtz resonators.

A model of a TMD is shown in Figure 1, where $m_1$ is the mass of the damper, $k_1$ is the stiffness, $F_1$ is the harmonic force directly applied to the mass, which is typically zero, and $F_c$ is the force at the attachment point between the spring and the structure. Note at this stage, the model does not have any damping included; however damping terms can be added later without complication. The equations of motion for the TMD can be written succinctly in matrix form as

$$
\begin{bmatrix}
  k_1 - m_1\omega^2 & -k_1 \\
  -k_1 & k_1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_c
\end{bmatrix}
= \begin{bmatrix}
  F_1 \\
  F_c
\end{bmatrix}
$$

(6)

where $k_1$ is the spring stiffness, $m_1$ is the mass, $F_1$ is the force applied directly
Fig. 1. Model of a one degree of spring-mass system, to be used as a Tuned Vibration Damper (TMD).

to the TMD which is usually $F_1 = 0$, $\omega$ is the driving frequency in radians / second, $x_1$ is the displacement of the TMD, $x_c$ is the displacement of the connection of the isolator to the structure, and $F_c$ is the force applied by the structure to the TMD, which is equal and opposite to the force from the spring.

One can assume that at the point of attachment, the displacement of the structure and the position of the end of the spring are the same. If the dynamics of the structure are introduced into Eq. (6) by using the theory from Section 3.1, then the equations of motion for multiple TMDs attached to a structure can be written as

$$\ddot{w}_p + \omega_p^2 w_p = \frac{S}{\Lambda_p} \sum_n p_n C_{np} + \frac{F_p}{\Lambda_p} + \frac{1}{\Lambda_p} \left\{ [\psi_J]^T k_{J}^{\text{TMD}} (x_{J}^{\text{TMD}} - [\psi_J]w_p) \right\}$$

(7)

for the equation of motion of the structure and

$$-\omega^2 m_J x_J^{\text{TMD}} + k_{J}^{\text{TMD}} (x_{J}^{\text{TMD}} - [\psi_J]w_p) = 0$$

(8)

for the equation of motion of the TMD. Consider the case for the structural domain only for the moment, by omitting the acoustic coupling term $C_{np}$. 
Eqs. (7) and (8) can be written in matrix form as

\[
\begin{pmatrix}
k_J^{\text{TMD}} - \omega^2 m_J^{\text{TMD}} & -k_J^{\text{TMD}}[\psi_J] \\
-[\psi_J]^T k_J^{\text{TMD}} & \Lambda_p (\omega_p^2 - \omega^2) + [\psi_J]^T k_J [\psi_J]
\end{pmatrix}
\begin{pmatrix}
x_J^{\text{TMD}} \\
w_p
\end{pmatrix}
= \begin{pmatrix}
0 \\
F_p
\end{pmatrix}
\]  

(9)

where \([\psi_J]\) is the structural mode shape vector evaluated at the \(J^{th}\) connection point of the TMD to the structure, \(^T\) is the matrix transpose operator. Note that the matrix \([\Psi]\) contains all the mode shapes for all the nodes and has dimensions \((N_{\text{nodes}} \times N_s)\) - the number of structural nodes \(\times\) the number of structural modes. Hence, each row of the matrix contains \(N_s\) entries for the normal structural displacement at each mode, and so the vector \([\psi_J]\) has dimensions \((1 \times N_s)\). Eq. (9) can be extended to the general case, where there are multiple TMDs that each have multiple degrees of freedom, as shown in Refs. [8, 9].

The equations derived thus far have not included damping terms. Damping can be included by using a hysteretic structural loss factor, so that the stiffness value for the TMD becomes a complex number. Hence the complex stiffness can be written as \(k_J^{\text{TMD}} = k_J^{\text{TMD}}(1 + j\eta)\), where \(\eta\) is the structural loss factor.

3.3 Helmholtz Resonators

A Helmholtz resonator can be thought of as a piston attached to a cavity, with the dynamics of an equivalent mass-spring system, as shown in Figure 2.

The equation for the coupled response of the fluid described in Eq. (5) includes a term for the application of acoustic sources \(Q_n\). The modal volume velocity sources inside the cavity comprise the point sources from Helmholtz resonators, \(Q_{HR}^j\), and from internal sources such as loudspeakers or other noise generating devices \(Q_{\text{int}}^j\). The total volume velocity inside the cavity is given
Fig. 2. A Helmholtz resonator attached to a cavity can be thought of as a mass-spring system.

by

\[ Q_T = \sum_{J=1}^{N_{HR}} Q_{J}^{HR} + \sum_{K=1}^{N_{int}} Q_{K}^{int} \]  \hspace{1cm} (10)

where \( N_{HR} \) is the number of Helmholtz resonators, and \( N_{int} \) is the number of interior acoustic sources.

The equation of motion for the simple mass-spring system shown in Figure 3 is

\[ -\omega^2 m_{J}^{HR} x_{J}^{HR} = F_{J}^{HR} - k_{J}^{HR} x_{J}^{HR} \]  \hspace{1cm} (11)

where \( F_{J}^{HR} \) is the force applied by the back pressure from the cavity, \( k_{J}^{HR} \) is the equivalent spring stiffness of the compliant volume, and \( m_{J}^{HR} \) is the equivalent mass of the volume of fluid inside the neck of the resonator. These terms are given by [12]

\[ F_{J}^{HR} = P_{J} A_{J}^{HR} \]  \hspace{1cm} (12)

\[ k_{J}^{HR} = \frac{\rho c^2 A_{neck}^2}{V_{HR}} \]  \hspace{1cm} (13)

\[ m_{J}^{HR} = \rho A_{neck} L_{eff} \]  \hspace{1cm} (14)

Fig. 3. A Helmholtz resonator attached to a cavity can be thought of as a mass-spring system.
where $A_{HR}^J$ is the area associated with the node attaching the spring to the acoustic cavity, $P_J$ is the nodal pressure at the attachment point, $V_{HR}$ is the volume of the Helmholtz resonator, $A_{neck}$ is the cross sectional area of the neck, $L_{eff}$ is the effective length of the neck given by $L_{eff} = L_{neck} + 1.7a$, where $L_{neck}$ is the length of the neck, and $a$ is the radius of the neck.

The volume acceleration from the Helmholtz resonator is given by

$$\dot{Q}_{HR}^J = -\omega^2 A_{HR}^J [\phi_J]^T x_J^{HR}$$  \hspace{1cm} (15)

Hence the equation for the response of the fluid can be written as

$$\ddot{p}_n + \omega_n^2 p_n = -\left(\frac{\rho_0 c^2 S}{\Lambda_n}\right) \sum_p \dot{w}_p C_{np}$$
$$+ \left(\frac{\rho_0 c^2}{\Lambda_n}\right) \left(\dot{Q}_n - \omega^2 A_{HR}^J [\phi_J]^T x_J^{HR}\right)$$

and the equation of motion for the HR can be written as

$$-m_J^{HR} \omega^2 + k_J^{HR} (x_J^{HR} - A_J^{HR} [\phi_J]) = 0$$  \hspace{1cm} (17)

Consider the case for the acoustic domain only for the moment, by omitting the coupling term $C_{np}$. Eqs. (16) and (17) can be written in matrix form as

$$\begin{bmatrix}
\dot{Q}_{HR}^J - \omega^2 A_{HR}^J [\phi_J] & -A_{HR}^J [\phi_J] \\
-\omega^2 A_{HR}^J [\phi_J]^T \frac{\Lambda_n}{\rho_0 c^2} (\omega_n^2 - \omega^2)
\end{bmatrix}
\begin{bmatrix}
x_J^{HR} \\
p_n
\end{bmatrix}
= 0$$

\hspace{1cm} (18)
The equations for the fully coupled vibro-acoustic system, including the effects of the TMDs and HRs can be formed into a matrix equation using Eqs. (7), (8), (16), and (17) as

\[
\begin{bmatrix}
A_{11} & A_{12} & 0 & 0 \\
A_{21} & A_{22} & 0 & A_{24} \\
0 & 0 & B_{33} & B_{34} \\
0 & B_{42} & B_{43} & B_{44}
\end{bmatrix}
\begin{bmatrix}
x^{\text{TMD}} \\
w_p \\
x^{\text{HR}} \\
p_n
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
F_p \\
0 \\
\dot{Q}_n
\end{bmatrix}
\tag{19}
\]

where \(A_{11}\) is a \((N_{\text{TMD}} \times N_{\text{TMD}})\) matrix that has elements only on the main diagonal of

\[
A_{11} = 
\begin{bmatrix}
k_1^{\text{TMD}} - m_1^{\text{TMD}} \omega^2 \\
& \ddots \\
& & k_J^{\text{TMD}} - m_J^{\text{TMD}} \omega^2
\end{bmatrix}
\tag{20}
\]

\(A_{12}\) is a \((N_{\text{TMD}} \times N_s)\) matrix given by

\[
A_{12} = 
\begin{bmatrix}
-k_1^{\text{TMD}} \psi_{11} & \cdots & -k_1^{\text{TMD}} \psi_{1N_s} \\
& \ddots & \ddots \\
-k_J^{\text{TMD}} \psi_{J1} & \cdots & -k_J^{\text{TMD}} \psi_{JN_s}
\end{bmatrix}
\tag{21}
\]
\( \mathbf{A}_{21} \) is a \((N_s \times N_{TMD})\) matrix given by

\[
\mathbf{A}_{21} = \begin{bmatrix}
- [\psi_{11}]^T k_1^{TMD} & \cdots & - [\psi_{J1}]^T k_J^{TMD} \\
\vdots & \ddots & \vdots \\
- [\psi_{1N_s}]^T k_1^{TMD} & \cdots & - [\psi_{JN_s}]^T k_J^{TMD}
\end{bmatrix}
\]  

(22)

\( \mathbf{A}_{22} \) is a \((N_s \times N_s)\) symmetric matrix given by

\[
\mathbf{A}_{22} = \Lambda_p (\omega^2_p - \omega^2) + [\psi_J]^T k_J^{TMD} [\psi_J]
\]  

(23)

\( \mathbf{B}_{33} \) is a \((N_{HR} \times N_{HR})\) matrix that has entries along the diagonal of

\[
\mathbf{B}_{33} = \begin{bmatrix}
k_1^{HR} - m_1^{HR} \omega^2 & \cdots & \vdots \\
\vdots & \ddots & \vdots \\
k_J^{HR} - m_J^{HR} \omega^2
\end{bmatrix}
\]  

(24)

\( \mathbf{B}_{34} \) is a \((N_{HR} \times N_a)\) matrix given by

\[
\mathbf{B}_{34} = \begin{bmatrix}
-A_1^{HR} \phi_{11} & \cdots & -A_1^{HR} \phi_{1N_c} \\
\vdots & \ddots & \vdots \\
-A_J^{HR} \phi_{J1} & \cdots & -A_J^{HR} \phi_{JN_c}
\end{bmatrix}
\]  

(25)
$B_{43}$ is a $(N_a \times N_{HR})$ matrix given by

$$B_{43} = 
\begin{bmatrix}
-\omega^2A_1^{HR}[\phi_{11}]^T & \cdots & -\omega^2A_J^{HR}[\phi_{J1}]^T \\
\vdots & \ddots & \vdots \\
-\omega^2A_1^{HR}[\phi_{1N_c}]^T & \cdots & -\omega^2A_J^{HR}[\phi_{JN_c}]^T
\end{bmatrix}
$$

(26)

$B_{44}$ is a $(N_a \times N_a)$ matrix that has entries only along the diagonal of

$$B_{44} = \frac{\Lambda_n}{\rho_0 c^2} (\omega_n^2 - \omega^2)$$

(27)

The two remaining elements account for the cross coupling between structure and fluid. $A_{24}$ is a $(N_s \times N_a)$ matrix given by

$$A_{24} = -SC_{np}$$

(28)

$B_{42}$ is a $(N_a \times N_s)$ matrix given by

$$B_{42} = -\omega^2S[C_{np}]^T$$

(29)

One can see that the matrix in Eq. (19) is almost symmetric. The properties of the sub-elements are

$$A_{21} = [A_{21}]^T$$

(30)

$$B_{43} = \omega^2[B_{34}]^T$$

(31)

$$B_{42} = \omega^2[A_{24}]^T$$

(32)

Hence if the sub-elements on the fourth row of the matrix in Eq. (19) are divided by $\omega^2$ then the matrix will be symmetric.
The acoustic potential energy is a measure of the acoustic energy contained within a cavity and is given by [6]

\[ E_p = \frac{1}{4\rho_0 c_0^2} \int_V |p(\vec{r})|^2 dV \]  

(33)

which can be implemented in a finite element formulation as

\[ E_p = \frac{1}{4\rho_0 c_0^2} \sum_{n=1}^{N_a} p_n^2 V_n \]  

(34)

where \( p_n \) is the pressure at the \( n^{th} \) node and \( V_n \) is the volume associated with the \( n^{th} \) node. This equation can be re-arranged so that the acoustic potential energy is calculated in terms of the modal pressure amplitudes as

\[ E_p = \sum_{n=1}^{N_a} \Lambda |p_n|^2 \]  

(35)

\[ = p^H \Lambda p \]  

(36)

where \( \Lambda \) is a \( (N_a \times N_a) \) diagonal matrix of which the diagonal terms are

\[ \Lambda(n, n) = \frac{\Lambda_n}{4\rho_0 c_0^2} \]  

(37)

where \( \Lambda_n \) is the modal volume of the \( n^{th} \) cavity mode.
4 Condor Software and Genetic Algorithms

4.1 Introduction

Genetic Algorithms (GAs) have been used for many acoustic and vibration optimisation problems such as finding optimal locations for vibration control actuators to reduce radiated noise [13–15], and optimisation of truss structures to reduce the transmission of vibration [16, 17].

Genetic algorithms are well suited to a parallel computational framework, where the cost function evaluations are conducted by ‘worker’ computers and a ‘master’ computer is responsible for managing the population by conducting the selection, recombination and mutation processes.

In the simplest form, all the cost functions are evaluated at the same time, and the master processor waits until all the results are returned from the worker processors. This scheme is called a synchronous master-slave GA [18, p208], which is relatively easy to implement but suffers from the drawback that it requires all the worker processors to complete their tasks before the master processor can update the population. If one of the worker processors fails, then the population cannot be updated. In addition, the GA can only update at the speed of the slowest worker processor. A variation of this scheme is the semi-synchronous master-slave [18, p208] or asynchronous GA, where the master process does not depend on the completion of all the worker processors, and instead selects members ‘on the fly’. The main disadvantage of this scheme is that more software programming is required for communicating between computers and dealing with faults. An asynchronous parallel GA was developed here and is explained in more detail in Section 4.3 of this paper. The GA implementation whereby the ‘master’ computer sends jobs to ‘worker’ computers is managed for the distributed computing environment using a software
package called Condor.

4.2 Condor Software

The Condor software package is publicly available and can be used to create a distributed computing environment [19]. It is best suited to ‘embarrassingly parallel’ problems, where there is no inter-processor communication. This type of framework is also suitable for parallel GAs where a master processor is responsible for ‘managing’ the population, conducting the breeding and mutation processes and distributing the parameters for the cost function evaluations to ‘worker’ processors. The worker processors receive the jobs from the master processor, evaluate the cost function, and return the result, which in this case is the acoustic potential energy within the cavity.

The Condor system was installed on 180 Pentium 4 computers and one computer was assigned as the master processor and distributed the jobs to the worker computers. This formed the distributed computing network referred to as the Condor pool.

An attractive feature of the Condor system is that the master processor responsible for the distribution of tasks can look at the usage of any particular computer in the network and determine if someone is using a ‘worker’ machine. If the ‘worker’ machine is being used, then that computer is marked as unavailable. The system was set up so that computers that have been left idle for 15 minutes are marked as available to the Condor pool. One limitation of the current version of Condor for computers running Microsoft Windows XP, is that if a ‘worker’ computer is used by a person whilst it is processing a Condor job, the computer will halt the Condor job and find another computer to run the job, but it will re-start the job from the beginning instead of continuing from where the interruption occurred. This is not a significant problem
because people rarely use the computers in the Condor pool overnight.

The Matlab compiler [20] was used to create a binary executable file for the cost function evaluation. The advantage of creating a stand-alone executable file compared to running a Matlab script is that a Matlab license is not required for the calculations and is available for other users of the computer network. The executable file reads in Matlab .mat files and extracts the parameters for the PVADs and then uses those parameters to calculate the acoustic potential energy, which is then written to an output Matlab .mat file. The GA reads the results from the cost function evaluation and performs the selection, recombination and mutation processes.

4.3 Details of the Asynchronous Parallel GA

Hansen et. al. [13] describes the use of an integer formulation for the use of a genetic algorithm to optimise the location of control actuators for active-structural-acoustic-control (ASAC) (see also Refs [14, 15]). Their conclusion was that an integer representation for the location of the actuators provides faster convergence than a binary string representation. With a binary representation of the chromosomes, the cross-over process swaps bits (as in a 1 or 0) of the string between children, which can cause a large change in the location of the actuators, thereby destroying any information gained in the population from the evolutionary process. Whereas for an integer representation of the chromosomes, the cross-over process swaps the entire integer string between children and hence retains the position information in the population. The mutation of the chromosome string (which means selecting a random number to replace the existing number) is the process responsible for evaluation of alternative actuator locations.

The results presented here confirm the recommendation from Refs [14, 15],
with the convergence rate being about twice as fast when using the integer string representation than when using the binary string representation.

The Genetic Algorithm Toolbox [21] was used as a basis for developing an asynchronous parallel GA. In this application the location, mass, stiffness, and damping of the TMDs and HRs was encoded using either an integer, or binary scheme. Each parameter was assigned a finite number of choices that corresponds to a physical value. For example, the damping was permitted to have 11 possible values in the linear range from 5% to 25%. If the GA selected a value of 4, then the corresponding damping value used in the evaluation of the acoustic potential energy would be 11%.

Previous work showed that the best results occurred when the mass of the TMD was maximised so as to maximise absorption [22], and when the mass of the HR was minimised to achieve the best coupling between the HR and the cavity [4]. Note that the mass of the HR is the effective mass of the air in the neck of the resonator.

The asynchronous GA was developed with guidance from Stanley and Mudge [23], and operates as follows:

1. The GA generates a random initial population
2. The cost functions are evaluated for the initial population using the Condor pool.
3. The GA checks how many jobs are already being calculated by the worker processes and if it has not exceeded a chosen value (it is possible to have more jobs in the Condor queue than the number of computers in the pool), then
   a. The results from the cost function evaluations are sorted in rank order based on their fitness. In this case the lower the Acoustic Potential Energy the higher the rank.
   b. A new set of individuals is created by performing selection, mutation
and recombination operations.

(c) A file containing the parameters for the PVADs (chromosomes) is generated for submission to the Condor pool. The files are placed in a unique directory, where all the files relevant to that particular cost function evaluation reside. The job is then submitted to the Condor pool.

(4) The directory structure is checked for the existence of a file called success.sub, which indicates that the cost function evaluation was completed successfully. The jobs that are ready to have their results read back into the GA are formed into a queue, with the oldest jobs at the front of the queue.

(5) The results from the cost function evaluation from the oldest job in the queue are retrieved.

(6) The new population is inserted into the old population by replacing the chromosomes in the old population that had the worst fitness, in this case the highest acoustic potential energy.

(7) The process repeats from step 3 until a predetermined number of iterations is reached.

5 Example Problem

5.1 Introduction

An example problem is shown here to demonstrate the effectiveness of the parallel GA and the distributed computing network. The example is taken from a research project concerned with the optimisation of the placement and parameters of integrated tuned mass absorbers and Helmholtz resonators attached to the fairing of a launch vehicle used to launch satellites. The excessive noise levels inside the payload bay of the launch vehicles is blamed for many satellite failures. It is claimed that 40% of the mass of a satellite is present just
to enable the satellite to survive the harsh launch environment [24]. Reducing the mass of a satellite has obvious financial benefits.

One of the advantages of the techniques described in this paper is that the shapes of the structural and acoustic models are not restricted. Provided structural and acoustic finite element models can be constructed of the system under investigation, then the techniques described here can be used to calculate the vibro-acoustic response.

5.2 Finite Element Model

The model used to demonstrate the optimisation techniques is a scale model of a launch vehicle fairing; a large cylindrical structure made of composite material. The cylinder is approximately 2.56m long, 2.46m diameter, and 2mm thick walls with clamped rigid end caps made of wood. Future work will consider experimental and theoretical models of a Representative Small Launch Vehicle Fairing (RSLVF). However, here only a composite cylinder is considered. Figures 4 and 5 show the complete finite element acoustic model, and a cross section of the finite element structural model, respectively, both created using the ANSYS software package. The nodes for the structural model form part of the nodes for the acoustic model, with an average mesh density spacing of 0.08m. Based on the rule-of-thumb guideline for acoustic finite element analyses, one should have an element spacing of at least 6 elements per wavelength [25, 26]. Using this guide, the model should be suitable for analyses up to (speed of sound / element spacing = 344/(0.08 × 6) =) 716Hz. This element mesh density was selected as a balance between the accuracy of results and the time taken to solve for the coupled vibro-acoustic response. As the analysis was conducted many times during the optimisation process, it was important to reduce the time taken to solve the cost function.
Fig. 4. ANSYS model of the cylinder showing the acoustic space.

Fig. 5. ANSYS model of the cylinder, showing a cross section through the structure.
5.3 Acoustic Loading

The acoustic loading on the exterior of the fairing originates from the noise produced by the rocket exhaust. The sound field is very complex and dependent on the layout of the launch site. The acoustic loading is influenced by the orientation of the exhaust buckets which changes the directivity of the sound source, the proximity of the environmental control shelter and the launch control tower which influences the acoustic reflections, and many other factors. The noise is broad-band and is incoherent between any two points on an imaginary surface.

Estéve et. al. [27] have assumed that a plane wave is incident on a cylinder from an elevated angle, and have calculated the acoustic loading by using the theory from Morse and Ingard [28, Ch 8-p400, Ch9-p511]. Gardonio et. al. [29] assume a similar loading condition.

For the work presented here it is assumed that the acoustic loading is a harmonic plane wave that strikes the exterior of the fairing perpendicular to the axis of the cylinder. Future research work will focus on more representative loading conditions, and this is not the focus of the work presented here.

In a finite element sense, the surface of the cylinder can be discretised into small elements that have nodes at the corners of the area of each element. As the acoustic plane wave reaches the surface of the cylinder, the pressure acts on an elemental area. This pressure can be converted into a nodal force, based on the area associated with each node, which can be applied to the modal model. The pressure that acts on the surface of the cylinder is a function of the area that faces the incoming plane wave. Hence the nodes that have normals facing the impinging direction have the greatest pressure excitation amplitude, and the nodes that are at 90 degrees to the impinging direction have zero excitation pressure, as the plane wave is travelling tangentially to
the surface of the cylinder. It is assumed that there is no diffraction around the cylinder, and only half of the cylinder is loaded. In reality, diffraction would be expected to occur around the cylinder; however it may be assumed that this is of less importance than the waves that directly strike the cylinder and thus it will be ignored. Another effect that is ignored is the external re-radiation from the cylinder, where the vibration of the structure generated by the incident sound field causes sound to radiate away from the structure (not into the cavity), hence removes vibrational energy from the structure and thereby provides a form of damping. This effect can be included at a later stage by adding radiation damping terms to the matrices [10, p466].

Figure 6 shows a diagram of the important features of the mathematical model for the pressure loading on the cylinder. The pressure is assumed to vary cosinoidally around the circumference of the cylinder, in proportion to the area that is normal to the impinging direction. Hence the nodal pressure is given by

$$F_{\text{node}} = P(\theta)A_{\text{node}} \cos \theta$$

(38)

where $F_{\text{node}}$ is the nodal force, $P(\theta)$ is the pressure at angle $\theta$, and $A_{\text{node}}$ is the nodal area.

The distance travelled by a plane wave between the point where the plane wave first strikes the cylinder and another point on the circumference of the
cylinder is

\[ x_{\text{delay}} = R(1 - \cos \theta) \]  

(39)

The wavelength of sound is \( \lambda = c/f \), where \( c \) is the speed of sound, and \( f \) is the frequency in Hertz. If it is assumed that the spatial sound pressure level variation is sinusoidal, and that the maximum pressure amplitude \( P_0 \) occurs when the plane wave first strikes the cylinder, then the instantaneous pressure at other locations on the cylinder will vary as

\[
P(\theta) = P_0 \left[ \cos \left( \frac{2\pi x_{\text{delay}}}{\lambda} \right) + j \sin \left( \frac{2\pi x_{\text{delay}}}{\lambda} \right) \right] \]  

(40)

It was assumed that the incident sound pressure level was 140dB re 20\( \mu \)Pa, which corresponds to \( P_0 = 200\)Pa. Hence the nodal force is found by combining Eqs. (38), (39), and (40) to give

\[
F_{\text{node}} = P_0 A_{\text{node}} (\cos \theta) \left[ \cos \left( \frac{2\pi R(1 - \cos \theta)}{\lambda} \right) 
+ j \sin \left( \frac{2\pi R(1 - \cos \theta)}{\lambda} \right) \right] \]  

(41)

The distribution of nodal loads around the circumference of the cylinder is calculated by using Eq. (41), and is shown in Figure 7 when looking along the axis of the cylinder. The lengths of the arrows are proportional to the magnitude of the force, and all the forces are acting normal to the surface of the cylinder. Note that it is assumed that the plane wave is perpendicular to the axis of the cylinder, so the nodal forces through one diametrical slice of the cylinder are the same along the entire length of the cylinder.
Fig. 7. Plot of the nodal loads on the cylinder that accounts for the nodal area and incidence angle. The lengths of the arrows are proportional to the magnitudes of the nodal forces.
The results presented here are not intended to reveal new vibro-acoustic phenomena, but to demonstrate that the genetic algorithm optimisation using the distributed computing network can be used successfully.

### 6.1 Performance vs Number of PVADs

Several optimisations were conducted to determine the variation of the acoustic potential energy within the cavity when varying the number of PVADs. Optimisations were conducted over 18,000 cost function evaluations, using 10, 20 and 30 PVADs, where they could be located anywhere on the circumference of the cylinder, and the parameters were allowed to vary as listed in Table 2. The masses of the TMDs and HRs were fixed at 0.45kg and 0.01kg, respectively. Hence, the greater the number of PVADs, the greater the added mass attached to the structure.

<table>
<thead>
<tr>
<th>PVAD parameter</th>
<th>Min</th>
<th>Max</th>
<th>No. Values</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVAD position</td>
<td>1</td>
<td>4032</td>
<td>4032</td>
<td>Circumferential nodes</td>
</tr>
<tr>
<td>Mass-spring frequency</td>
<td>11</td>
<td>510</td>
<td>500</td>
<td>[Hz]</td>
</tr>
<tr>
<td>Mass-spring damping ($\eta$)</td>
<td>0.01</td>
<td>0.25</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Acoustic resonator frequency</td>
<td>11</td>
<td>510</td>
<td>500</td>
<td>[Hz]</td>
</tr>
<tr>
<td>Acoustic resonator damping ($\eta$)</td>
<td>0.01</td>
<td>0.25</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Table 2

There are many interesting optimisations that could be conducted such as establishing a 'mass budget' and determining an optimum number of PVADs for a given added mass, investigating the placement of the PVADs on circumferential stiffening rings, optimising the locations of the TMDs and HRs separately, etc. However, the example presented here is used merely to illustrate that the optimisation routine can work successfully.
Table 3 lists the population size used for each of the optimisations.

<table>
<thead>
<tr>
<th>No. PVADs</th>
<th>Population Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>30</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3
The population size for each of the optimisations

Figure 8 shows the variation in acoustic potential energy versus frequency calculated for 10, 20 and 30 PVADs. Figure 9 shows the average acoustic potential energy calculated within the cavity, over the frequency range of 50 - 300Hz.

Fig. 8. Variation in acoustic potential energy for no PVADs (Base), 10, 20 and 30 PVADs.
Fig. 9. Average acoustic potential energy for no PVADs (Base), 10, 20 and 30 PVADs.
6.2 Binary vs Integer Formulation

Optimisations were conducted using the genetic algorithm and the parallel computing network to compare the formulation of the chromosomes with binary and integer representations. Optimisations were conducted using 20 and 30 PVADs attached to the circumference of the cylinder.

Figure 10 shows the evolution of the value of the cost function for the optimisation of 20 PVADs attached to the cylinder. The results show that the integer representation evolved about twice as fast as the binary representation. For example, after 3000 generations (6000 cost function evaluations) the acoustic potential energy is -14.25dB for the integer representation of the chromosomes. To achieve the same value of the cost function for the binary representation takes about 6000 generations (12,000 cost function evaluations), or twice as long.

Figures 11 shows the evolution of the value of the cost function for the optimisation of 30 PVADs attached to the cylinder. The results show that after 9000 generations the value of the cost function for the integer representation is still declining and has not yet reached an optimum solution, whereas the binary representation appears to be stuck in a local optimum.

Figures 12 and 13 show the locations of 20 and 30 PVADs on the cylinder, respectively, after 18,000 cost function evaluations for the binary and integer representations. The results shows that the locations determined by each method are quite different.
Fig. 10. Evolution of the genetic algorithm’s cost function over 18,000 evaluations for 20 PVADs.

Fig. 11. Evolution of the genetic algorithm’s cost function over 18,000 evaluations for 30 PVADs.
Fig. 12. Location of the 20 PVADs after optimisation for 18,000 cost function evaluations, as if the surface of the cylinder had been unwrapped.

Fig. 13. Location of the 30 PVADs after optimisation for 18,000 cost function evaluations, as if the surface of the cylinder had been unwrapped.
The results in Figures 10 and 11 show that the integer representation for the chromosomes evolves about twice as fast as the binary representation.

The other interesting result is that the final optimal distribution of the PVADs was different, depending on whether the integer and binary representation was used. Some researchers have an undocumented hypothesis that for low numbers of PVADs (about 5), the optimum location of the absorbers is fixed. However, as the number of PVADs increases (more than 20), then the locations of the absorbers are not critical and there are many ‘optimal’ locations. The structure and dampers are likely to approach a ‘fuzzy structure’ that has been discussed in the literature [30–37]. This hypothesis is the subject of current work being undertaken and is expected to be reported in another journal paper by the author.

7 Conclusions

The work presented in this paper has demonstrated the time saving that can be achieved using a distributed computing network for the optimisation of a vibro-acoustic system. An asynchronous parallel genetic algorithm was used to optimise the locations, stiffnesses and masses of tuned mass dampers and Helmholtz resonators, with the use of a distributed computing network created using the software Condor.

The results have shown that the integer formulation for the chromosomes converges about twice as fast as the binary representation.
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