Active control of sound using a parametric array

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ABSTRACT

The parametric array uses two ultrasonic sources to generate an audible sound wave by non-linear interaction. The frequency of the audible sound wave is equal to the difference between the two ultrasonic carrier frequencies. This scattered wave is far more directional than would be generated by a conventional transducer operating at the same frequency. This paper is reports a theoretical study into the use of the parametric array as a secondary source for active noise control. It is shown that a highly directional zone of quiet can be produced. A study of the required source strengths and carrier frequencies with respect to secondary source authority and frequency bandwidth is reported. The high levels of ultrasound required to create useful secondary field levels are noted and the safety of such fields discussed.

1 THE PARAMETRIC ARRAY

The parametric array is created by the non-linear interaction between two high level ultrasonic sound fields. The resulting scattered sound field contains components at the sum and difference of the carrier wave frequencies. The sum component is quickly absorbed in air, being of a much higher frequency, however the difference component forms a highly directional beam which propagates. The beam could be used as a secondary source to create local zones of quiet which can then track an individual as they move through the sound field, while not adversely affecting the majority of the field outside the directional beam. This approach relies on the following concepts:

1. The interaction between the secondary and primary fields remains linear at the zone of quiet.
2. The scattered component of the secondary field is linear.
3. The phase and amplitude of the field scattered at the difference frequency can be accurately controlled by the carrier wave characteristics.
4. The sum and difference fields are only created in the desired beam.
5. The sum field does not interfere with the difference field.

This paper shows the results of a theoretical study into the behaviour of the parametric array (or audio spotlight) as a secondary source for active noise control. Earlier experimental work by Brooks \textit{et al.} [1] has shown that only low levels could be produced by the audio spotlight and that the signal quality was poor, often displaying non-stationary characteristics. This paper aims to show the theoretical bounds for the device.

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1.1 Models of the scattered pressure.

The non-linear interaction of two sound waves can be modeled using Lighthill’s approach [2]. The key to Lighthill’s acoustic analogy is that the sound generated by aerodynamic forces is equivalent to that generated by a distribution of quadrapole sources. The stresses in the fluid effectively change the momentum, which is equivalent to a force. It is important to note that Lighthill splits the stresses into *acoustic* stresses, those for which the resulting pressure is given by \( p = c_0^2 \rho \) and the Reynolds stress within the fluid, \( \rho v_i v_j \). From these components Lighthill creates a stress tensor

\[
T_{ij} = \rho v_i v_j + p_{ij} - c_0^2 \rho \delta_{ij}
\]  

which is equivalent to the difference in the stresses on a stationary element of fluid \( \rho v_i v_j + p_{ij} \) and the stresses that would act on the boundary of an element of fluid due to acoustic changes in density \( \rho c_0^2 \delta_{ij} \). A general equation of motion for the fluid can then be written as

\[
\frac{\partial^2 \rho}{\partial t^2} = -c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
\]  

Westervelt[3] made second order approximations of the terms in Equation 2 and so derived an equivalent source strength due to the interaction between two waves.

\[
q = \frac{1}{\rho_0 c_0^4} \frac{\partial}{\partial t} (p_i^2) \left[ 1 + \frac{\rho_0}{2c_0^2} \left( \frac{d^2 p}{d\rho^2} \right)_{\rho=\rho_0} \right]
\]

where \( q \) is the source strength, and \( \rho_0 \) and \( c_0 \) are the fluid density and sound speed. The pressure of the two waves is denoted by \( p_i \), \( i = 1, 2 \) Note that for adiabatic expansion[4],

\[
\left( \frac{d^2 p}{d\rho^2} \right)_{\rho=\rho_0} = \frac{\gamma - 1}{\rho_0} c_0^2
\]

Berktay[5] used this model, and showed that if the carrier waves are plane and colimated

![Figure 1: Geometry used by Berktay to solve for the wave scattered by two plane waves interacting.](image)

then the scattered pressure in the far-field can be written as Equation 4.

\[
p_s(R, \theta) = \frac{p_1 p_2 Q^2 S}{4\pi \rho c^4 R} \exp(-\alpha R) \frac{\sin(bk \sin \theta)}{bk \sin \theta} \frac{1}{\sqrt{A^2 + 4k^2 \sin^4(\theta/2)}}
\]
where $S$ is the area of the beam and $b$ its width in the plane of $\theta$. The absorption at the scattered frequency, $\Omega$ is denoted $\alpha_s$. The angle from the axis of the array beam is $\theta$ which is assumed to be small so that the length of the region of interaction, $x$, is much less than the distance to the observation point $r$, i.e. $x \ll r \rightarrow R \approx x \cos(\theta) + r$.

This minimum distance can be written in terms of the absorption coefficient of the carrier waves $\alpha_c$. It is assumed that the carrier waves only interact over a finite region that extends to the point where the carrier waves have been significantly attenuated by absorption therefore the length of this virtual source is given by $L = 1/2 \alpha_c$. Using the absorption coefficient given by Bass et al. [6], a carrier wave frequency of 50kHz yields an absorption of 0.23m$^{-1}$ and therefore a source length of 4.35m.

The value $A$ is a function of the absorption of the medium and is given by Equation 5

$$A = \alpha_1 + \alpha_2 - \alpha \cos(\theta)$$

If $A \ll 2k$ and the width of the transducer is small compared to the wavelength of the scattered frequency then the 3 dB beam width is given by $2\theta_d = 4\sqrt{A/2k}$. A large transducer size imparts an additional directivity due to an aperture effect of a $\sin(x)/x$ form.

Expressions for the scattered field when the carrier waves are cylindrical or spherical waves were also given by Muir[7]. The expression for interaction of spherically spreading carrier fields are obviously more complex and have been solved numerically by Muir et al. [7] and have been verified by experiments in air [8] and water [7].

In the current paper, only the interaction of plane waves is considered for simplicity, however the numerical approach is used to obtain the response in the near field, rather than the expressions provided by Berktay (Equation 4) which are not valid in the near field, (which is the region of interest here) because assumptions used in their derivation do not hold.

Note that although Equation 4 and the underlying assumptions do not hold in the near field, the expression can still be used to serve as a guide for trends in the response.

2 OVERVIEW OF THE PARAMETRIC ARRAY PROPERTIES

With reference to Equation 4 for the parametric array, the source strength is proportional to the time derivative of the product of the carrier wave pressure amplitude, $p_i$: $q \propto \partial p_i^2 / \partial t$, and so the scattered wave will contain frequency components at the sum and difference of the carrier wave frequency. There is also a significant frequency weighting to the magnitude of the source strength.

The source strength is also proportional to the second derivative of the pressure to the ambient density, $q \propto d^2p/d\rho^2 |_{\rho=\rho_0}$. This is a function of the ratio of specific heats. If adiabatic compression is assumed $d^2p/d\rho^2 |_{\rho=\rho_0} = (\gamma - 1)c_0^2/\rho_0$. Figure 2 shows the ratio of the product of the carrier wave amplitudes to the amplitude of the resulting scattered wave as a function of frequency. One carrier wave frequency was fixed at 50kHz, the other varied to create the difference frequency wave. It can be seen that the ratio decreases with frequency, implying the parametric array becomes more efficient at higher frequencies. In the audio range the difference in level between the carrier and scattered wave is of the order of 80 dB.
Bellin [9] and Westervelt’s[10] work on end fire virtual arrays assumes that the transducers dimensions are much smaller than the wavelength of the scattered wave [5]. For typical frequencies at which active noise control is applied in air, (<1 kHz) this requires the transducer to be \(\ll 0.34 \text{ m}\).

The 3 dB beam width shown in Figure 3, is \(4\sqrt{(\alpha_1 + \alpha_2 - \alpha_s)/2k}\). Figure 3 shows an almost constant beam width of 11° vs frequency if the carrier waves are at approximately 50 kHz, and it can be seen that the parametric array creates a source that is far more directional than would be created by a simple piston at the scattered frequency. An equivalent array length that has the similar directional response is given by Berktay as \(L = 1/2\alpha\).

The parametric array has near and far field behaviour, the boundary between these two regimes is given by a fraction of the Rayleigh distance, \(ka^2/2\), where this fraction is usually between 1 \(\rightarrow 3/4\pi\).

Figure 4 shows the on-axis amplitude of the scattered wave versus distance. The vertical lines on the figure indicate the Rayleigh distance, \((a^2/\lambda)\), Berktay's minimum distance, \((k_s/\alpha^2)\) and the length of the equivalent array that has the same directionality, \((1/2\alpha_c)\). The results were calculated for a carrier wave of 50 kHz and a scattered wave of 1 kHz.

3 OTHER METHODS FOR MODELING THE NON-LINEAR INTERACTION

The methods of Westervelt and Berktay rely on making approximations that simplify the process of substituting linear approximations to the non-linear interactions that are described by the Lighthill analogy. The methods are limited by the increasingly complexity required by including higher order terms and by the convergence requires that stipulate subsequent terms in the expansions must be smaller than the previous terms. This limits this approach to weak nonlinear interactions.
Figure 3: The 3 dB beam width vs frequency for a carrier wave of 50 kHz.

Figure 4: The scattered on-axis pressure amplitude vs distance. The vertical lines indicate the Rayleigh distance, Berktay’s minimum valid distance, and the equivalent source length.
3.1 The KZK equation

As stated previously Westervelt’s and Berktay’s models are based on a far-field assumption, which for ultra-sound sources in air is not often satisfied within practical distances from the parametric array. The KZK (Khokhlov-Zabolotskaya-Kuznetsov) equation models the non-linear response of a fluid at all ranges. The model includes the effects of relaxation phenomena, thermo-viscosity and non-linearity. The numerical algorithm used was developed by Lee[11] and augmented by Cleveland[12]. The algorithm is time domain based.

The KZK equation is

\[ \frac{\partial^2 p}{\partial z \partial t'} = \frac{c_0}{2} \left( \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right) + \frac{D}{2c_0^3} \frac{\partial^3 p}{\partial t'^3} + \frac{\beta}{2\rho_0 c_0^2} \frac{\partial^2 p}{\partial t'^2} \]

where \( t' = t - \frac{z}{c_0} \), the retarded time. \( D = \rho_0^{-1}[(\zeta + 4\eta/3) + \kappa(1/c_v + 1/c_p)] \) is the diffusivity of the medium, \( \beta \) is the non-linear coefficient and \( z \) is the distance from the source.

Figure 7 shows the scattered pressure and carrier wave pressure vs distance as predicted by the KZK equation. The parameters for the simulation were:

- Carrier Wave: \( p_0 = (\sin(\omega_0 t) + \sin(1.02\omega_0 t))H(t) \), where \( H(t) \) is a Hanning window and \( \omega_0 = 100\pi \ \text{rads}^{-1} \).
- Expected difference frequency: \( 0.02\omega_0 = 2\pi\text{kHz} \)
- Time step: 0.2\( \mu s \)
- Transducer radius: 0.1m
- Fluid properties: \( \rho=1.2 \ \text{kgm}^{-1}, \ c_0=343 \ \text{ms}^{-1} \).
- Non-linear coefficient: 1.0

The transducer radius of 0.1m gives a Rayleigh distance of 4.35m.

The KZK equation allows the time history of the pressure to be predicted as a function of range as shown in Figure 5. It can be seen from the figures that the high frequency content is attenuated for distances greater than the Rayleigh distance. The pressure spectra close to the source (\( z = 0.02m \)) and further away, (\( z= 3.02 \)) are shown in Figure 6. The peak amplitude of the pulse is greatly reduced.

The spectrum of the pressure close to the source contains a lot of energy at the source frequencies of 50 kHz and 50.1 kHz. As the range is increased part of this energy is transferred to the difference frequency, at 1 kHz.

Figure 7 shows the scattered pressure amplitude and the carrier wave amplitude as a function of distance. It can be seen that the level of the scattered wave is low compared to the source waves and that it varies with distance approximately in agreement with the trends shown in Figure 4 after the minimum Berktay distance. Note that the Berktay approximation showed in Figure 4 overestimates the pressure amplitude up to a distance of 10m from the source. This highlights the importance of using a numerical approach such as solving the KZK equation to accurately predict the level.
Figure 5: The time history of the pressure wave generated by the audio-spotlight. Note the reduction in level as the high frequency component is absorbed.

Figure 6: The spectrum of the pressure at \( z = 0.022 \) m and \( z = 3.02 \) m.
Figure 7: The SPL of the source, (dotted lines) and scattered (solid line) waves from the audio spotlight vs distance. Note the low level of the 1 kHz scattered wave compared to the level of the 50 kHz source waves.

4 ACTIVE CONTROL OF A MONOPOLE SOURCE

In this section we use a C++ implementation of the Fortran code written by Lee[11] to solve the KZK equation. From these values we can predict the zone of quiet produced when this field interacts with a monopole source.

The numerical routine to solve the KZK equations is computationally intensive as it is a time domain solution. It iteratively calculates the pressure along lines of $\hat{r}$, where $\hat{r}$ is a transformed radial coordinate. The step size in both time and $z$, the axial distance must be small to ensure convergence. Also the time history of the disturbance signal must be long enough to include the propagation delay. Therefore for calculation at large values of $z$ a large number of points must be used. In this paper a time history of 1000 cycles was used. There were 100 steps per cycle. The step size in $\sigma$, the normalised $z$ coordinate was 0.02. Data was collected at 100 logarithmically spaced $\sigma$ values, for 10 values of $\hat{r}$. The simulation took approximately 10 hours on a G4 MacMini.

4.1 The primary and secondary fields

Figure 8 shows the sound fields created by a monopole and a parametric array at 1 kHz.

The left hand side of the plot shows the monopole field, it can be seen that it is constant with angle and decreases with distance according to the free field Greens function. The right hand plot shows the field created by the parametric array with the properties listed previously. Note that both fields have been normalised so that the level on-axis at 1m is 94 dB. It can be seen that the parametric array field falls off with angle and has a local maxima in the $z$-direction.
4.2 The combined field

To calculate the control gain required to cancel the primary (monopole) field, two data points on-axis in both fields were taken. These are considered the disturbance and control signals. Two data points were taken so that the system would not be square which would result in perfect cancellation and would not give an informative result.

The optimal gain to be applied to the parametric array was calculated as

\[ g = -\vec{x} \vec{y}^{-1} \]  

where \( \vec{x} \) is the vector containing the two primary field data points and \( \vec{y} \) contains the two secondary field data points. The sum of the primary and amplified secondary fields is plotted in Figure 9 and Figure 10.

The left plot in Figure 9 shows the controlled and uncontrolled sound fields. In this case the error sensors were at 0.9m and 1.0m. The right plot in Figure 9 shows the on-axis pressure for the uncontrolled and controlled field. The sharp dip at 1m can be seen. Note the increase in level over the range 2m to 12m. Because the sound field from the parametric array does not decay in its near field in the same way as the free space Greens function, matching the level at 1m creates a large zone of spill over. Compare this to Figure 10, where the error sensors are placed at 4.5m and 5m. The increase in spacing is a consequence of the logarithmic spacing of the grid, this may account for the smaller reduction level at 4.5m to 5m. However there is also less spill over as by this point the parametric array field is closer to the free space Greens function.

The surface plots in Figure 9 and Figure 10 show local minima a points other than the locations of the error sensors. This is to be expected. However in general the level of the sound field not at the sensors increases. In these simulations the monopole and the parametric array are collocated; if the parametric array behaved as a monopole we
could expect global control. As we try to control points further from the origin where the parametric array sound field is similar to the free field Greens function we would expect this to happen. As the monopole is separated from the parametric array we would expect more focussed zones of quiet as the phase of the sound fields is even less likely to be similar over any appreciable distance.

For both cases the optimal gain was on the order of 190 dB. This implies that very high amplitude ultrasound would have to be produced. Discussion of the difficulties involved is outside the scope of this paper. However attention should be drawn to a review of the international standards for ultrasound levels [13].

![Graph](image)

Figure 9: The controlled, (left surface) and uncontrolled (right surface) sound field error sensors at 0.9m and 1.0m. The on-axis pressure for the uncontrolled, (dashed line) and controlled, (solid line) sound field.

5 CONCLUSIONS
This paper has reviewed the theoretical and numerical predictions of the output of the parametric array v.s. distance. It was observed that the Berktay approximation over estimates the sound field close to the source and does not hold for distances that would be practical for noise control in air at the 1 kHz. The KZK equations were solved and were shown to predict a sound field that exhibited local maxima close to the source.

Based on Berktay’s approximation the parametric array output v.s the carrier wave pressure was shown to be a strongly frequency dependent function. Very large carrier wave amplitudes are required to created significant levels at the difference frequency. However the 3 dB bandwidth was found to be a fairly constant function of frequency.

The control of a monopole sound field by the parametric array was predicted. It was shown that local zones of quiet with little spill over could be produced at distances greater than the Rayleigh distance. When the sound field close to the source was minimised the spill over was significant.
In conclusion, the parametric array greatest drawback is its low output, but the advantage is the directivity achieved from a compact source.

REFERENCES


