Impedance correction for a branched duct in a thermoacoustic air-conditioner

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ABSTRACT

A thermoacoustic air conditioner that utilises a branched duct network is described in this paper. It was found that the standard four-pole methods to describe the acoustics in the branched duct did not match the predictions from finite element analyses. Additional correction terms to describe lumped inertances at the branch connection are required to properly describe the acoustic behaviour of the system. Branched ducts of square, rectangular, triangular, and semi-circular shapes are considered, and the corresponding terms for the lumped impedances are given.

INTRODUCTION

The focus of this paper is the development of a mathematical model to describe an end-correction factor for a branched duct in a thermoacoustic air-conditioning system proposed by Swift (2002, p195) shown in Figure 1. The system is a novel implementation of a flow-through style thermoacoustic system and comprises two resonating closed-open tubes placed one on top of the other. The purpose for this concept air-conditioner is to eliminate the use of all heat-exchangers, thereby reducing manufacturing costs. For more details about the thermoacoustic operation of the air-conditioner, the reader is referred to Swift (2002). The focus of the work presented here is the acoustics within the ducts.

An acoustic driver on the left hand end of the duct provides an oscillating harmonic acoustic pressure and is superimposed with a steady flow as a cross-flow, from two fan-forced transverse inlets that draw ambient air into the two resonating close-open tubes. Fans on the inlet and exhaust tubes attached to the resonator tube are adjusted in such a way that there is a mean flow out the end of the resonator tube, leading to a flux of cool air, after having been chilled as the air passes through the stack. The acoustic driver placed in the mid-wall acts as an acoustic dipole, as shown in the graphs on the right of Figure 1, as the pressure in the upper tube is 180° out of phase with the pressure in the lower tube. The midwall is ¾ of a wavelength of the acoustic drive frequency, which ensures that the cross-flow streams and cool exit stream are positioned at pressure nodes, to minimise sound transmitted from the cold air duct. Four transverse tubes (two inlet, and two outlet) are connected to the resonator tube at a pressure node, permitting ambient air to enter the resonator tube and exhaust the resonator tube at an elevated temperature. These four tubes are positioned at the pressure node to prevent the high-amplitude sound in the resonant cylinder from ‘leaking’ into the surrounds. Hence, it is important that these tubes are placed precisely at the pressure nodes, and the estimation of the location of this pressure node is part of the focus of this study. It should be noted that during operation of the thermoacoustic air-conditioner, a temperature gradient will exist along the axis of the device. Hence the speed of sound and the corresponding wavelength will also change, thereby altering the location of the pressure node within the cylinder. However, for the work presented here, it is assumed that the speed of sound of the air within the cylinder is constant.

The acoustics of branched ducts is important in applications such as air-conditioning, car manifolds, car exhaust systems, lung function tests, and more recently, using acoustic methods to identify the structures of cave networks. Standard textbooks, such as Blackstock (2000), and Kinsler et al (2000, p290), consider a branched network shown in Figure 2, where an upstream duct of impedance Z1 is bifurcated into two branch impedances Z2 and Z3.

The combined acoustic impedance is calculated as

\[(1/Zp1) = (1/Z2) + (1/Z3)\]  

(1)

The same result is utilised in four-pole transmission line theory of acoustic ducts (Munjal 1987). Numerous researchers have omitted the impedance at the joint of the branches [Abom 1988, Desantes 2005, Griffin 2000, Boonen 2002, Munjal 1988, Selamet 1997, Blackstock 2000, Kinsler 2000]. It has been found from the work presented here, that an additional ‘end-correction’ term is required to accurately model the acoustics within the duct network.

Tang (2004) discusses the correction factors for T-branch pipe networks as an additional length correction at the junction. Tang used a finite element method programmed in Mat-
The acoustic impedance of the branched duct considered here is examined using four-pole transmission line theory and is discussed in the next section.

ACOUSTIC MODEL OF A BRANCHED DUCT

The thermoacoustic system proposed by Swift (2002) can be considered as a branched duct shown in Figure 3, and the equivalent circuit of the duct is shown in Figure 4.

It is possible to write down four-pole expressions for the pressure and velocity in each of the three ducts as (Beranek & Ver 1992)

\[
\begin{bmatrix}
P_A \\
\rho \cdot S_A \cdot U_A \\
P_B \\
\rho \cdot S_B \cdot U_B \\
P_C \\
\rho \cdot S_C \cdot U_C
\end{bmatrix} =
\begin{bmatrix}
T_{A11} & T_{A12} \\
T_{A21} & T_{A22} \\
T_{B11} & T_{B12} \\
T_{B21} & T_{B22} \\
T_{E11} & T_{E12} \\
T_{E21} & T_{E22}
\end{bmatrix}
\begin{bmatrix}
P_{AC} \\
V_{AC} \\
P_{BC} \\
V_{BC} \\
P_{E} \\
V_{E}
\end{bmatrix}
\]

(4)

(5)

(6)

where, \( \rho \) is the density of air, \( S \) is the cross-sectional area of the duct, \( U \) is the gas particle velocity, the volume velocity is given by \( V_d \) (similar expressions can be written for \( V_B \) as well).

\[
\begin{bmatrix}
P_{AC} \\
V_{AC} \\
P_{BC} \\
V_{BC} \\
P_{E} \\
V_{E}
\end{bmatrix} =
\begin{bmatrix}
W_{A11} & W_{A12} \\
W_{A21} & W_{A22} \\
W_{B11} & W_{B12} \\
W_{B21} & W_{B22} \\
W_{E11} & W_{E12} \\
W_{E21} & W_{E22}
\end{bmatrix}
\begin{bmatrix}
P_A \\
\rho \cdot S_A \cdot U_A \\
P_B \\
\rho \cdot S_B \cdot U_B \\
P_C \\
\rho \cdot S_C \cdot U_C
\end{bmatrix}
\]

(7)

(8)

(9)

(10)

The impedance \( Z \) is the equivalent circuit of the duct shown in Figure 4.

\[
P = Z \cdot V
\]

(11)

(12)

(13)

The equivalent radius for a non-circular tube is given by Beranek and Ver (1992) as \( a = 2S / \pi \).
\[ Z = -j \frac{\cot(k \Delta l)}{S} \]  

(14)

where \( \Delta l \) is an additional length of duct to account for the branch. Pierce (1991, p348) discusses effective neck lengths for Helmholtz resonators, which is an additional length \( \Delta l \) that is required to be added to the actual length of a Helmholtz resonator neck to account for the added inerterance of entrained fluid. He suggests that the opening of the neck can be considered as pipes with a flanged termination, in which case the additional neck length is \( \Delta l = 0.6 \alpha a \). It seems reasonable that these two extremes bound the problem considered here. It was found from the comparison of the results from the ANSYS analysis and the theoretical four-pole model that a suitable end correction factor for this particular design of branched duct is

\[ \Delta l = 0.75a \]  

(15)

The effect of the branch impedance could be considered as a distance, effectively lengthening the upper and lower sections of the ducts. A transmission matrix \( T_C \) can be written as

\[
T_C = \begin{pmatrix}
1 & Z_s \\
0 & 1
\end{pmatrix}
\]  

(16)

and can be used to modify the response in each duct. Hence the equations for the upper duct \( A \) can be written as

\[
\begin{pmatrix}
P_A \\
\rho S_A U_A
\end{pmatrix} = \begin{pmatrix}
T_{A11} & T_{A12} \\
T_{A21} & T_{A22}
\end{pmatrix} \begin{pmatrix}
1 & 1 \\
0 & 1
\end{pmatrix} \begin{pmatrix}
P_{AC} \\
\rho S_{AC} U_{AC}
\end{pmatrix}
\]  

(17)

and similarly for the lower duct \( B \). Equation (17) can be used to derive a modified four-pole matrix \( T_A \) by using the results in Eqs. (11)-(13) and substituting the known boundary conditions for the volume velocities, the pressures within the duct can be calculated.

**RESULTS**

A parametric finite element model of the duct networks considered here was constructed using the ANSYS software package. Ducts with square, rectangular, isosceles triangular and semi-circular cross sections were examined. The ‘duct height’ \( H \) was a parameter in the model and could be varied so that analyses of ducts of various sizes could be investigated.

Figure 5 shows ANSYS wire-frame models of the square, semi-circular, and triangular duct.

Figure 6 shows the sound pressure level (SPL), with the contours indicated SPL in dB inside the duct for a square duct with \( H = 0.2 m \). The white contours indicate the loudest SPL and the black contours indicate the lowest SPL. Note that SPLs lower than the minimum contour level are shown as grey! It can be seen that the maximum sound pressure level is at the locations of the acoustic source, labelled as ‘MX’. Where the upper and lower ducts join at 3/4\( \lambda \), the sound fields cancel and it can be seen that the SPL decays with the minimum SPL labelled ‘MN’ at the end of duct where the cooled air would exit.

Figure 7 shows the results from predictions using ANSYS of the normalised SPL versus the normalised axial distance in a square duct for varying duct heights. The normalised SPL is calculated as the difference in the maximum SPL in the duct and the SPL at an axial position in the duct. The normalised axial position is the axial position along the duct divided by the wavelength. It can be seen that the location of the minimum SPL varies around 1/4\( \lambda \) as the duct height changes.
were not considered in the analyses conducted here, and it should be noted that viscous losses at the pipe openings could be another correction factor that should be included in the theoretical model. It is difficult to model viscous losses in acoustic analyses in ANSYS, hence it was not possible to deduce suitable correction factors for viscous losses.

### CONCLUSIONS

A theoretical model for the acoustics of a branched duct used in a thermoacoustic air-conditioner is presented here. The theoretical model was developed using four-pole transmission line theory. Standard acoustic text books that consider the acoustics of branched ducts require pressure continuity and volume velocity continuity across the joint of the branch, however it was found that does not accurately model the acoustics of the duct. After conducting numerous finite element analysis of the branched duct, it was found that the location of the pressure node within the duct varies with cross sectional area / duct height, whereas conventional four-pole analyses would find that the location is independent of duct height. The results presented here highlight the need to include an additional inerter term at the junction of branched ducts. This additional inerter term was modelled as an end-correction factor for a pipe, and it was found that an end correction length of $\Delta = 0.75a$ resulted in a favourable comparison of theoretical predictions and predictions using ANSYS.

### REFERENCES


Beranek, L.L. and Ver, I.L., 1992, Noise and vibration control engineering, principles and applications, John Wiley & Sons,


A. Selamet and V. Easwaran, Modified Herschel-Quincke tube: attenuation and resonance for n-duct configuration,

<table>
<thead>
<tr>
<th>Table 1. Values used for radius and area</th>
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<tr>
<td>Radius</td>
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<td>Semi-circle</td>
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<td>Triangle</td>
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<td>Rectangle</td>
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Figure 8 shows the comparison of the position of the minimum SPL in the duct versus the duct height, predicted using the ANSYS software and shown as the point markers, and predicted using the theoretical model and shown as the lines. Note that the term ‘duct height’ $H$ has different meanings for each duct shape, and the relationship between the duct height and area is described in Table 1. It can be seen that all the results for the ANSYS predictions are very similar, and likewise the results for the theoretical predictions are also very similar. An additional line is drawn in Figure 8 for the theoretical prediction of the position of the minimum SPL in the duct if the correction factor proposed here had not been applied, and is a constant at $1/4\lambda$. The theoretical results presented in Figure 8 can be normalised by the wavelength and are shown in Figure 9.

Figure 8: Comparison of ANSYS and theoretical results. The markers indicate predictions using ANSYS and the lines indicate theoretical predictions.

Figure 9: Normalised results from Figure 8.

It should be noted that viscous losses at the pipe openings were not considered in the analyses conducted here, and could be another correction factor that should be included in the theoretical model.