Implementation of active noise control
in a multi-modal spray dryer exhaust stack

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Abstract

Tonal noise emitted from large-diameter spray dryer exhaust stacks used in the dairy industry can give rise to complaints from nearby communities. In many cases, the tone at the fan blade passing frequency is characterized by a frequency above the first mode cut on frequency of the exhaust stack and both its amplitude and the frequency are time varying. The variation in amplitude is a result of turbulence and temperature variations in the duct which cause angular variations in the nodal plane of modes with diametrical nodes. This in turn results in large fluctuations in sound pressure with time at any specified location in the duct, thus presenting a significant challenge for an ANC system with fixed control source and error sensor locations. In many food processing industries, the use of sound absorptive materials in silencers is not acceptable and, particularly when the fan speed is variable, it is difficult to achieve an acceptable passive solution at a reasonable cost. Here, the design and implementation of an active noise control system for tonal noise propagating above the cut-on frequency of the first higher order mode in large size cylindrical industrial exhaust stack is discussed, where the frequency and amplitude vary significantly and relatively rapidly with time. Physical system design principles and control algorithm optimization for a practical active noise control system are presented. Finally, real time control results which were achieved by a prototype installation on a large-diameter, in-service exhaust stack are given. Significant noise reductions were achieved in the community.

1. Introduction

Traditional means of controlling noise propagating in ducts involves implementation of passive mufflers, which for large-diameter exhaust stacks are expensive and often only partially effective for low frequency tonal noise which varies in frequency over time. Although active noise control may appear to be a promising alternative due to its successful application in many instances [1-4], the large diameter of the stack in the case considered here and the resulting propagation of higher order modes at the frequency to be controlled, mean that the application of active noise control is particularly challenging. The high level of turbulence and large temperature variations in the stack further complicate the problem by causing fluctuations in the angular orientation of the nodal plane of higher order modes with diametrical nodes. This fluctuation, in turn, leads to large fluctuations in the sound

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pressure at any given location in the duct, which causes immense difficulties for active noise control systems. The only previously reported successful installation of an ANC system to control a higher order mode in a duct was implemented by Digisonix [5], but they had the luxury of low turbulence levels and relatively uniform temperature in the duct, so sound levels at any particular location did not vary too rapidly.

Zander and Hansen [1] developed a theoretical model for analyzing the effectiveness of active noise cancellation on reducing the energy transmission associated with higher order acoustic modes. In their work, the effects of source size, location and strength on the control performance were evaluated and the results showed that total sound power reduction was dependent on the relative location of the control sources. Alternative arrangements of error sensors for the active control of tonal noise radiated from turbofan engines was investigated by Joseph et al. [2,3]. Chan and Elliott [4] discussed the performance of active noise control using remote sensors for canceling higher order acoustic duct modes.

More recently, several publications [6-8] have discussed various aspects of the practical implementation of active control of tonal noise propagating in a large-diameter, in-service exhaust stack. It was demonstrated experimentally that it was possible to actively control tonal noise propagating as higher order acoustic modes in a cylindrical duct with optimally located error sensors and control sources [6,7]. The robust design of the electronic controller for such an application has also been discussed [8]. Here, the physical system design and the control algorithm optimization for the ANC system are discussed in detail. Results from real time control on in-service equipment are also reported.

2. System design considerations for active control of noise in cylindrical duct

2.1 Sound propagating in cylindrical ducts

The sound field in a cylindrical duct with uniform axial mean flow can be expressed in the frequency domain as [9]:

\[
p(\theta, r, z, f) = \sum_m \sum_n P_{mn}^+ (f) \Psi_{mn} (\theta, r)e^{-jk_{mn}z} + P_{mn}^- (f) \Psi_{mn} (\theta, r)e^{jk_{mn}z}
\]

(1)

where \( P_{mn}^+ \) and \( P_{mn}^- \) are modal amplitudes of the sound pressure corresponding to a mode propagating in the forward and reflected directions respectively, \( k_{mn}^+ \) and \( k_{mn}^- \) are the axial modal wave numbers in the forward and reflected directions respectively, \( \Psi_{mn} \) is the modal shape function (eigenfunction) corresponding to \( mn^{th} \) mode, \( m \) and \( n \) are mode orders in the circumferential and radial directions respectively, \( j \) is the square root of \(-1\), \( f \) is the frequency and \((\theta, r, z)\) are the coordinates defined in Fig. 1.
The axial wavenumber $k_{mn}$ in Equation (1) can be expressed as [9,10]

$$k_{mn}^\pm = \left( \pm \alpha_{mn} - M \right) \frac{\omega}{c_0} \left( 1 - M^2 \right)^{1/2}$$

where $\omega$ is the radian frequency, $c_0$ is the sound speed in the fluid, $M$ is the Mach number of the flow and the parameter $\alpha_{mn}$ is given by

$$\alpha_{mn} = \left[ 1 - \left( \kappa_{mn} / k \right)^2 \left( 1 - M^2 \right) \right]^{1/2}$$

In Equation (3), $\kappa_{mn}$ is an eigenvalue characterized by the duct cross-section corresponding to the $mn$th mode and it can be obtained from:

$$J_m' (\kappa_{mn} a) = 0$$

where $a$ is the radius of the duct, $J_m(x)$ is a Bessel function of the first kind of order $m$ and the prime denotes the first order derivative with respect to $x$.

As is well known, for subsonic modes, the cut-on frequency of mode $mn$ for a circular duct with rigid internal surfaces is given by [11],

$$f_{mn,c} = \frac{c_0}{2\pi} \kappa_{mn} \left( 1 - M^2 \right)^{1/2}$$

and the modal shape functions, $\Psi_{mn}$, can be expressed as [11]:

$$\Psi_{mn}^{\pm} = \frac{\cos(m\theta)}{\sin(m\theta)} J_m (\kappa_{mn} r)$$

Equation (6) indicates that for each mode with $m$ different from zero, there is a pair of modes (degenerate): one for $\Psi_{mn}^c = \cos(m\theta) J_m (\kappa_{mn} r)$ and the other for $\Psi_{mn}^s = \sin(m\theta) J_m (\kappa_{mn} r)$.

For higher order acoustic modes propagating in cylindrical ducts, the modes can be classified as circumferential - spiral modes $(m \neq 0, n=0)$ or radial - doughnut modes $(n \neq 0, m=0)$ or the combination of these modes $(m \neq 0, n \neq 0)$. For the spiral modes or the combining modes, the number of threads is equal to the circumferential mode index $m$ and the pitch of thread is equal to the axial wavelength [10]
as illustrated in Fig. 2. If the frequency of interest reduces toward the cut-on frequency the axial wavelength decreases and the spiral wave will be spinning at right angles to the axis of the duct.

![Fig. 2 Spiral (spinning) mode propagation above the duct cut-on frequency](image)

2.2. Effect of the error sensor locations on the control performance in the far field

For spiral wave propagation in a cylindrical duct, the location of the nodal line is a function of frequency. In addition, any small temperature variations (which are exacerbated by turbulence) have a large effect on the location of the nodal line for any given axial location, which in turn causes large sound pressure fluctuations at any fixed location on the circumference. However, the optimal locations shift with time so the best results are achieved by using more than the theoretical optimum number of control source and error sensors.

In this section, the effect of error sensor locations for control of a stationary (1,0) spiral acoustic mode in a cylindrical duct is demonstrated experimentally.

**Control setup**

For evaluation of the control system prior to its installation on in-service equipment, a steel cylindrical duct of diameter 0.8 m and length 9 m, representative of a half size in-service spray dryer exhaust duct, was constructed. A radial type fan with 10 blades, powered by a 55 kW motor, was driven at a speed to produce a BPF of 368 Hz (twice that of the full scale spray dryer exhaust). The flow velocity in the duct was measured using a pitot tube. The Mach number of the flow was then calculated as 0.08 and the first cut-on frequency of the (1,0) mode was 251.2 Hz. The BPF of the fan was therefore above the cut on frequency of first higher order mode so that the propagation energy was not simply contained in a plane wave but also involved two higher order, degenerate duct modes.

Generally, the control source numbers and locations will determine the achievable noise reduction, assuming that an ideal error sensor arrangement exists. The optimization of the error sensor arrangement is directed at achieving the maximum reduction in the cost function set by the control source arrangement. In other words, the maximum achievable noise reduction is determined by the control source arrangement first and the error sensor arrangement second. Here, to control tonal noise propagating in the duct above the cut-on frequency of the (1,0) mode, three speakers (the theoretical minimum required number) are optimally located in the duct wall as control sources [12]. Five microphones are mounted downstream in the duct. For active control, three of the five microphones are used as error sensors and the control results achieved using different combinations of the error sensors are evaluated in the far-field. Fig. 3 shows the arrangement of the control speakers and the microphones on the duct wall.
To perform real time control, a multi-channel feedforward controller with 3 error input channels and 3 control output channels was used. The error signals were measured using electret microphones mounted flush with the interior duct wall and these signals were then fed through pre-amplifiers into the controller. The controller outputs were amplified to drive the loudspeaker control sources. A tachometer signal was used as a reference signal for feedforward control, where a timing disk mounted on the fan shaft was used to generate an impulse signal. The impulse signal was then sent through a timing sensor into a programmable tacho signal generator, which converted the impulse signal into a sine wave signal at the BPF. Fig. 4 shows the test configuration.

![Diagram of real time control setup](image)

**Fig. 4. Real time control setup for the test rig**

**Control results** To evaluate the effect of the error sensor positions on the control performance, testing was performed using two error sensor configurations, as shown in Table 1 and with reference to Fig. 3.
Here in the configuration 1 the error sensor locations were chosen to minimize the higher order modes, in contrast to the configuration 2 in which error sensor locations were arbitrarily chosen, with no consideration for controlling the higher order modes [12].

Tables 2 and 3 show the noise levels at the error sensors before and after implementing ANC with the error sensor configuration 1 and configuration 2 respectively. In both tables it is shown, that as expected, the noise levels at the error sensors were mostly reduced to the background noise level (levels at adjacent frequencies) at the frequency of interest (BPF) regardless to the locations of the error sensors.

Table 2 Noise reductions at error sensors using configuration 1

<table>
<thead>
<tr>
<th>Location</th>
<th>Before ANC (dB)</th>
<th>After ANC (dB)</th>
<th>Noise reduction (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error 1</td>
<td>14.7</td>
<td>4.0</td>
<td>10.7</td>
</tr>
<tr>
<td>Error 2</td>
<td>8.3</td>
<td>3.0</td>
<td>5.3</td>
</tr>
<tr>
<td>Error 3</td>
<td>7.5</td>
<td>2.6</td>
<td>4.9</td>
</tr>
</tbody>
</table>

Table 3 Noise reductions at error sensors using configuration 2

<table>
<thead>
<tr>
<th>Location</th>
<th>Before ANC (dB)</th>
<th>After ANC (dB)</th>
<th>Noise reduction (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error 1</td>
<td>14.8</td>
<td>5.4</td>
<td>9.4</td>
</tr>
<tr>
<td>Error 4</td>
<td>12.2</td>
<td>4.0</td>
<td>8.2</td>
</tr>
<tr>
<td>Error 5</td>
<td>12.5</td>
<td>2.1</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Figures 5 and 6 show the noise spectra at the far-field measurement locations before and after implementing ANC using error sensor configurations 1 and 2 respectively. Figure 5 demonstrates that the noise levels at two far-field measurement locations were reduced mostly to the background noise levels using the error sensor configuration 1. However, Fig. 6 provides evidence that when consideration is not given to the optimal location of the error sensors, a significant level of sound still propagates from the exhaust outlet to the community despite, local cancellation at the error sensor locations. The tonal peak remains dominant with less than 5 dB reduction at the monitoring locations in the far field. In other words, propagation of the higher order mode energy is not minimized and provides the mechanism for sound propagation through the duct and out into the far field. Here it should be emphasized that the sound pressure reduction measured at two discrete points in the far field does not necessarily represent the overall sound power reduction. However, it is expected that these measurements are representative of what would be expected as far field pressure reductions. It is interesting to note the increase in level at some frequencies other than the control frequency. This is not a result of the control system as the frequencies are unrelated to the reference signal. It is possibly produced by turbulence induced noise, which varies randomly with time. As can be seen in later figures, the apparent increase at these frequencies does not occur consistently.
It was shown experimentally that active noise control can always provide cancellation at the error sensor locations in a fully determined system, such as the control system configuration presented here; that is, a system with 3 error sensors and 3 control sources. However, this phenomenon does not imply that the noise will be globally reduced, especially external to the duct. To achieve a substantial reduction in the tonal noise emitted from the outlet of a duct with higher order acoustic modes propagating, both the control source locations and error sensor locations must be optimised. Where optimisation is not possible due to variation with time of the higher order mode nodal planes, it is necessary to use more control sources and error sensors than theoretically require.
2.3. **Some considerations for robust design of the control algorithm**

One of the problems associated with the tonal sound field that is to be controlled in the dairy factory, is its rapidly varying nature, both in magnitude and phase relative to a fixed location. Fig. 7 shows a measurement of amplitude and phase as a function of time at 184 Hz which corresponded to the fan blade passing frequency. The measurement was taken at a location in the in-service exhaust stack and shows a large variation with time at a specific location, a phenomenon not observed in the half scale experimental duct.

![Figure 7](image)

**Fig. 7. Variation in the magnitude and phase of the fan noise at the BPF in the in-service exhaust stack**

Fig. 7 shows that the magnitude and phase of the noise spectrum at the BPF varied randomly and the variations of magnitude and phase are as large as 6 dB and 40 degrees respectively. The variation of the sound field may be caused by small changes in temperature, wind flow across the duct outlet and production load on the exhaust fan.

Another serious issue was the rapidly varying amplitude and phase of the transfer function as a function of frequency measured between a microphone and a control speaker in the exhaust stack as can be seen by inspection of Fig. 8.

![Figure 8](image)

**Fig. 8. Spectrum of a cancellation path transfer function in the duct**
From the figure, it can be seen that the phase of the cancellation path transfer function at the BPF will change by a very large amount as a result of only small changes in the fan speed. By observing the measured data (not presented here) it was found that the cancellation path transfer functions at the BPF vary considerably with time in a similar way as the primary sound. The amplitude variation of the cancellation path transfer function can be as large as 6dB and the phase variation can be as large as 60 degrees. It was also observed from the measured data that when the fan speed changes, the amplitude and phase of the cancellation path transfer function at the BPF frequency changes by a very large amount.

The robustness of the control algorithm under these sound field uncertainties, which include plant cancellation path and primary disturbance uncertainties was investigated numerically. To make this possible, the primary noise field at 8 error microphones and the cancellation path transfer functions from 4 control speakers to the 8 error microphones were measured. All speakers and microphones were mounted on the in-service exhaust stack [13].

With reference to previous work [14,15], two kinds of uncertainties were simulated with the purpose of representing the effects of either or both the measurement error and turbulence.

1. **Structured uncertainty** - the primary sound field and cancellation path transfer functions change from a nominal state to a new state due to a physical change in the system.

2. **Unstructured uncertainty** - the variation in the primary sound or in the cancellation path transfer functions are assumed to be independent of each other and are simulated by adding a particular magnitude of random amplitude and phase.

Based on measured data, the amplitude variation for both kinds of uncertainties was limited to 6dB and the phase variation is limited to 60 degrees. The unstructured uncertainty in amplitude means a random amplitude change within $\pm 3$dB, and the unstructured uncertainty in phase means a random phase change within $\pm 30$ degrees. The control algorithm was then evaluated using these configurations in the frequency domain and time domain respectively.

**Frequency domain.** At a single frequency, the complex error vector $e$ is derived from the sum of the primary sound vector $p$ and the control sound vector. The control sound vector is generated by a complex control force vector $f$ via the complex matrix $Z$, representing the cancellation path transfer function at the same frequency. Therefore,

\[
e(k) = p(k) + Zf(k) \quad (7)
\]

where $k$ is the iteration number, and the complex multiple error LMS algorithm with leakage is given by [16-21]

\[
f(k+1) = (1 - 2\mu\alpha)f(k) - 2\mu Z_0^H e(k) \quad (8)
\]

where $\alpha$ is the leakage coefficient, $\mu$ is the convergence coefficient and $Z_0$ is the estimated nominal cancellation path transfer function.
Fig. 9 shows the calculated noise reduction averaged over the eight error sensors (with four control sources operating) without uncertainty in either the primary sound or the cancellation path. Fig. 10 shows the results for the same configuration with structured uncertainty in the primary sound and the cancellation path respectively.

Fig. 9. Estimated noise reduction for different convergence and leakage coefficients, with no uncertainty in either the primary sound or the cancellation path.
Results in Figs. 9 and 10 demonstrate that the variation of the primary sound does not significantly affect the convergence properties of the system; however, the control outputs need to be adapted to match the change in the primary sound. For the case of structured uncertainty in the cancellation path, a small convergence coefficient is necessary to keep the system stable under all situations. When there are structured uncertainties in both the cancellation path and the primary sound, simulations show similar results to the case of changes only in the cancellation path. However, these results are not shown here.

Fig. 11 shows the simulation results obtained when there is unstructured uncertainty in the primary sound amplitude. It can be also seen that a small convergence coefficient can lead to better convergence and hence better performance. If the uncertainty is in the phase of the primary sound, rather than the amplitude, similar results are found in the simulations, which are not shown here.

Fig. 11. The noise reduction with unstructured uncertainty in the primary sound amplitude, (a) $\mu = 0.2$ with average NR=6.7dB, (b) $\mu = 0.05$ with average NR=8.1dB
Fig. 12 shows the simulation results with unstructured uncertainty in the cancellation path transfer function. In the simulations, it was found that if the uncertainty is in the magnitude of the cancellation path transfer function, the convergence coefficient needs to be very small to keep the system stable. The effect of the uncertainty in the cancellation path phase is even worse, as can be seen by comparing Figs. 12(a) and (c). Figs. 12(b) and (d) show the effects of applying the leakage for the cases in (a) and (c) respectively. It can be seen that a little leakage can help to stabilize the system and hence increase its performance, although normally leakage is used to stabilize the system at the cost of performance. Similar remarks were made in reference [15], where it was found that most of the degradation of the performance of their ANC system was due to the influence of the original unstable system and the use of leakage was found to be essential to obtain some reduction.

**Fig 12.** Noise reduction with unstructured uncertainty in the cancellation path (a) in amplitude, $\alpha = 0$ and $\mu = 0.1$, NR=0.63, (b) in amplitude, $\alpha = 0.05$ and $\mu = 0.1$, NR=4.4dB (c) in phase, $\alpha = 0$ and $\mu = 0.01$, NR keeps getting worse (d) in phase, $\alpha = 0.1$ and $\mu = 0.01$, NR=3.8

**Time domain simulations.** For an ANC system with $K$ control sources, $M$ error sensors and a sampling rate of 2000Hz, at sample $n$, the $m^{th}$ error signal $e_m(n)$ comes from the sum of the primary sound $p_m(n)$ and the control sound $s_m(n)$ at that error sensor. The control sound is generated by the linear convolution of the control signal $y_k(n)$ ($k=1, \ldots, K$) and the transfer function from the $k^{th}$ control source to the $m^{th}$ error sensor. Therefore,
\[ e_m(n) = p_m(n) + \sum_{k=1}^{K} Z_{km} \ast y_k(n) \]
\[ y_k(n) = w_k(n) \ast r(n) \]

where \( w_k(n) \) are the weights of the \( k^{th} \) control filter, \( r(n) \) is the reference signal, and the multiple error filtered-X LMS algorithm with leakage is given by [17-20]

\[ w_k(n+1) = (1 - 2\mu\alpha)w_k(n) - 2\mu \sum_{m=1}^{M} (Z_{km} \ast r(n))e_m(n) \]

where \( Z_{km} \) is the estimated nominal impulse response of the transfer function from the \( k^{th} \) control source to the \( m^{th} \) error sensor.

Table 4 shows the maximum convergence coefficient (without leakage) and the number of samples needed for the fastest possible convergence of a stable ANC system when there is structured uncertainty in the system, which just happens once at the 8000\(^{th} \) sample. The sample number shown in Table 4 does not include the first 8000 samples before the structured uncertainty happens, so the third column indicates the tracking speed of the system.

<table>
<thead>
<tr>
<th>Case</th>
<th>Maximum ( \mu )</th>
<th>Sample number</th>
<th>NR(dB)</th>
<th>Uncertainty description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.075</td>
<td>n/a</td>
<td>9.0</td>
<td>No uncertainty</td>
</tr>
<tr>
<td>2</td>
<td>0.075</td>
<td>3.4e+3</td>
<td>9.0</td>
<td>6dB in primary amplitude</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
<td>7.2e+3</td>
<td>9.0</td>
<td>60 degrees in primary phase</td>
</tr>
<tr>
<td>4</td>
<td>0.075</td>
<td>7.5e+3</td>
<td>9.0</td>
<td>Both cases 2 and 3</td>
</tr>
<tr>
<td>5</td>
<td>0.039</td>
<td>4.3e+3</td>
<td>9.0</td>
<td>6dB in cancellation path TFs' magnitude</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>( \infty )</td>
<td>0.0</td>
<td>60 degrees in cancellation path TFs' phase</td>
</tr>
</tbody>
</table>

For case 6 in Table 4, when there is a phase change of 60 degrees in the cancellation path, the system does not converge with even a very small convergence coefficient. A 60 degrees difference between the cancellation path transfer functions and their models is too large to maintain system stability in this case. However, a small leakage coefficient is able to stabilize the system. For example, with \( \mu = 0.005, \alpha = 0.18 \), the system can converge to a stable value with an NR of 1.2dB following introduction of the 60 degree uncertainty in the cancellation path transfer function. Simulations were also carried out for a cancellation path phase changes of 50, 40, 30, -30, -60 degrees. For these situations, it is still possible to have a noise reduction of 9.0dB with different convergence coefficients.

Table 5 shows similar results to Table 4 when unstructured uncertainties are introduced in the system. When the unstructured uncertainty is in the primary sound, it can be shown that a smaller convergence coefficient normally results better noise reduction, but a slower tracking ability. For example, for case 2 in Table 5, if the convergence coefficient is 0.025, then the final noise reduction can be as large as 8.0dB, which occurs after the 15000\(^{th} \) sample. Fig. 13 shows the results when there are unstructured uncertainties in both the amplitude and phase of the primary sound, where the convergence coefficient is 0.025, much smaller than the limit in Table 5.
Table 5. The convergence behaviour of the ANC system with unstructured uncertainty

<table>
<thead>
<tr>
<th>Case</th>
<th>Maximum $\mu$</th>
<th>Sample number</th>
<th>NR(dB)</th>
<th>Uncertainty description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.075</td>
<td>4.7e+3</td>
<td>9.0</td>
<td>no uncertainty</td>
</tr>
<tr>
<td>2</td>
<td>0.075</td>
<td>5.8e+3</td>
<td>5.0</td>
<td>$\pm$3dB in primary amplitude</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
<td>1.7e+3</td>
<td>3.8</td>
<td>$\pm$30 degrees in primary phase</td>
</tr>
<tr>
<td>4</td>
<td>0.075</td>
<td>2.8e+3</td>
<td>1.7</td>
<td>Both cases 2 and 3</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>$\infty$</td>
<td>0.0</td>
<td>$\pm$3dB in cancellation path TFs' magnitude</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>$\infty$</td>
<td>0.0</td>
<td>$\pm$30 degrees in cancellation path TFs' phase</td>
</tr>
</tbody>
</table>

Fig. 13. Case 4 in Table 2, $\mu = 0.025$, $\alpha = 0$, unstructured uncertainty in the primary sound, average NR = 7.9dB

It should be noted that the total error levels in the figures represent the average of the total squared pressures at the error sensors normalized by the sum of squared primary signals at the error sensors.

The zero maximum $\mu$ in cases 5 and 6 in Table 5 means that for unstructured uncertainty in the cancellation path, even for a very small convergence coefficient, the system cannot remain stable without using leakage. Fig. 14 shows the simulation results when the uncertainty is in the magnitude of the cancellation path transfer function. Uncertainty in the phase of the cancellation path transfer function gives similar results. However, if the uncertainty is not as large as that in Table 5, a small convergence coefficient and leakage coefficient should be able to keep the system stable. Fig. 15 shows an example.
Fig. 14. Case 5 in Table 3, unstructured uncertainty in the cancellation path magnitude (a) $\mu = 0.005$, $\alpha = 0.05$, average NR = 4.2dB (b) $\mu = 0.005$, $\alpha = 0.00$, showing the total error slowly reducing

Fig. 15. Unstructured uncertainty in cancellation path ($\pm0.5$dB in magnitude, 5 degrees in phase) (a) $\mu = 0.05$, $\alpha = 0$, average NR = 7.3dB (b) $\mu = 0.025$, $\alpha = 0$, average NR = 8.2dB

**Summary for control algorithm considerations**  
From the preceding simulations, it can be seen that a number of considerations should be taken into account when designing a robust ANC system.

1. Before applying an active noise control system on a practical problem, the primary noise and the cancellation paths should all be characterized. Then, by using the measured data, the maximum achievable noise reduction and the convergence time can be predicted.

2. By measuring the variation of the primary sound and the cancellation path, the performance of the adaptive system under these situations can be simulated. After extensive simulations, a suitable convergence coefficient and leakage coefficient can be selected which guarantee the stability of the system under most situations.

3. Simulations show that it is apparently better to use the frequency domain algorithm in the system. By comparing the frequency domain and the time domain simulation results, it can be shown that the frequency domain algorithm converges faster than the time domain FXLMS algorithm.
example, when there is no cancellation path delay in the system, about 4000 samples are needed to make the time domain algorithm converge, while in the frequency domain, only 200 iterations are needed. If each iteration needs 11 samples (the period for 180Hz), 200 iterations are just 2200 samples. When there is a cancellation path delay of 30 samples, the time domain algorithm needs 80,000 samples to converge, while the frequency domain only needs about 8200 samples (obtained from \((11+30)\times 200\)). It is also found that for the case discussed here the frequency domain algorithm is more robust to uncertainties in the primary noise and the cancellation path. The disadvantage with the frequency domain algorithm is that more processor power is needed to obtain the frequency components of all input signals and to reconstruct the time domain control signal.

3. **Real time control of tonal noise emitted from a large in-service exhaust stack**

The aim of this work was to reduce noise emitted into the surrounding community from a dairy industry spray dryer stack. The spray dryer exhaust stack has a diameter of 1.6 m and a length of 18 m from the base where an exhaust fan is mounted. Due to production variations, the temperature in the stack varies between 50°C and 80°C and the exhaust fan speed varies between 960 rpm and 1200 rpm corresponding to a BPF variation between 160 Hz and 200 Hz (10-bladed fan). Fig. 16 shows a typical A-weighted noise spectrum measured in the community surrounding the exhaust stack. From the figure it can be seen that the spectrum is characterized by several tonal peaks so that control of these tones would have an important impact on the community noise. The first peak at 184 Hz with an amplitude of 38.3 dB(A) corresponds to the BPF of the exhaust fan of the spray dryer. Other peaks correspond to other equipment.

![Fig. 16. A noise spectrum measured in the community](image)

In this section the active control of the tone at the BPF of the exhaust fan, which is more than 15 dB above the noise levels at adjacent frequencies, is discussed.

3.1. **Control system design strategy**

**Physical system consideration.** Corresponding to the stack diameter of 1.6 m, the first and second cut-on frequencies of the exhaust stack are 135.5 Hz and 224.5 Hz at 60°C respectively. The range of BPFs of the exhaust fan between 160 Hz and 200 Hz are therefore above the cut on frequency of the
first (degenerate) higher order mode so that propagation was not simply by a plane wave but also via two higher order duct modes (with diametral modes). In general, the optimal locations and numbers of the error sensors and the control sources depend on the sound field distribution in the stack, so it is hard to find a configuration that is suitable for optimally controlling the sound field over the frequency range between 160 Hz and 200 Hz. Past experience on a half scale model indicated that to achieve good control of noise emitted from the exhaust stack into the community the number of control sources needed to be approximately twice the number of cut-on modes in the duct. Thus in this application it was necessary to use 12 error microphones and 6 control speakers to maintain the robustness and performance of the control system.

**Electronic controller unit.** Real time control on this particular problem was carried out using a multi-channel adaptive feedforward controller (12 input channels and 6 output channels) using an FXLMS algorithm with the parameters optimized for this particular problem [8]. The error signals were measured using calibrated electret microphones which were mounted on the stack wall. These signals were fed through pre-amplifiers and signal conditioning filters into the controller. The control signals were sent through signal reconstruction filters and power amplifiers to the control speakers. A tachometer signal was used as a reference signal for adaptive feedforward control. Fig. 17 shows the system configuration.

![Block diagram of on-site ANC system set-up](image)

In the figure the signal conditioning filter unit contains 12 modules of the band-pass filter with a bandwidth from 140 to 300 Hz and gain adjustor. The high-pass filter is used to filter out the high level noise which is mainly caused by turbulent flow in the low frequency region. The low-pass filter is used as an anti-aliasing filter and the gain adjustor is used for calibration of the microphone. The signal reconstruction filter unit used for the output control signals contains 6 modules of the low-pass filter with a cut-off frequency of 300 Hz.

To be useful for industrial noise control purposes, the control system was designed to be self-reliant. During operation, if some part of system fails (i.e. a control output overflow or an error input overload occurs), the control system will pause rather than stop. In this case, the control system will attempt to recover itself after a recurrence interval. This feature is included to avoid controller failure caused by some ‘unpredictable’ transient disturbance; for instance, an error input overload caused by a transient noise disturbance from another noise source. If the control system is still unstable after a specified number of rescue attempts, it is assumed that this unexpected disturbance is not transient and it does significantly affect the noise field to be controlled. Alternatively, it could indicate that too many
control source or error sensor channels had failed. The controller will then stop. Other features include an automatic start up sequence following a power outage and indicators on the front panel of failed control source and error sensor channels.

3.2. Real time control results
Real time control was performed with the system set-up shown in Fig. 17. To evaluate the control performance surrounding the facility in the community, spectra in the community were measured with and without ANC switched on. When the control performance at the error sensors had stabilised for a particular test, the control system adaptation was turned off and then the noise levels at the evaluation locations in the community were measured. Thirteen locations were selected surrounding the facility to properly evaluate the noise control performance in the community. Fig. 18 shows A-weighted spectra with and without ANC switched on, measured at several locations in the community.

Fig. 18. Noise spectra measured in the community with and without ANC switched on

From Fig. 18 it can be seen that significant reductions in the tonal level at the BPF were achieved at all locations in the community. In fact the results demonstrated that wherever it could be
measured in the community, the tonal noise emitted from the exhaust stack at the fan BPF (above the first mode cut on frequency of the exhaust stack) was reduced almost to the background noise level as a result of ANC. It should be noted that the significant high peak at 216 Hz was due to another noise source in the facility.

4. Summary

The active control of tonal noise varying over time in amplitude and phase and propagating above the cut-on frequency of the first higher order mode in a cylindrical industrial exhaust stack was shown to be feasible provided sufficient in-duct control sources and error sensors were used. System design considerations, such as physical system arrangement and control algorithm optimization, were investigated. The effect of location of the error sensors on the active control of tonal noise, propagating as both plane wave and higher order modes, in a circular half scale test rig was evaluated with optimally located control speakers. To control two cut-on higher order modes in the duct, in addition to the plane wave, 3 control speakers and 3 error microphones were used. It was shown experimentally that active noise control can always provide cancellation at the error sensor locations in this fully determined system. The test results demonstrate that when the error sensors are optimally located, the far field noise is also reduced as a consequence of minimization of the sound field at the error sensor locations. However, minimization of the sound field at indiscriminate error sensor locations may not necessarily reduce the far field noise. This demonstrates the necessity for carefully locating error sensors to ensure control of the higher order modes propagating in the duct, if noise radiated from the duct outlet is to be minimized.

For designing a robust control algorithm for an active noise control system, there are a number of benefits that arise from a knowledge of the primary disturbance and the plant (cancellation path between the control sources and error sensors) to be controlled in practical situations. For example, the upper physical limit of the possible attenuation, the control output power requirement, the optimum control algorithm and its parameters can all be obtained. This has been demonstrated here by simulating the performance of an ANC system on a spray dryer exhaust for various plant and disturbance uncertainties.

Real time control was demonstrated on a large-diameter in-service exhaust stack and the corresponding control performance was evaluated in the community. The results demonstrated that the tonal noise emitted from the exhaust stack at the BPF where it is above the first mode cut on frequency of the exhaust stack was reduced almost to the background noise level using the optimized ANC system, even though the blade passing frequency varied significantly in amplitude, phase and frequency over short periods of time. The current system has now been operational for more than a year and current work is directed at adapting the ANC system to a spray dryer with a wet scrubber and a corresponding very humid environment.
References


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