Development of a Transducer for Active Vibration Isolation Using Translational and Rotational Power Transmission as a Cost Function

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Abstract

When conducting active vibration control experiments it is often necessary to omit the measurements of the contribution of power transmission due to rotational moments because of the lack of suitable transducers. Here, a transducer is described which can be used to measure the translational and rotational power transmission from a source to a receiving structure. A description of the procedure used to calibrate the device is also included. The results from the calibration show that whilst the amplitude of the forces, moments, translational and rotational displacements can be measured accurately, it is the phase accuracy of these measurements that limit the accuracy of measurements of power transmission.

Keywords: Active vibration isolation, structural intensity, force transducer, moment transducer, power transmission

1 INTRODUCTION

The use of vibrational power as a cost function to be minimised for active vibration isolation is considered to be theoretically attractive because this metric captures the kinetic and potential energy flow into or through sub-structures. Hence the minimisation of vibrational power will reduce the vibration levels in the entire structure. However, the experimental measurement of power transmission is fraught with difficulty for numerous practical reasons such as the phase accuracy of transducers and the measurement of rotational power transmission.

Here the development and testing of a transducer which is capable of measuring motion, forces and moments for all six degrees of motion, is described. The transducer is used to measure power transmission along translational and rotational degrees of freedom and consists of an array of strain gauges mounted to an aluminium mandrel to measure forces and moments, and an array of accelerometers to measure the translational and rotational motion of the force transducer.

The motivation for this work was for active vibration isolation experiments to demonstrate some of the practical difficulties associated with the use of power transmission as a cost function and the power circulation or negative power transmission phenomenon. The results from these experiments are described in Howard and Hansen (2006).

The original contributions of the work presented in this paper, which expand on current knowledge are:

- the development of a novel transducer which is used to measure power transmission in both translational and rotational degrees of freedom
- the experimental measurements involving the use of this transducer to measure translational and rotational power transmission from a vibrating mass into a simply supported beam

2 PREVIOUS WORK

The practical measurement of power transmission in vibrating structures can be classed into two categories: those that involve measuring the power transmission between a
source and receiver, or those that involve estimating the vibrational power that was transferred to the receiving structure by measuring the vibration response of the receiver and using an estimate of the damping properties of the receiver, such as obtained from mobility measurements of the receiver.

The following review discusses prior work concerning methods used to measure vibrational power transmission and associated accuracy, the benefit of measuring power transmission compared to point measurements of acceleration, multi-axis force and moment transducers, and experiments that have been conducted that involve the measurements of rotational moments.

2.1 Measurement of Vibrational Power Transmission

The technique for measuring vibrational intensity in beams and plates was introduced by Noiseux (1970). The typical method uses an array of accelerometers to approximate gradients of vibration parameters. This was discussed in detail by Pavić (1976), for one and two dimensional flexural wave harmonic and narrow band fields. Since this development, there have been many experimental studies using the technique.

To measure the power transmission in structures which carry different types of waves, for example, longitudinal and flexural, techniques have been developed that use arrays of accelerometers (Horner and White, 1990; Troshin et al., 1990), or laser transducers (Baker et al., 1990; Fuller et al., 1990; Hayek et al., 1990; Lee et al., 1990; McDevitt et al., 1990).

The ideal method of measuring power transmission from a rigid body into a flexible structure is to measure it using an accelerometer and force transducer at the connection between the two systems. This is not always practical and a high degree of phase accuracy between the transducers is required to obtain an accurate measurement of power transmission. This has led to development of practical techniques that use only acceleration signals. Pinnington (1986) derived and experimentally verified a technique for measuring the power absorbed by finite structures. His technique uses the measured acceleration cross spectral density between any two points on the structure, and an envelope function which passes through the peaks in the transfer functions between these points. The limitations of this technique are:

- if an accelerometer is placed on a nodal point, then no measurement of power can be made;
- the measurement of power is inaccurate at frequencies between the resonance peaks on the acceleration cross spectral density;
- the measurement of power is inaccurate for structures with high modal density (e.g. cylinders) or heavy damping; and
- no measure of power transmission can be made for power transmission by moments.

Following this work, Pinnington (Pinnington, 1987) used the technique to measure the power transmitted from an externally excited rigid body (DC electric motor), through four vibration isolators, to a heavily damped beam stiffened plate.

Several researchers (Bardou et al., 1997; Gialamas et al., 1996; Gibbs and Yap, 1998; Howard and Hansen, 1997, 1999; Ji et al., 2003; Koh and White, 1997a,b,c; Mondot and Moorhouse, 1996; Moorhouse, 1999, 2002; Petersson, 1991; Petersson and Gibbs, 1990; Petersson, 1993a,b; Sanderson et al., 1995; Sanderson, 1996; Shepard Jr., 2002; Yap and Gibbs, 1996, 1999a,b) have theoretically demonstrated that power transmission by mo-
ments in vibration isolation systems plays a vital role in vibration transmission even at low frequencies. Yap and Gibbs (1999a,b) experimentally measured the translational and rotational power transmission from a machine into a concrete floor. However, they did not directly measure the rotational power transmission, as they did not have suitable transducers, and instead inferred the contribution of rotational power transmission from measurements of translational mobility. The work presented here expands from Yap and Gibbs’ work, as rotational power transmission is measured directly. Bardou et al. (1997) show the theoretical importance of considering the minimisation of both translational and rotational power transmission. However, there is little experimental work presented in the literature which attempts to actively minimise vibration transmitted by moments. A companion paper (Howard and Hansen, 2006) describes experiments in which the power transmission by rotational moments is actively minimised.

2.2 Phase Accuracy of Measurement Transducers

The problems associated with structural intensity measurements have been investigated by numerous authors (Carroll, 1990; Ohlrich and Nojgaard, 1991; Redman-White, 1983; Taylor, 1990; Troshin et al., 1990) and are mainly concerned with the phase accuracy of the measurement process.

To obtain accurate measurements of power transmission into structures, the phase difference between the measured velocity and force must be determined precisely. Small errors in the phase measurements can lead to enormous errors in the measurement of power. The time averaged vibrational power transmission for tonal vibrational is given by (Skudrzyk, 1968):

\[
\langle FV \cos(\phi) \rangle_t = \frac{1}{2} FV \cos(\phi)
\]

where \( F \) and \( V \) are the amplitudes of the force and velocity response, and \( \phi \) is the phase angle between the force and velocity signals. Consider the velocity at a point on a structure to be 90° out of phase from the driving force, then using Eq. (1) the power transmitted into the structure is zero. If the instruments measuring the force and velocity result in a 1° error in phase, then the percentage error from the measured power to the actual power is infinite!

Ohlrich (1995) has suggested that to obtain a reasonable measurement of power transmission, the phase errors in the measuring chain should be less than 0.2°. Gade et al. (1993) suggest that relatively large phase errors between a force transducer and accelerometer pair (±10°) only have a small influence on the calculation of power transmission (0.1dB). When two accelerometers are used to measure structural intensity, the measured phase angle between the two acceleration measurements should be at least 5 times greater than the phase mismatch, so that the error in the intensity measurement will be less than 1dB. Clearly the effect of an error of this size is dependent on the ratio of active to reactive power that is transmitted (or the absolute phase difference between the force and velocity). When the receiving structure is only lightly damped, the resulting power transmission is dominated by reactive power and any phase errors have a much larger effect on the measured active power. Henriksen (1996) and Gardonio et al. (1997) showed that phase errors associated with the measurement of intensity when used as a cost function to be minimised in an active vibration isolation system can cause the vibration response to be worse than when passive control only is used.
Carroll (1990) performed measurements with various types of accelerometers and showed that some have phase errors greater than 1° below 100 Hz. The phase limitation was found to be due to spurious strains on the piezoelectric element which are mechanically generated by cable motion.

Horner and White (1990) showed that phase errors associated with the transducers and amplifiers can be corrected by digital compensation. If the phase errors can be accurately measured then a correcting electronic filter can be applied using digital electronics, to remove the phase errors. Here, the phase errors associated with the transducers and amplifiers are measured and improved using digital filters, which results in an effective phase accuracy of ±2°.

2.3 Multi-Axis Force Transducers

To measure power transmission along several axes, a multi-axis force transducer and accelerometer array is needed. The calculation of power transmission requires the measurement of force and velocity at the same location, which can often be difficult to measure in practice because different types of transducers are needed to measure force and velocity. The measurement of the velocity in the transducer described here is achieved by predicting the velocity of the material beneath strain gauges, which measure the force of the mandrel, using accelerometers that are remote from the strain gauges. Further details about how this is practically implemented is described in section 3.4.

The use of an accelerometer array to measure vibration along several axes is common and need not be reviewed in detail. A lengthy derivation of the mathematics on the use of accelerometer arrays is presented by Dimasi (1995), where a nine accelerometer array is used to measure the kinematics of a dummy head in automobile crash testing.

Since the beginning of the 1980’s several types of 6 axis force transducer have been commercially available for use on the end of robot arms used in manufacturing industries. However, for the commercial transducers examined, it was found that they were not suitable for active control experiments because the transducers do not have suitable analog outputs which can be used to calculate power.

A company from the United Kingdom assembles off-the-shelf force transducers into a package to measure forces along 6 axes. These force transducers are used in automobile crash testing where the loads are impulsive and extremely high and therefore unsuitable for active control experiments where the loads are continuous and small compared to impact testing.

Engeler and Giorgetta (1995) describe a 6 axis force transducer that uses specially shaped piezo-electric crystals intended for use inside a joystick. The adaptation of this design to measure forces between a vibration isolator and receiving structure would be too difficult, as special purpose piezo-electric crystals would be needed.

Kaneko (1996) gives a good overview of the development of 6 axis force transducers and describes some commercially available products. He used two 3 axis force transducers to make a 6 axis force transducer. The 3 axis force transducers were packaged as an integrated circuit. This product sounded extremely attractive, but after contacting the manufacturers of the integrated circuit (and their competitors), this type of product was found not to be commercially available. No reasons were given by the companies.

Due to the lack of suitable commercial 6 axis force transducers, and the high cost of commercially available 3 axis force transducers, it was decided to develop a custom
built 6 axis force transducer for the experimental work presented in this paper.

Quinn and Mote Jr. (1990) describe a 6 axis force transducer to measure the force that a cyclist applies to pedals while cycling. The ingenious design uses strain gauges mounted to shear panel elements. The shear panel elements were used to reduce the cross axis sensitivity of the sensor. A design using this method was examined by the authors but it was found that large displacements of the shear panels would occur which would cause resonance problems.

In section 3, a force transducer is described which uses 24 strain gauges mounted to a cylindrical structure to measure forces and moments along 6 axes. A similar design of a six axis force transducer was not found in the literature, but it is unlikely this design is unique. The cross axis sensitivity can be vastly improved by careful orientation of the strain gauges, yielding better results than a single strain gauge measuring strain along a single axis. This force transducer is combined with an accelerometer array that measures the motion of the transducer, so that vibratory power transmission can be measured along 6 axes, and is further described in section 3.

3 SIX AXIS FORCE TRANSDUCER

3.1 Power Transmission

The instantaneous measure of power transmission for tonal (harmonic) vibration, $P$, is given by (Skudrzyk, 1968) as $P = f\dot{v}$ where $f$ is the force and $\dot{v}$ is the measured velocity at the same location where the force is measured. The time averaged measurement of power transmission for harmonic motion is given by $\langle P \rangle_t = \langle f\dot{v} \rangle_t = \frac{1}{2} \Re(f\dot{v}^*)$, where $\Re$ denotes the real part of the expression, and $\dot{v}^*$ is the conjugate of velocity. This expression is equivalent to Eq. (1). Similar expressions can be written for the power transmission along rotational axes where the forces are replaced by moments, and translational velocity replaced by rotational velocity. The total power transmission is calculated by the addition of power transmission from all translational and rotational axes.

In order to measure power transmission, the force and the velocity of the material where the force is measured must be acquired. The next following sections describe the design of the force transducer to measure translational forces and rotational moments, the calibration of the transducer, and the method used to measure the velocity of the material beneath the strain gauges that measure the force.

3.2 Design

Six axis force transducers are made by a few manufacturers for use on robot arms. These transducers are expensive and do not provide analog signals which can be used for active control experiments. This led to the development of a 6 axis force transducer which is shown in figure 1. This force transducer uses 24 strain gauges mounted to a cylindrical tube to measure forces and moments along all six axes ($F_x$, $F_y$, $F_z$, $M_x$, $M_y$, and $M_z$). Figure 1(b) shows a close up view of the strain gauges mounted to the cylindrical tube. The outer dimensions of the sensor are 70mm outside diameter and 50mm high.

A group of four strain gauges are combined to form a full bridge Wheatstone circuit, which measures the force or moment along a single axis. The full bridge circuit
has advantages in increased sensitivity (4 times greater compared to a quarter bridge), negligible thermal effects and decreased cross axis sensitivity. Each group of four strain gauges is orientated to reject off-axis loads, which is a standard technique for the arrangement of strain gauges (Hoffmann, 1976). The arrangement of the strain gauges around the column of the transducer is shown in Figure 2, where the labels $F_x$, $F_y$, etc. indicate the direction of force or moment that the strain gauge measures, and the numbers $1 \ldots 4$ next to each strain gauge indicates the bridge number on the Wheatstone circuit that it forms. An advantage of using a force transducer made with strain gauges compared to piezo-electric crystals is that the transducer can be calibrated using static loads. Force transducers that use piezo-electric crystals can only be calibrated with dynamic loads. The disadvantage of using a force transducer utilising strain gauges is that the device must have some flexibility so that strain in the body can be measured, which has the potential of introducing unwanted resonances into the system under investigation. A force transducer utilising piezo-electric sensing elements to measure the strain is substantially stiffer than a device using strain gauges, and hence less likely to introduce additional resonances. Hence care must be taken when designing a force transducer utilising strain gauges, so as not to introduce resonances in the frequency range in which the transducer is to be used. Further details about the design of this
3.3 Calibration

The transducer was calibrated using static and dynamic calibration methods. The static calibration involved loading the transducer with known loads along each axis and comparing the known load with the measured response. Dynamic calibration was only attempted along the axial direction of the transducer, as dynamic calibration of moments is not easily realised. Instead, it is assumed that the dynamic phase response for the five axes, other than the axial direction, would exhibit the same trend in the phase response as the axial direction. This assumption is reasonable given that the same type of strain gauges and instrumentation amplifiers were used for all axes. This assumption was tested by conducting an experiment that involved applying force and moment excitation through the transducer into a simply supported beam, and is further discussed in section 3.5.

Figure 3 shows how the 6 axis strain gauge force transducer was calibrated with static loads. The force transducer was mounted to an angle plate and secured to a table. A beam was bolted to the top flange of the transducer and a mass was hung from a hook on the end of the beam. Several masses and orientations of the transducer were used to determine the on-axis and cross axis sensitivities of the sensor. The cross-axis sensitivity of each axis is approximately -20dB relative to the axial sensitivity. Typical piezo-electric type multi-component force transducers have cross axis sensitivities of -40dB (1%).

Power transmission is calculated by using the force measured in the cylindrical body multiplied by the velocity at the point on the cylindrical body where the force is measured.

An experiment based on Newton’s second law, the acceleration of a mass is proportional to the applied force, was performed to measure the phase accuracy of this 6 axis force transducer. The experimental setup is shown in figure 4. For this test, the 6 axis force transducer was attached to a steel mass which was hanging vertically. A Brüel and Kjær Type 8200 force transducer was attached to the 6 axis force transducer and a Brüel and Kjær Type 4393 accelerometer was attached to the back of the hanging mass. The Brüel and Kjær transducers were electrically connected to Brüel and Kjær Type 2635 charge amplifiers. The system was vibrated horizontally with band limited random excitation.

Figures 5 and 6 shows the comparison of the amplitude and phase accuracy of the 6 axis force transducer compared with an accelerometer mounted to the rigid mass. The results show that there is a variation in amplitude of about ±0.1dB and a variation in phase of about ±1°, with a bias offset of about +1°. The bias phase error is caused by the filters in the analog strain gauge amplifiers which can be corrected by digital filtering (Horner and White, 1990). The experimental results presented here which use the force signals from this transducer, have been corrected to take account of this bias phase error. The random phase errors cannot be corrected.

The phase accuracy of the transducers is important to the overall accuracy of the measured value of power transmission. When power transmitted into a lightly damped structure is measured, the phase angle between force and velocity signals are almost 90° apart, hence the cosine of the angle is close to zero. Small phase errors will result in
Figure 3: Static calibration of the 6 axis force transducer.

Figure 4: Experimental setup to measure the phase accuracy of the 6 axis force transducer.

Figure 5: The amplitude accuracy of the 6 axis force transducer along the axial direction.
a large error of the measured power transmission. Conversely, when the phase angle between force and velocity is small (say 10°) in a heavily damped structure, the phase errors become less important.

Power transmission is calculated by using the force measured in the cylindrical body multiplied by the velocity at the point on the cylindrical body where the force is measured. An ideal transducer used to measure power transmission would be infinitely stiff, have zero inertia, perfect phase accuracy, and be extremely small and inexpensive. In the present case, when an external force is applied to the body of the cylindrical tube of the force transducer, the tube elongates and is measured by the strain gauges to provide values of force and moments along the 6 axes of motion. The elongation of the tube is undesirable because it can introduce resonance problems and more importantly the velocity measured at the base of the force transducer may not necessarily be the same as the velocity of the material beneath the strain gauge.

3.4 Measurement of Velocity

The target application of the transducer described in this paper was for active vibration isolation experiments described in Howard (1999) and Howard and Hansen (2006). Preliminary investigations showed that the most important axes which need to be considered are the axial $Z$ and the two rotational axes $\theta_x$ and $\theta_y$. When horizontal forces are applied to the top of the force transducer, moments are generated on the cylindrical tube and are measured by strain gauges which measure bending moments. Therefore the measurement of the shear forces $F_x$ and $F_y$ and the corresponding accelerations were not required. The measurement of the torsional vibration was also unnecessary because the torsional vibration does not couple well with the transverse vibration of the beam system investigated in this paper.

In order to measure power transmission between two sub-system, the force and the velocity of the material at the location where the force is measured must be acquired. The question arises: where should the accelerometers be placed to measure the velocity of the material beneath the strain gauges? To answer this question, a finite element model was constructed of one experimental setup used in this paper. Figure 7 shows the finite element model and figure 8 shows the corresponding sketch of the model. A vibrating rigid body was attached to a visco-elastic spring which isolated the vibration.
Table 1: Parameters used in the isolator and beam system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length</td>
<td>1.495m</td>
</tr>
<tr>
<td>Beam thickness</td>
<td>0.025m</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>207 GPa</td>
</tr>
<tr>
<td>Beam density</td>
<td>7800 kg/m³</td>
</tr>
<tr>
<td>Isolator stiffness $k_z$</td>
<td>45870 N/m</td>
</tr>
<tr>
<td>Isolator stiffness $k_{\theta y}$</td>
<td>216 N/rad</td>
</tr>
<tr>
<td>Top mass</td>
<td>7.4 kg</td>
</tr>
<tr>
<td>Beam width</td>
<td>0.025m</td>
</tr>
<tr>
<td>Isolator location</td>
<td>0.750m</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$1.6 \times 10^{-5}$ m⁴</td>
</tr>
<tr>
<td>Beam damping</td>
<td>$7.48 \times 10^{-6}$ sN/m</td>
</tr>
<tr>
<td>Isolator damping $c_z$</td>
<td>140 sN/m</td>
</tr>
<tr>
<td>Isolator damping $c_{\theta y}$</td>
<td>140 sN/rad</td>
</tr>
<tr>
<td>Bottom mass</td>
<td>8.2 kg</td>
</tr>
</tbody>
</table>

from a simply supported beam. At the bottom of the vibration isolator is a lumped mass which is attached to the 6 axis force transducer. The force transducer sits on a 1mm thick washer and is attached to the simply supported beam. The washer is used to reduce the area of the transducer which is in contact with the beam, so that a more concentrated point load can be applied to the beam. The 6 axis force transducer was modelled as a cylindrical tube using shell elements, and plates attached to each end of the tube, again modelled using shell elements. The loads from the lower mass and the beam were transferred to the finite element model of the force transducer using mass-less rigid link elements arranged in a spider. The cylindrical tube had an outside diameter of 22mm, an inside diameter of 20mm and a length of 40mm. The top and bottom plates have an outside diameter of 60mm and a thickness of 15mm. The parameters for the beam and isolator system are described in Table 1.

This finite element model was used to compare the velocity of the material beneath the strain gauges on the 6 axis force transducer with the velocity measured at a practical location, such as on the beam or on the plates of the force transducer. The top mass of the finite element model was excited with a harmonic vertical force of amplitude $F_z = 1$N and a harmonic rotational moment around the $\theta_y$ axis of amplitude $M_y = 0.002$Nm over a frequency range of 2-200Hz.

It was hypothesised that the axial velocity of the 6 axis force transducer could be measured on the base plate of the force transducer, as the axial stiffness of the 6 axis force transducer is much greater than the stiffness of the receiving structures examined in this paper. Figure 9 shows a comparison of the velocities predicted using the finite element model at the location of the strain gauges, the top plate and the bottom plate.

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Figure 7: Finite element model of the 6 axis force transducer attached to a simply supported beam.
The figure shows that the axial velocity beneath the strain gauge can be accurately approximated by measuring the velocity at either the top and bottom plate.

The measurement of the rotational velocity at the location of the strain gauges on the force transducer is made difficult by the low bending stiffness of the cylindrical tube compared to the bending stiffness of the simply supported beam. A practical location to measure the rotational velocity would have been to mount accelerometers along the axial direction on both sides of the attachment point of the force transducer to the beam. Finite element modelling showed that this was a poor approximation to the measurement of rotational velocities. A better approximation of the rotational velocity at the strain gauge can be made by measuring the angular velocities of the top and bottom plates and interpolating between the two measurements. Figure 10 shows a diagram of the magnitudes of the velocities measured by the accelerometers $A_1$ and $A_2$ at the edges of the plate. The translational velocity of the plate is calculated as the average of the velocity measurements, $z = (A_1 + A_2)/2$. The angular velocity of each plate can be measured by using two accelerometers placed on opposite edges of plate, as shown in Figure 8, by calculating the difference between the two translational (complex) velocities and dividing by the distance which separates them, $\theta = (A_2 - A_1)/L$. Accelerometer $A_1$ has components $a$ from the translational motion and $c$ from the rotational motion, hence $A_1 = a + c$. Similarly, accelerometer $A_2$ has components $b$ and $d$ from the translational and rotational motion, respectively. The translational components can be written as $z = a = b$ and the rotational component are $\theta L/2 = d = -c$, which means that $c$ and $d$ are 180° out of phase with each other. Proceeding with the calculation of the rotational velocity,

$$\theta = \frac{[A_2 - A_1]}{L}$$

$$= \frac{[(b + d) - (a + c)]}{L}$$

$$= \frac{[(b + d) - (b - d)]}{L}$$

$$= \frac{2d}{L} \quad (2)$$

Although the measurement of the rotational velocity involves calculating the difference between two (complex valued) velocity measurements to resolve relatively small magnitudes, which has the potential for an ill-conditioned problem, as the pertinent components are 180° out of phase with each other the difference calculation results in a doubling of the pertinent component, as shown in Eq. 2 and hence ill-conditioning is

![Figure 8: Schematic of the finite element model used to determine suitable locations for the accelerometers to measure the velocity of the 6 axis force transducer.](image-url)
Figure 9: Predicted axial velocity at the location of the strain gauges, top plate and bottom plate.

Figure 10: Diagram illustrating the calculation of rotational velocity of the top or bottom plate.
less likely.

Figure 11 shows a comparison of the angular displacements predicted using the finite element model at the location of the strain gauge, the top plate, the bottom plate and the difference in angular velocities between the top and bottom plates. The results from the finite element analyses show that the angular displacement at the strain gauge can be reasonably approximated by the difference in the angular displacement of the top and bottom plates.

3.5 Experimental Verification

An experiment was undertaken to verify that the 6 axis force transducer could be used with an accelerometer array to measure power transmission into a simply supported beam. Figure 12 shows the experimental setup. The six axis force transducer was bolted at 0.75m along a simply supported beam of dimension 1.5m length, 25mm square and a steel washer placed in between the force transducer and the beam. The washer is used to reduce the area of the transducer which is in contact with the beam, so that a more concentrated point load can be applied to the simply supported steel beam.
beam. An aluminium bar was attached the top of the force transducer which simulates a cantilever. The axis of the aluminium bar was in parallel with the axis of the simply supported steel beam. Five accelerometers were attached to the beam to measure an approximation of the kinetic energy of the beam. The accelerometers were located at 0.30m, 0.35m, 0.40m, 0.45m and 0.50m from the end of the beam. The frequency range of interest is between 0-200Hz, which corresponds to the first 3 vibration modes of the beam. A shaker was attached to the end of the cantilevered beam through a Brüel and Kjær Type 8200 force transducer which was used to measure the force applied by the shaker. Accelerometers were attached to the top and bottom plates on the force transducer as shown in figure 8.

A Finite Element Model was constructed using the software package ANSYS of the experimental setup described above. The model was constructed so that predictions could be made of the forces, displacements, power transmission from the shaker into the beam, and kinetic energy in the beam, which could be compared with the experimentally measured values. The model is similar to the one shown in Figure 7, only a cantilever beam was attached to the force transducer instead of the vibration isolator. In addition, the results from this experiment are used to justify the method used to predict the velocity of the material beneath the strain gauges, which is used in the calculation of the vibratory power transmission.

Figure 13 shows the predicted and measured displacements on the top and bottom plates of the force transducer. The predicted results were obtained from the finite element modelling. Using the displacement measurements on the top and bottom plates of the force transducer, the vertical and angular displacements of the top and bottom plates can be calculated, which can be used to predict the vertical displacement and the angular displacement of the material beneath the strain gauges used to measure the axial force and bending moments. Figures 14 and 15 show the vertical and rotational displacements of the force transducer, respectively. The results presented in figures 13 to 15 compare favourably. The calculated and measured resonance peaks in the graphs do not exactly match, but this is due to the non-ideal boundary conditions in the experimental setup. The FEM assumes that the cantilevered beam is fully clamped to the top of the force transducer. In reality, the clamping arrangement permits some rotational motion at the bolted connection and is therefore less stiff than assumed in the finite element model. The lower stiffness results in a lower resonance frequency of the cantilevered beam.

The force along the vertical Z axis and the bending moment along the $\theta_y$ axis are presented in figures 16 and 17 respectively. The experimental results compare favourably with the theoretical predictions.

The kinetic energy in the simply supported beam is theoretically calculated by the integral of the squared translational and squared rotational velocities over the length of the beam. In practice, only an approximate measure of the kinetic energy is ever obtained by using a finite number of accelerometers attached to the beam and in this case, 5 accelerometers were used. Figure 18 shows the theoretically predicted and experimentally measured sum of the squared accelerations at the 5 accelerometer locations. This measure will be called the kinetic energy in the beam. This result shows there is good agreement between the theoretically predicted and experimentally measured kinetic energy.

The experimental results presented so far in figures 13 to 18 are for amplitude measurements. The phase information in the signals has only been required to calculate
Figure 13: Displacements predicted using FEM on the top and bottom plates of the force transducer compared with experimentally measured results for the (a) Bottom plate accelerometer 1, (b) Bottom plate accelerometer 2, (c) Top plate accelerometer 3, (d) Top plate accelerometer 4.

Figure 14: Comparison of the theoretically predicted and experimentally measured displacement along the vertical Z axis.
Figure 15: Comparison of the theoretically predicted and experimentally measured angular displacement of the material beneath the strain gauge used to measure bending moment.

Figure 16: Comparison of the theoretically predicted and experimentally measured force along the vertical $Z$ axis.

Figure 17: Comparison of the theoretically predicted and experimentally measured bending moment along the $\theta_y$ axis.
the angular displacement of the force transducer. The calculation of power transmission into the simply supported beam requires a high degree of phase accuracy because the beam is very lightly damped. Figures 19 and 20 show the predicted and measured power transmission into the beam along the vertical $Z$ axis and around the $\theta_y$ axis respectively.

It can be seen in figure 20 that there is good agreement between theoretical predictions and experimental results above about 50Hz. The results for the amplitude of force, moment, translational and rotational displacement shown in the preceding graphs show good agreement between theory and experiment in the 10-200Hz frequency range. The discrepancy in the results can be traced to the phase angle for the rotational motion, as shown in Figure 21. These results show there is good agreement between theory and experiment in the frequency range between 50-200Hz, which corresponds to the frequency range where the rotational power measurements were reasonably accurate. It can be seen that between 0-50Hz, the phase angle passes through $90^\circ$ phase shifts which corresponds to a rotational resonance and is not taken into ac-
Figure 20: Comparison of the theoretically predicted and experimentally measured power transmission into the beam along the $\theta_y$ axis.

Figure 21: Comparison of the theoretically predicted and experimentally measured phase angle for the rotational displacement along the $\theta_y$ axis.
count by the theoretical model. The resonance was identified from the shaker support structure.

The force transducer was rotated 90° and the experiment was repeated so that the rotational power transmission along the $\theta_x$ axes could be examined. Similar results were obtained as presented here for the rotational power transmission along the $\theta_y$ axis.

4 CONCLUSIONS

A transducer was constructed that measures vibratory power transmission along 6 axes and utilizes a force transducer, which can measure forces and moments along several axes, and an accelerometer array mounted to the top and bottom flanges of the force transducer, which measures translational and rotational motion. The force transducer consisted of 24 strain gauges mounted to a cylindrical tube. Sets of four strain gauges were combined into a single full bridge Wheatstone circuit which could be used to determine the force or moment along an axis. The orientation of the strain gauges was important in minimizing cross axis loads. Experiments were conducted to determine the magnitude and phase accuracy of the force transducer. It was found that the force transducer had good amplitude accuracy, however the phase accuracy of less than 2 degrees was insufficient for highly accurate measurements of power transmission in lightly damped structures.

In a companion paper (Howard and Hansen, 2006), a single axis vibration isolator and the 6 axis force transducer described in this paper are used in a comparison of results from active vibration isolation experiments with theoretical predictions.
References


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