Modelling of delamination damage in composite beams

Manudha T. Herath¹*, Kaustav Bandyopadhyay¹ and Joshua D. Logan¹

¹ School of Mechanical Engineering, University of Adelaide, SA, Australia
*Corresponding author. Email: manudha.herath@student.adelaide.edu.au

Abstract: Composite structures have gained importance in engineering applications due to their favourable mechanical properties, for example laminate materials are widely used in many applications. However, for laminate materials, the presence of the layers makes the structures prone to delamination damage, which can be hard to detect with conventional detection techniques. An important aspect of any damage detection technique is an understanding of the mechanical behaviour of the ‘damaged’ material or component. This can be best achieved through theoretical modelling. This paper focuses on modelling delamination in composite beams using finite element analysis model and an analytical Euler-beam model. Several beam models with different delamination sizes were developed and analysed using FEA software ANSYS. The curvature plots obtained were analysed using the mathematical formulations of the governing equations. Finally, a simple 1-D beam model with delamination was developed to understand the mechanical behaviour of the delaminated component. The developed theory is later intended to be integrated with a new damage detection technique using a scanning laser doppler vibrometer.

Keywords: composites, delamination, Euler-beam model, finite element analysis

1 Introduction

Over the past few decades composite structures have gained importance due to their favourable mechanical properties compared to traditional monolithic materials. Properties such as high specific strength, light weight, excellent corrosion resistance, durability and low maintenance costs have made composites the material of choice in an ever increasing range of applications, such as aerospace components, boat and scull hulls, bicycle frames, and high performance motor vehicles [1]. Layered composites, or laminates, are the most commonly used form of composite in many of the aforementioned applications. The superior properties of laminate composites are governed by their layered structure which leads to non-homogenous composition and anisotropic material properties. However, the presence of multiple layers and reinforcements can make damage modelling and detection in composite materials and structures significantly challenging.

Delamination damage is one of the most common structural damage types in composites, and can result from impact, overload, or fatigue crack growth from defects along or near the adhesion layer. Delamination leads to a reduction in stiffness and strength of the composite structure and potentially to catastrophic failure. Early detection of delamination damage is therefore vital for high risk and high value assets such as aircraft and civil infrastructure. Existing non-destructive damage detection techniques (NDDT) to detect structural damage in composite materials include eddy current methods, radiography, ultrasound, dye penetrant inspection and techniques implementing strain or displacement measurements [2, 3, 4, 5]. In recent years Scanning Laser Doppler Vibrometry (SLDV) has been employed in NDDT to measure the displacement, or strain, of surfaces on a structure [6, 7]. Of particular interest is a recently developed strain measurement based technique as seen in work by Wildy et al. [4,5,8], which utilises 3D SLDV and the governing equations of continuum mechanics (i.e. equilibrium and compatibility).

An important aspect of any damage detection technique, especially displacement and strain based approaches, is an understanding of the mechanical behaviour of a delaminated composite component under a wide range of load and geometry conditions. This knowledge is achieved through theoretical modelling such as by simplified beam or plate models as seen in Wildy et al. [8] or extensive finite element analysis (FEA) [9]. Theoretical modelling can also provide ‘ideal’ comparisons for undamaged and damaged components and can be used to evaluate the success and range of applicability of new damage detection methods. Simple beam or plate models are particularly good for this as they allow for a wide variation of input parameters without the computational and time constraints of FE methods.
The paper discusses the development of an analytical beam model for delaminated composite beams. This model is accompanied by FEA simulation of a delaminated composite beam under quasi-static loading condition. The purpose of an analytical model aided with FEA is to investigate general mechanics of a delaminated beam and to see the effect of the size and location of the delamination damage. The developed model can also be used to determine the applicability and limits of the damage detection methods and to aid in experimental design.

2 Finite element modelling for the development of damage detection technique

This section discusses the Finite element model simulation using ANSYS APDL Version 12.0.1. The aim of the FEA is to investigate the mechanical behaviour of delaminated composite beam including the effects of contact and shear deformation. This is to validate the assumptions made for analytical model which is discussed in Section 3. The FEA model was also used to investigate the range of applicability of the simple model i.e. by determining the delamination sizes which are detectable. Boundary conditions were chosen to be appropriate to an experimental study currently being undertaken by Wildy et al. [8], though the trends and effects observed are applicable to a range of loading and structural boundary conditions.

2.1 FEA specifications

Several assumptions were made in order to simplify the finite element model to reduce the computational and theoretical effort. Firstly, 2-D finite model with plane stress with thickness was analysed. In accordance with the dimensions in Figure 1, a thickness of 60 mm was specified. The reason to use plane stress with thickness model was that Euler-Beam theory which forms basis for the governing equation in later section does not take out of plane stress into account. Also, there are no restraints in z-direction and the length to thickness ratio is 10:1 which is very small, thus assuming plane stress condition is acceptable. Also, using a 2-D model is valid, as the delamination modelled is along the length of the beam. In addition, the width of the beam is significantly smaller than the length of the beam. Sandwich composite layers are usually glued together using special adhesives, such that all the layers are bonded together. Due to finite strength of the adhesive layer, under load application, the layer may have considerable deformation and can even lead to failure. However, such effects will not be revealed in the ANSYS simulation and almost perfect behaviour at the adhesive layers was expected. Nevertheless, it can be argued that these effects are negligible as the applied loads are considerably small. Finally, the material properties of the composite are considered to be isotropic. The composite materials used in the accompanying experimental study are 90° cross ply laminates thus, the grid pattern of reinforcement fibres, it can be expected that the isotropic properties will be maintained to a certain extent. The material properties of glass fibre (E-glass) reinforced plastic (GFRP) which are modelled as laid in grid-like pattern (0° and 90° pattern) in order to use isotropic material properties. The material properties were derived based on composite fibre volume fractions [10] and are summarised below,

\[
\begin{align*}
\text{Modulus of elasticity, } E &= 50.204 \text{ GPa} \\
\text{Poisson ratio, } \nu &= 0.19 \\
\text{Density, } \rho &= 2.004 \text{ gcm}^{-3}
\end{align*}
\]

The FEA model was created based on the geometry and properties of the experimental specimens used by Wildy et al [8]. Figure 1 illustrated the specimen geometry and the dimensional specifications. The specimen is 600 mm long, 60 mm thickness and each layer has a height of 3 mm.

![Figure 1: Dimensions of a composite layer (not to scale)](image)

As shown in the Figure 1, there are two layers in the model of the composite structure and the delamination is situated between these two layers. The analysis was carried out for different ranges of delamination sizes and positions in order to develop the theory of the simple beam model discussed in
Section 4. Also, FEA acts as a tool to determine the limitation of the simple model, i.e. the models validation. The delamination sizes and locations modelled in FEA are specified in Table 1. The delamination location is given the fixed end of the beam. The delaminations were chosen to be at 4.5mm from the base at the middle of the beam to give the FEA simulation a more generalised approach.

<table>
<thead>
<tr>
<th>Delamination length</th>
<th>Delamination location</th>
</tr>
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<tbody>
<tr>
<td>300 mm</td>
<td>120 mm</td>
</tr>
<tr>
<td>150 mm</td>
<td>200 mm</td>
</tr>
<tr>
<td>60 mm</td>
<td>200 mm</td>
</tr>
<tr>
<td>6 mm</td>
<td>200 mm</td>
</tr>
</tbody>
</table>

**Table 1**: Delamination sizes and locations

Furthermore, 2-D element, Plane 183 with thickness effect was used to mesh the model. In accordance with the dimensions in Figure 1, a thickness (in z-direction with reference to Figure 5) of 60 mm was specified. The contact pairs were created at the adjacent layers to the delamination with a coefficient of friction of 0.3. The element profile of the beam is illustrated in Figure 2.

The boundary conditions applied to the FE model simulates the experimental loading conditions. The load was applied in terms of displacement as it is directly related to the deflection and the resulting curvature. Hence, two boundary conditions were applied to the beam, one being clamped-end boundary conditions and the other at the opposite end as a force of 12.5 N.

![Figure 2: The element profile of the beam after being meshed](image)

### 2.2 FEA results

As stated in Section 2, the curvature of the beam can be accurately approximated as the Second Derivative of deflection with respect to distance along the beam. A straight path from the clamped end to the free end (excitation end) of the beam was defined. The lower surface was selected to define the path, as in the experimental set up SLDV lasers are installed to scan the lower surface of the beam. At this stage, it was possible to import the results to Excel or MATLAB and numerically differentiate the results two times in order to obtain the curvature along the beam, as this will be the approach in actual experiments. However, it was witnessed that such an approach introduced large numerical errors in the 2nd derivative. Hence, it was decided to perform differentiation using ANSYS.

The FEA results for the curvature of the beam are illustrated in Figure 3. The graphs illustrate the curvature of the beam along the lower surface, where the ordinate is the curvature of the beam in m⁻¹ and abscissa is the length of the beam.
Figure 3: Curvature profile of the beam with delamination of (a) ½ of beam length, (b) ¼ of beam length, (c) 1/10 of beam length (d) 1/100 of beam length

It is clear that, for every delamination length, there is a clear discontinuity in the curvature with varying magnitude. Therefore, if a delamination detection method were formulated based on SLDV and curvature profile of the beam, the resolution of delamination detection will be governed by the accuracy of the curvature measured by the SLDV system.

In addition, the contact pressure was also investigated, in order to determine the amount of contact that occurs between the delaminated layers (Figure 4).

Figure 4: Contact pressure distribution along the delamination under static loading

In Figure 4, the region with the larger lines emanating from the delamination interface represents high contact pressures and considerable contact. Thus, it is clear that the amount of contact occurring in the delaminated region is negligible. Furthermore, the contact occurs at the process zone of the tip of delamination. Therefore, even this occurrence of contact can be neglected as it is a zone with considerable singularities. Thus, it can be argued that contact elements are not necessary when modelling a delamination using Finite Element Method under static loading conditions.
3 Mathematical modelling of the delaminated beam

An analytical beam model was developed in order to investigate the behaviour of a delaminated beam under static loading conditions.

Consider a delaminated cantilever beam of length $L$ loaded by a point load $F$ at the free end (figure X). The beam contains a single delamination of length $L_2$ located $L_1$ distance from the fixed end and $h_3$ from the lower beam surface. The formulation can be readily extended to distributed-loading scenarios, other structural boundary conditions and multiple delaminations, though this is not of concern for the current paper.

The analytical model is based on Euler-Bernoulli beam theory and is thus limited to long slender beams where shear deformation is negligible. For simplicity, the formulation also assumes a constant cross-section and constant modulus of elasticity across each section. Material properties were taken as isotropic and homogenous, although the formulation can be extended to orthotropic and non-homogenous cases. Non-linear behaviour such as buckling and contact between the delaminated surfaces is also neglected. Through the accompanying FE simulations, these assumptions were found to be appropriate for the beam geometries under consideration (discussed further in the previous section).

Note that the definitions of the variables are as given by Figure 5; where, $L$, $h$, $V$ and $M$ stand for the length, height (thickness), shear force and moment. $T$ and $C$ stand for the tensions and compression generated within the delaminated region.

The beam layout can be solved by considering the static equilibrium of a portion created by an imaginary cut through the delaminated zone as illustrated in Figure 6.

By simple static equilibrium it can be proven that tension is equal to compression. In addition, a moment equilibrium equation can be written relating $M_{2(x)}$, $M_{3(x)}$ and $T$. The tension can be calculated considering the compatibility of axial displacements of beam segments 2 and 3 as shown by Figure 7.
It can be proven that, where, $M_{(x)} = F(L - x)$,

$$T = \left(\frac{h_1}{2}\right) \left(\frac{\int_0^{L_2} M_{(x)} \, dx}{E_1} - \frac{\int_0^{L_1+L_2} M_{(x)} \, dx}{E_1}\right)$$

(1)

In addition, by assuming that deflections, thus curvatures, of layers 2 and 3 are equal, another relation was formed between $M_2$ and $M_3$. This has been assumed by many previous authors in their work related to delaminated structures [11]. This was observed when investigating the contact pressure of the contact elements used within the delaminated zone.

The method of static equilibrium, explained above, provides a simple method to evaluate complete moment profile of the delaminated segment of a delaminated beam; which can then be used to evaluate curvature, slope and deflection profiles along the beam. Alternate approaches such as minimum potential energy theory may also be used to solve this system.

In addition, a more generalized method was formulated to solve the afore-mentioned problem. The compound beam was separated into four segments (Figure 8) and the continuity of deflection, slope, shear forces and moments between each beam segment in conjunction with Euler Beam Theory was used as the basis for formulating the system.

Euler beam theory states that,

$$E_1 w''' = q_{(x)} = 0$$

(2)
where, $E$ is the Young’s Modulus, $I$ is the Area Moment of Inertia about the bending axis, $w$ is the deflection of the beam in $z$ direction and $q(x)$ is the distributed loading and is 0 for a point loaded case. This can be used to formulate a system with 11 equations and 11 unknowns which describe the deflections, slopes and curvatures of the four beam segments. The system can then be solved either symbolically or numerically using any standard method to obtain the response of each beam segment under static point loading. This system, when solved symbolically, shows that the deflections, slopes and curvatures of beam segments 2 and 3 are identical. Thus, the assumption made in the equilibrium method can be validated. The results obtained using Finite Element Analysis was then validated by comparing the curvature profile obtained using FEA and analytical methods explained in the discussion (Figure 9). Based on this comparison, it can be concluded that both finite element technique and analytical modelling techniques presented in this paper are accurate.

![Figure 9: Comparison of Hand Calculated Results to FEA Results](image)

### 4 1-Dimensional beam model

Based on the generalized 2-dimensional beam model explained in the previous section, a 1-dimensional beam model can be developed in order to further simplify the delaminated beam. Here, the beam will divided into three segments, with the mid-portion representing the delaminated zone and the other two segments representing the non-delaminated zones (Figure 10).

![Figure 10: (a) Actual beam model and (b) Simplified 1-D beam model](image)

From the mathematical relations in Section 3 it can be proven that for a static point-loaded case the curvature of the delaminated segment deviates slightly from the simple $EIw''(x) = M(x) = F(L-x)$ relation due to the internal tension and compression generated inside the delaminated layers. The final result is of the form,

$$ (EI)_{eff}w''(x) = F(k-x) $$

(3)
Where,

\[ (EI)_{\text{eff}} = 1 - 3m + 3m^2 \]  \hspace{1cm} (4)

\[ k = L[1 - 3(1 - n)(1 - m)] \]  \hspace{1cm} (5)

\[ m = \frac{h_2}{h_1} \]

\( n \) is the normalized distance to the midpoint of delamination from the fixed end,

\[ n = \frac{L_1 + \frac{L_2}{2}}{L} \]

Note that, in the model, the usual curvature response given by \( EIw''(x) = F(L - x) \) is valid for the non-delaminated segments of the beam. Thus, the curvature response of the compound 1-D beam can be modelled. Once the curvature behaviour of the model has been developed, the response can be readily expanded to obtain slope and deflection behaviours by integrating and calculating the integrating constants using appropriate boundary conditions.

This model was proved to be extremely accurate by performing validations using FEA models. A comparison between the FEA and simplified 1D beam model is shown in Figure 11.

![Comparison between 1D Beam Model and FEA Results](image)

**Figure 11**: Comparison between 1D Beam Model and FEA Results

In addition, it is clear from Equation (4) that the ratio between effective bending stiffness of the delaminated segment of the beam to the non-delaminated beam depends only on the location of the beam in terms of thickness of the beam. In other words, it is not governed by the location, in terms of length, of the delamination. Several FE models with different delamination lengths and thicknesses were constructed to test this result. Figure 12 shows the comparison between FE models and the curve generated by Equation (4). Thus, it is clear that the 1D beam model is of extremely high accuracy and can be easily used to model the static behaviour of a delaminated beam.
5 Conclusion

The paper presented an accurate and less complicated method for modelling delaminated beams analytically and using FEA. The Finite Element Analysis was performed using the most generalized approach without making any radical assumptions. The analytical modelling was presented in three forms, a static equilibrium form, a continuity form and a simplified 1-dimensional form. These forms can be easily employed in determining the static behaviour of a delaminated composite beam in order to determine the behaviour if the severity of the damage is known or to determine the severity of damage based on the static behaviour. Although the formulations were based on static point loading conditions with one delaminated layer, the formulation can be readily extended to distributed loading scenarios and multi-layered delaminations. It is intended to extend this work to more generalized dynamic loading conditions such that it can be used effectively in SLDV based damage detection methods.

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References


