Sound Pressure at the Surface of a Cylinder Due to a Point Source

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Abstract

The prediction of the sound pressure field at the surface of a cylinder is of interest in many fields of acoustics, especially for investigating the acoustic loads induced on the payload fairing during launch of a space vehicle. For the launch environment of a space vehicle, the source position and strength determine the external sound pressure excitations on the vehicle. Unfortunately, existing analytical derivations for the acoustic field around a cylindrical geometry are not able to consider the source position, and the source is typically assumed to be at an infinite distance from the cylinder. There is also no scope in these theories to consider the decay of the source strength due to wave propagation. Therefore, effort has been spent here to modify the existing theories to make them applicable to a finite distance between the source and the cylinder and to allow for the decay of the source strength due to wave propagation.

The theory has been used to check the accuracy of the Boundary Element Method (BEM) for calculating acoustic loading at the surface of a cylinder. In addition, the analytical and numerical models have been verified experimentally with measurements of sound pressure patterns at the surface of a cylinder at various frequencies due to a point source positioned at a finite distance from the cylinder surface.

Keywords Point Source, Boundary Element Method, Sound Pressure, Acoustic Scattering.

1. Introduction

The prediction of the acoustic loading at the surface of rocket fairings is challenging because of the complexity of the fairing geometry. The incident waves that strike the circumference of the fairing produce complicated patterns of scattered waves, which make the problem even more complex. However, for simplicity, the majority of previous analytical investigations of external acoustic loading have been conducted using a cylindrical geometry. Theoretical investigations of the external acoustic loading at the surface of a cylinder can be found in previously reported work [1-10]. However, all of these previous analyses are based on the analyses presented by Morse [11], Morse & Feshbach [12] and Morse & Ingard [13]. All of these analyses are either limited to high or low frequency approximations or the governing equations require that the source position be assumed to be at an infinite distance from the cylinder. Thus these previous analyses are not able to model either a source position located at a finite distance from the cylinder or the source strength.

To overcome the aforementioned limitations of previously published work concerned with acoustic loading on a cylinder, the work reported here establishes theoretical and numerical 2-D models for the prediction of the external sound pressure loading at the surface of a cylinder, due to wave propagation from a point source normal to the circumference of the cylinder as shown in Fig.1.

2. Theory

To predict the overall external sound pressure at the surface of a cylinder, it is necessary to consider the pressure due to both incident and scattered waves; the latter occur due to the reflection of waves from the surface. The total external sound pressure at the surface is the superposition of these two waves. In this instance, to calculate the total sound pressure at the surface of a cylinder, the following assumptions have been made: (a) the incident waves are plane waves; (b) the cylinder is infinite in length; and (c) the cylinder wall is hard so that all of the scattered waves proceed outward from the surface.

If all the plane waves traveling normal to the cylinder axis $z$ and in positive $x$ direction as shown in Fig 1 (the directions $\phi=180^0$ and $\phi=0^0$ from the positive $x$ axis will be considered as the front and back of the cylinder respectively), then the incident and scattered sound pressures can be calculated as a function of cylinder radius $a$, wave number $k$ and azimuthal angle $\phi$ as [12]:

$$ p^i(a, k, \phi) = p^o \sum_{m=0}^{M-1} e_m i^m \cos(m\phi) J_m(ka), \quad (1) $$

$$ p^s(a, k, \phi) = -p^o \sum_{m=0}^{M-1} A_m \cos(m\phi) H_m(ka), \quad (2) $$

where $m$ and $M$ represent the term number and total number of terms required in the series summation respectively, $p^o$ is the incident wave amplitude, $e_m = 1$ if $m = 0$ and 2 if $m > 0$, $J_m$ is the Bessel function of the first kind and $H_m$ is the Hankel function of $m$th order for variable coefficients $k$ and $a$. The coefficients $A_m$ and the phase angles $\gamma_m$ can be determined as described in [13]:

$$ A_m = e_m \sin \gamma_m $$

when $m = 0$
where $R = \sqrt{a^2 + r^2 - 2ar \cos \phi}$, $\omega$ is the angular frequency, $\rho_o$ is the density of air and $Q_i$ is the source strength. In general, the source strength is a known or estimated parameter but the unknown is the resultant sound pressure at the surface of the cylinder due to a point source. Hence, the total resultant sound pressure at the surface of the cylinder due to a point source can be expressed using Eqs. (1), (2) and (5) as [15]:

$$p_i(R,a,k,\phi, Q_i) =$$

\[
p_i(R) \sum_{m=0}^{M-1} \varepsilon_m \imath^m \cos(m\phi) \left\{ J_m(ka) - \imath e^{-i\gamma_m} \sin \gamma_m H_m(ka) \right\}
\]  \hspace{1cm} (6)

The experimental arrangement inside the 4.79m x 3.90m x 3.94m anechoic chamber is shown in Fig. 3. A hard cylindrical PVC tube was used and mounted on a turntable which was used to rotate the cylinder 360° about its longitudinal axis. A B&K 4133 half-inch microphone was placed half way along the cylinder length and inside the experimental cylinder such that the microphone face was flat with the outer surface of the cylinder, as shown in Fig. 3. A loudspeaker was placed approximately 4.1m away from the cylinder front face and the speaker centre was approximately at the same height as the microphone position. In the experiment the orientation of the microphone directly towards the speaker was chosen to be 180°/180° or the front side of the cylinder where the sound waves impinge directly from

**4. Experimental Work**

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the speaker. The specifications of this experimental rig are reported in Table 1. The input signal for the loudspeaker generated using a Function Generator (Hewlett Packard 3325B) and provided to the loudspeaker through a Power Amplifier (Playmaster Proseries Three). The output signal of the microphone was calibrated using a Pistonphone and Frequency Analyser (B&K 2120) and measured using Data Acquisition Software known as TracerDAQ for 360° rotation of the cylinder and the embedded microphone.

![Picture of the experimental setup inside the anechoic chamber](image1)

**Fig. 3:** (a) Picture of the experimental setup inside the anechoic chamber, (b) Picture showing the placement of the microphone inside the experimental cylinder.

<table>
<thead>
<tr>
<th>Object</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>Overall length</td>
<td>1.446 m</td>
</tr>
<tr>
<td>Cylinder</td>
<td>Outside diameter</td>
<td>0.114 m</td>
</tr>
<tr>
<td></td>
<td>Wall thickness</td>
<td>0.0035 m</td>
</tr>
<tr>
<td>Microphone</td>
<td>Height (position from the floor)</td>
<td>( \approx 1.13 \text{ m} )</td>
</tr>
<tr>
<td>Speaker</td>
<td>Height (speaker centre from the floor)</td>
<td>( \approx 1.13 \text{ m} )</td>
</tr>
<tr>
<td></td>
<td>Distance from front face of the cylinder</td>
<td>( \approx 4.1 \text{ m} )</td>
</tr>
</tbody>
</table>

5. **Experimental Results and Comparisons**

The sound pressure at the surface of the experimental cylinder (the length to diameter ratio is 12.7) was measured for excitation frequencies of 700 Hz, 1.5 kHz and 3 kHz, respectively. The measurements were taken using TracerDAQ (Data Acquisition Software) for each 0.01s interval for a total time of 81 seconds, which corresponded to one rotation of the cylinder and the embedded microphone. The sample rate was 100Hz and a total of 8,100 samples were taken for each applied frequency. The results for the various frequencies are shown and compared with the analytical and numerical results in Figs. 4 to 6. The results shown are normalised relative to the measured sound pressure level at 180° or \(-180°\). For the analytical and numerical results, Eqs. (6) and (9) respectively, were used. For the numerical calculations, the cylinder surface was divided into 40 elements. Each element had two end nodes and one mid node, and there were in total 80 nodes on the surface, as shown in Fig. 7. As only relative sound pressure distributions were of interest, the source strength was not measured; hence the source amplitude \( 1 + i \text{ m/s} \) was used for the analytical and numerical calculations. From these comparisons it can be seen that the analytical and numerical results show very good agreement with the experimental results. The only difference between them is in the sound pressure magnitudes in some regions at the lower frequencies, especially at the back region of the cylinder. This is because at the lower frequencies a small amount of sound energy is reflected from the walls of the anechoic chamber and this impinges on the surface of the cylinder, which adds to total amount of reinforcement and cancellation that occurs between the two diffracted waves passing around each side of the cylinder. This results in increased sound pressure amplitudes compared with the analytical and numerical results for free field. For higher frequencies almost no sound energy is reflected by the walls and the experimental results show very good agreement with the analytical and numerical results. From these results it is interesting to see that the sound pressure amplitude is relatively smoothly varying at the front face (180° - 180°) of the cylinder and varies more aggressively at the back face (0°) of the cylinder. The reason for this is a result of positive and/or negative interference of the two diffracted waves travelling around the two sides of the cylinder. For small values of \( ka \), the sound pressure amplitudes vary less at the back of the cylinder due to a lesser amount of interference of the two diffracted waves travelling around the two sides of the cylinder, as shown in Fig. 4. On the other hand, when the value of \( ka \) is large, the sound pressure amplitudes vary aggressively at the back of the cylinder because the amount of interference of the diffracted waves increases as the value of \( ka \) increases, as shown in Figs. 5 and 6.

![Comparisons between the experimental, analytical and 2D BEM results for \( f = 700\text{Hz} \) and \( ka = 0.73 \)](image2)

**Fig. 4:** Comparisons between the experimental, analytical and 2D BEM results for \( f = 700\text{Hz} \) and \( ka = 0.73 \).
Comparisons between the experimental, analytical and 2D BEM results for $f = 1.5\text{kHz}$ and $ka = 1.57$.

Comparisons between the experimental, analytical and 2D BEM results for $f = 3\text{kHz}$ and $ka = 3.13$.

Fig. 7: 2D BEM surface discretization of a cylinder of radius $a$, contains 40 elements and 80 nodes on the surface. $X$ and $Y$ represent the coordinates of the cylinder respectively.

6. Conclusions

Comparisons between the analytical, numerical and experimental results presented in this work show that the theoretical model used to predict the sound pressure at the surface of a cylinder when a point source is positioned at a finite distance from the cylinder is valid. From the results it has been found that the sound pressure amplitude is relatively smoothly varying at the front face ($180^\circ/-180^\circ$) of the cylinder and varies more aggressively at the back face ($0^\circ$) of the cylinder. The reason for this is a result of positive and/or negative interference of the two diffracted waves traveling around the two sides of the cylinder. The sound pressure fluctuation increases as the value of $ka$ increases and decreases as the value of $ka$ decreases.

7. References