Two Dimensional and Three Dimensional Acoustic 
Loading on Cylinders Due to a Point source

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The prediction of sound pressure at the surface of a cylinder is of importance in many applications of acoustics, especially to investigate the acoustic loading on a rocket fairing structure. For simplicity, the majority of the investigations of the external acoustic loading on a rocket fairing structure have been conducted using a cylindrical geometry for which the theoretical analyses are available. However, it was found that these analyses are not either able to identify the source position or strength; also there is no scope in those theories to consider the decay of source strength due to wave propagation considering the oblique distance relative to a cylinder. Therefore, effort has been spent here to modify the existing theories to make them applicable for finite distance of source position and decay of source strength due to wave propagation. The Boundary Element Method (BEM) has been used to develop the numerical models for acoustic loading at the surface of a cylinder. The analytical and numerical results were verified with the experimental results, which show very good agreement between the results obtained analytically, numerically and experimentally.

Nomenclature

\[ A = \text{coefficient matrix} \]
\[ a = \text{cylinder radius} \]
\[ C = \text{diagonal matrix} \]
\[ C(Q) = \text{solid angle} \]
\[ G = \text{Green’s function} \]
\[ H_m = \text{Hankel function or Bessel function of the third kind} \]
\[ J_m = \text{Bessel function of the first kind} \]
\[ J_m' = \text{derivatives of the Bessel function of the } m\text{th order} \]
\[ k = \text{wavenumber} \]
\[ M = \text{total number of terms} \]
\[ m = \text{term number} \]
\[ N = \text{total number of surface elements} \]
\[ N(\xi) = \text{element shape function} \]
\[ N_m = \text{Neumann function or Bessel function of the second kind} \]
\[ P = \text{projection point on the cylinder; also, integration point on the boundary} \]
\[ P' = \text{observation point} \]
\[ p = \text{acoustic pressure} \]
\[ p^i = \text{incident sound pressure} \]
\[ p^o = \text{incident sound pressure amplitude} \]
\[ p^s = \text{scattered sound pressure} \]

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\( p_{\lambda} \) = total sound pressure at the cylinder surface  
\( p'(R) \) = spatially dependent incident sound pressure  
\( Q \) = field point or observation point  
\( R \) = distance between the integration point and observation point  
\( r \) = radial distance  
\( S \) = cylinder surface  
\( V_e \) = exterior volume  
\( V_i \) = interior volume  
\( x \) = node position along the \( X \) axis  
\( y \) = node position along the \( Y \) axis  
\( \Phi \) = azimuthal angle  
\( \epsilon_m \) = constant terms used in the equations  
\( \gamma_m \) = phase angle  
\( \rho_o \) = equilibrium density in the fluid  
\( \lambda \) = wavelength

I. Introduction

The prediction of the acoustic loading at the surface of rocket fairing is challenging because of the complexity of the fairing geometry. The incident waves that strike the circumference of the fairing produce complicated patterns of scattered waves, which make the problem even more complex. However, for simplicity, the majority of the previous analytical investigations of external acoustic loading have been conducted using a cylindrical geometry. Theoretical investigations of the external acoustic loading at the surface of a cylindrical geometry can be found in previously reported work (Potter\(^1\), Junger & Feit\(^2\), Gardonio \textit{et al.}\(^3\), Johnson \textit{et al.}\(^4\), Estève & Johnson\(^5\)\(^6\), Nye\(^7\), Friot & Bordier\(^8\) and Fuller\(^9\)\(^10\)). However, all of these previous analyses are based on the analyses presented by Morse\(^11\), Morse & Feshbach\(^12\) and Morse & Ingard\(^13\). In all of these analyses at least one or two of the following shortcomings are present, which are:

a) the analytical solutions of the governing equations are limited to high or low frequency approximations, or to high and low ratios of the diameter of the cylinder compared to the acoustic wavelength.

b) the governing equations require that the source position be assumed to be at an infinite distance from the cylinder and thus they are not able to model either a source position located at a finite distance from the cylinder or the source strength.

c) the governing equations do not allow consideration of the oblique distance between the source and the cylinder for three-dimensional acoustic loading.

Here, an alternative approach is presented that overcomes the aforementioned limitations. Analytical and numerical models are established for the prediction of both two dimensional and three dimensional external sound pressure loading at the surface of a rigid cylinder, due to wave propagation from a point source normal and oblique to the circumference of the cylinder. The Boundary Element Method (BEM) was used to build numerical models to predict the sound pressure loading as a result of normal and oblique incident waves from a point source incident on the cylinder. The BEM software was written in MATLAB and is publicly available software known as Open BEM, which
has been mainly developed by the Acoustic Laboratory, Technical University of Denmark\textsuperscript{14}. The Open BEM codes are able to treat 2D, 3D and axisymmetric problems by using any general computer. In the current work, both 2D and 3D techniques were used to solve the 2D and 3D problem geometries.

An experiment was conducted in an anechoic chamber to measure the sound pressure field at the surface of a cylinder due to a point source positioned at a finite distance from the cylinder surface for the purpose of verifying the analytical and numerical models developed for a point source. The experimental work confirmed the accuracy of the 2D analytical and BEM models for a point source at a finite distance from the cylinder. Both models were extended to a large composite cylinder to predict the circumferential pressure due to oblique incident waves. The large composite cylinder analyzed here was constructed by the Boeing Company for the US Air Force and tested extensively by the Vibration and Acoustic Laboratories at Virginia Tech in the USA\textsuperscript{15}.

II. Analytical Modeling

To predict the overall external sound pressure at the surface of a cylinder, it is necessary to consider the pressure due to both incident and scattered waves; the latter occur due to the reflection of waves from the surface. The total external sound pressure at the surface is the superposition of these two waves. In this instance, to determine analytically the total sound pressure at the surface of a cylinder the following assumptions have been made:

1) the incident waves are plane waves and spherical waves for 2D and 3D acoustic loading, respectively;
2) the cylinder is infinite in length for 3D acoustic loading to avoid the effects of diffraction of sound waves from the ends of the cylinder; and
3) the cylinder wall is hard for both 2D and 3D acoustic loading, so that all of the scattered waves proceed outward from the surface.

If all the plane waves traveling normal to the cylinder axis $z$ and positive $x$ direction as shown in Fig. 1 (the directions $\phi = 180^\circ$ and $\phi = 0^\circ$ from the positive $x$ axis will be considered as the front and back of the cylinder respectively), then the incident pressure can be calculated as a function of cylinder radius $a$, wave number $k$ and azimuthal angle $\phi$ as\textsuperscript{12}

\[
p^i(a, k, \phi) = p^0 \sum_{m=0}^{\infty} e_m l^m \cos(m\phi) J_m(ka). \tag{1}
\]

An approximation to equation (1) is

\[
p^i(a, k, \phi) = p^0 \sum_{m=0}^{M-1} e_m l^m \cos(m\phi) J_m(ka), \tag{2}
\]
where the plane waves are traveling from left to right assuming that the source is at an infinite distance, \( m \) and \( M \) are the term number and total number of terms required in the series summation respectively, \( \varepsilon_m = 1 \) if \( m = 0 \) and 2 if \( m > 0 \), and \( J_m \) is a Bessel function of the first kind of \( m \)th order for variable coefficients \( k \) and \( a \).

![Diagram of incident plane waves](image)

**Fig. 1** Two-dimensional incident plane waves travelling normal to the cylinder axis (positive \( z \) axis is out of the page).

To predict the scattered sound pressure, the distortion of sound waves due to the presence of the cylinder needs to be taken into account because it distorts the sound waves at the surface. The cylinder can be simulated as a source, where all the waves travel outwards from its surface, since the boundary is hard. The scattered sound pressure \( p_s \) can then be calculated as a function of cylinder radius \( a \), wave number \( k \) and azimuthal angle \( \phi \) as

\[
p_s(a, k, \phi) = -p_0 \sum_{m=0}^{M-1} A_m \cos(m\phi) H_m(ka),
\]

where \( H_m(ka) \) is the Hankel function or Bessel function of the third kind of \( m \)th order. The Hankel function is the combination of Bessel functions of the first and second kind (also called Neumann function, \( N_m \)) respectively. The coefficients, \( A_m \), satisfy the hard wall boundary condition at the surface by making \( U^i + U^s = 0 \), where \( U^i \) and \( U^s \) are the incident and scattered velocity respectively. The coefficients \( A_m \) can be determined as

\[
\begin{align*}
A_0 &= \varepsilon_0 e^{-i\gamma_0} \sin \gamma_0, & \text{when } m = 0 \\
A_m &= \varepsilon_m e^{i(m+1)\gamma_m} \sin \gamma_m, & \text{when } m > 0.
\end{align*}
\]

In Eq. (4) the sine and \( e^{-i\gamma_m} \) term have been chosen to ensure the correct combination of the incident and scattered waves and the phase angles, \( \gamma_m \), can be defined depending on the values of \( m \) as

\[
\tan \gamma_0 = -J_1(ka)/N_1(ka), \quad \text{when } m = 0
\]
\[
\tan \gamma_m = -J'_m(ka)/N'_m(ka) \quad \text{when } m > 0. \tag{5}
\]

The total sound pressure at the surface of a cylinder due to both incident and scattered waves can be calculated by simply adding Eqs. (2) and (3), and it can be written in the following form

\[
p'_m(a, k, \phi) = p^i(a, k, \phi) + p^s(a, k, \phi)
= p^0 \sum_{m=0}^{M-1} e_m l^m \cos(m\phi) [J_m(ka) - ie^{-im\gamma} \sin \gamma_m H_m(ka)]. \tag{6}
\]

Equation (6) is only valid for incident plane waves from a source located at an infinite distance and thus is not applicable to the situation for which the source position is at a finite distance from the cylinder. Therefore, in the following sections, the analytical framework has been modified for two dimensional and three dimensional acoustic loading considering a point source placed at a defined position with respect to the cylinder.

A. Monopole

For the following development, the use of a monopole or point source is essential. A point source occurs when the size of the source is small compared with the wavelength, that is \(ka \ll 1\). The sound pressure at a point due to a monopole or point source in an unbounded medium can be given using the free space Green’s function as

\[
G(R) = e^{ikR}/4\pi R, \tag{7}
\]

where \(R = |a - r|\) is the distance between the point source and observation point.

B. Two Dimensional Acoustic Loading

For the two dimensional case, consider a point source placed at a distance \(r\) from the origin of the cylinder as shown in Fig. 2. The first task is to find the distance between the point source and the observation point at the surface of the cylinder. Each observation point at the surface of the cylinder will have an azimuthal angle, \(\phi\). The distances between the point source and the observation points at the surface of the cylinder can be calculated by using a trigonometric formula considering a polar coordinate system as

\[
R = \sqrt{a^2 + r^2 - 2ar \cos \phi}. \tag{8}
\]

Now a spatially dependent factor can be considered, which can give the solution for the sound pressure produced by a point source, as a function of distance between the point source and observation points at the surface of the cylinder. This spatially dependent factor can be written as

\[
p'(R) = -i\omega \rho_o \frac{Q_s}{4\pi R} e^{ikR}, \tag{9}
\]
where \( \omega \) is the angular frequency, \( \rho_0 \) is the density of air and \( Q_s \) is the source strength. The source strength \( Q_s \) is defined as a complex volume velocity amplitude. In general, it is a known or estimated parameter but the unknown is the resultant sound pressure at the surface of the cylinder due to a point source. Hence, the total resultant sound pressure at the surface of the cylinder due to a point source can be expressed using Eqs. (6) and (9) as

\[
p_{r}(R, \alpha, \phi) = p'(R) \sum_{m=0}^{M-1} \varepsilon_m i^m \cos(m\phi) \left[ J_m(ka) - i e^{-ym} \sin y_m H_m(ka) \right]
\]

where the incident pressure amplitude \( p^o \) in Eq. (6) has been replaced by Eq. (9). Equation (10) is similar in form to Eq. (6). The difference is that Eq. (10) can consider a source at a finite distance from the cylinder.

C. Three Dimensional Acoustic Loading

For the three dimensional case, the methodology described above needs to be modified. Fig. 3 shows the geometry of obliquely incident waves impinging on a cylinder from a source located on the \( x \) axis. The cylinder axis is aligned with the \( z \) axis and the \( y \) axis is then perpendicular to these two axes. The angle \( \phi \) defines the position of the observation point at the surface of the cylinder. First, consider the pressure field due to incident waves propagating from a point source in a direction normal to the \( x \) axis. This follows the approach derived in preceding paragraphs. Since in the present case the incident waves are oblique to the cylinder, the resultant circumferential sound pressure at a point on the cylinder will depend not only on its location in the \( x-y \) plane but also its location along the \( z \) axis.

The approach used here considers a finite portion of the cylinder of length \( L \), and \( z \) is the elevation within the region of the cylinder where the resultant pressure is to be evaluated due to a point source positioned on the \( x \) axis as shown in the Fig. 3. Let the projection point of the point of interest \( P' \) on the \( x-y \) plane be \( P \). Then, the projected distance, \( R \), can be calculated using Eq. (8).
The resultant oblique distance can then be calculated as

$$R' = \sqrt{R^2 + z^2}.$$  \hspace{1cm} (11)

Substituting Eq. (11) into Eq. (9), the spatially dependent factor can be determined as

$$p'(R', z) = -i \omega \rho_o \frac{Q_i}{4 \pi R'} e^{ikR'}.$$ \hspace{1cm} (12)

Finally, using this spatially dependent factor, the total resultant pressure at the surface of the cylinder as a function of $z$ can be derived as

$$p'_e(R', \alpha, k, z; \phi, Q_i) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} p'(R', z) e^{-i m \gamma_n} H_m(k a) J_m(k a),$$ \hspace{1cm} (13)

where the first summation on the right hand side is for $N$ point sources and the second summation is in the same form as described in Eq. (10).

### III. Numerical Modeling

The use of the existing Open BEM software is discussed in the introduction section. For numerical modeling, consider an arbitrary shaped structure of volume $V_i$ surrounded by surface $S$ and placed in an acoustic domain of volume $V_e$, as shown in Fig. 4, where $P$ and $Q$ are two points at some distance $r_P$ and $r_Q$ respectively, from the centre of the body. In other words, one is the integration point on the boundary and the other is the field point or source point which may be placed in $V_e$, $V_i$ or on $S$. The free field Green’s function for the Helmholtz equation can be used as the solution for this particular case and can be written as$^{14,18}$
\[ (V^2 + k^2) G(r_p, r_Q) = -4\pi \delta(r_p - r_Q), \quad (14) \]

where \( k \) is the wave number and \( G \) represents Green’s function. To solve Eq. (14) an appropriate boundary condition is required. An evaluation of the unknown pressure gradients from the known velocity distribution of the sources along the surface \( S \), can be used to define the boundary condition. The relation between the velocity at the interface \( S \) to the gradient of pressure is as\(^{14,18}\)

\[
\frac{\partial p(r_p)}{\partial n} = -i\omega \rho_o u_n(r_p), \quad (15)
\]

where \( p \) and \( u_n \) are the acoustic pressure and outward normal particle velocity at point \( P \), respectively, and \( \rho_o \) is the medium density. The radiated and scattered acoustic field within an infinite acoustic domain, \( V_e \), exterior to a finite body of surface \( S \), satisfies Sommerfeld’s radiation condition which is\(^{19}\)

\[
\lim_{r \to \infty} [\gamma (\rho - \rho_o c u_r)] = 0, \quad (16)
\]

\[\text{Fig. 4 Geometry of the exterior boundary problem.}\]

where \( u_r \) is the particle velocity at infinite radial distance \( r \) from the radiator. Therefore, the value of the integral over the far field surface goes to zero for \( r \to \infty \) in Green’s second identity of the Helmholtz equation, which is\(^{14,18}\)

\[
- \int_V \left\{ G(r_p, r_Q) \frac{\partial^2 p(r_p)}{\partial n^2} - p(r_p) \frac{\partial^2 G(r_p, r_Q)}{\partial n^2} \right\} dV = - \int_S \left\{ G(r_p, r_Q) \frac{\partial p(r_p)}{\partial n} - p(r_p) \frac{\partial G(r_p, r_Q)}{\partial n} \right\} dS. \quad (17)
\]

Substituting Eqs. (14) and (15) into Eq. (17) yields

\[
4\pi \int_V p(r_p) \delta(r_p - r_Q) dV = \int_S \left\{ i\omega \rho_o u_n(r_p) G(r_p, r_Q) + p(r_p) \frac{\partial G(r_p, r_Q)}{\partial n} \right\} dS, \quad (18)
\]

which is the required Helmholtz integral equation for the Boundary Element Method (BEM). The integral on the left side of Eq. (18) depends on the position of point \( Q \) either inside, on the surface, or outside the surface.
Hence Eq. (18) can be rewritten in the form

\[ C(Q)p(Q) = \int_S \left\{ i\omega \rho_v u_v(P) G(R) + p(P) \frac{\partial G(R)}{\partial n} \right\} dS, \]  

(19)

where \( R = |r_P - r_Q| \) and the coefficient \( C(Q) \) is the solid angle measured from \( V \), where

\[ C(Q) = 4\pi + \int_S \frac{\partial}{\partial n} \left\{ \frac{1}{R(P, Q)} \right\} dS; \quad Q \in S. \]  

(20)

A. Reduced Integral Formulation for Scattering Problem

Eq. (19) can be used for various types of analysis such as sound radiation from a structure. In this case, the wave superposition technique is often used, where a continuous series of monopoles or dipoles or a combination of the two with known strengths can be distributed over the discrete surface to calculate the sound pressure at a field point. The use of this technique has been discussed in detail in the literature, such as Koopmann et al.\textsuperscript{24}, Fahnline & Koopmann\textsuperscript{25}, Song et al.\textsuperscript{26,27}, Jeans & Mathews\textsuperscript{28}, and Morgan et al.\textsuperscript{29}. However, since the aim of the current work is to investigate the scattering problem from a hard wall, only the second term of the integration in Eq. (19) is important, because the first term, which represents the velocity normal on the surface of the body, can be omitted for a rigid surface. Hence, for scattering from a rigid body, Eq. (19) reduces to

\[ C(Q)p(Q) = \int_S \left\{ p(P) \frac{\partial}{\partial n} \left( \frac{e^{-iKR(P, Q)}}{R(P, Q)} \right) \right\} dS, \]  

(21)

where \( G(R) = e^{-iKR(P, Q)}/R(P, Q) \); is the free field Green’s function. Finally, introducing an incident term, the total sound pressure can be written as

\[ C(Q)p(Q) = \int_S \left\{ p(P) \frac{\partial}{\partial n} \left( \frac{e^{-iKR(P, Q)}}{R(P, Q)} \right) \right\} dS + 4\pi p^i(Q), \]  

(22)

where \( p^i \) represents the incident sound pressure on the field points \( Q \).

B. Numerical Implementation

Eq. (22) can be evaluated numerically by discretizing the boundary surface into \( N \) surface elements. The discretization of the integral Eq. (22) can be approximated by the sum of integrals over the elements as

\[ C(Q)p(Q) = \sum_{i=1}^{N} \left[ \int_{S_i} \left\{ p_i(P) \frac{\partial}{\partial n} \left( \frac{e^{-iKR(P, Q)}}{R(P, Q)} \right) \right\} dS \right] + 4\pi p^i(Q), \]  

(23)

where \( p_i(P) \) are the constant nodal pressures for each element. The elementary solution can be given more precisely by introducing element shape functions as\textsuperscript{30}
\[ C(Q)p(Q) = \sum_{i=1}^{N} \left[ \int_{S_i} \left( p_i(P)N_i(\xi) \frac{\partial}{\partial n} \left( \frac{e^{-ikR(P,Q)}}{R(P,Q)} \right) \right) dS \right] + 4\pi p^i(Q), \]  

(24)

where \( N_i(\xi) \) are the element shape functions. The two-dimensional integral on the right-side of Eq. (24) may be solved by using the one dimensional Gaussian-Legendre quadrature formula. A detailed explanation may be found in the literature\(^{14}\). For a surface formulation where \( r_0 = r_p \), solving \( N \) calculations for all the elements on the surface, Eq. (24) can be written in the following matrix form\(^{30}\)

\[
[C][p] = [p][H] + 4\pi p^i, 
\]

(25)

where \([C]\) is equal to \(2\pi I_N \) (\(I_N\) is the \(N\)-by-\(N\) identity matrix) for the surface formulation, and \([H]\) is an \(H \times H\) coefficient matrix of the \(j\)th calculation points on the \(i\)th elements. Here both \([C]\) and \([H]\) are known but \([p]\) is unknown. Therefore to find \([p]\), Eq. (25) reduces to\(^{30}\)

\[
[A][p] = -4\pi p^i, 
\]

(26)

where \([A] = [H - C]\). On each node of the surface the incident sound pressure \(p^i\) of Eq. (26) can be calculated for a point source using Eqs. (9) and (12) for two dimensional and three dimensional cases respectively.

IV. Experimental Work

In the preceding sections, theoretical and numerical models were derived to predict the sound pressure pattern at the surface of a cylinder for a source positioned at a finite distance from the cylinder. To verify these techniques an experiment was conducted in the anechoic chamber at the University of Adelaide to measure the surface sound pressure patterns of an experimental cylinder for various frequencies due to a point source positioned at a finite distance. The purpose of this experiment was to obtain the sound pressure pattern on the cylinder surface only for comparison with the analytical and numerical results. Hence the source strength was not taken into account. The scope of this experiment was to determine whether it is possible to analytically and numerically predict the sound pressure patterns at the surface of a cylinder due to a source positioned at a finite distance from the cylinder.

A. Experimental Arrangement

The experimental arrangement inside the 4.79m x 3.90m x 3.94m anechoic chamber is shown in Fig. 5. A hard cylindrical PVC tube was used and mounted on a turntable which was used to rotate the cylinder 360° about its longitudinal axis. A B&K 4133 half-inch microphone was placed half way along the cylinder length and inside the cylinder such that the microphone face was flat with the outer surface of the cylinder, as shown.
in Fig. (5). A loudspeaker was placed approximately 4.1m away from the cylinder front face and the speaker centre was approximately at the same height as the microphone position. In the experiment the orientation of the microphone directly towards the speaker was chosen to be ±180° or the front side of the cylinder where the sound waves impinge directly from the speaker. The specifications of this experimental rig are reported in Table 1.

The input signal for the loudspeaker was generated by using a Function Generator (Hewlett Packard 3325B) and provided to the loudspeaker through a Power Amplifier (Playmaster Proseries Three). The output signal of the microphone was calibrated using a Frequency Analyser (B&K 2120) and measured using Data Acquisition System known as TracerDAQ for 360° rotation of the cylinder and the embedded microphone.

![Diagram of experimental setup](image)

**Fig. 5** (a) Picture of the experimental setup inside the anechoic chamber, (b) picture showing the placement of the microphone inside the experimental cylinder.

<table>
<thead>
<tr>
<th>Table 1 Physical parameters of the experimental setup.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Object</strong></td>
</tr>
<tr>
<td>Experimental cylinder</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td>Microphone</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Speaker</td>
</tr>
</tbody>
</table>

**B. Experimental Results and Comparisons**

The sound pressure at the surface of the experimental cylinder (the length to diameter ratio is 12.7) was
measured for single tonal excitation frequencies of 700 Hz, 1.5 kHz, 3 kHz and 5 kHz. The descriptions of each applied frequency in the experiment are given in Table 2. The measurements were taken using a microphone connected to an amplifier which had as its output, a signal proportional to the rms acoustic pressure. This signal was sampled using a Tracer DAQ (Data Acquisition System) at intervals of 0.01s for a total time of 81 seconds, which corresponded to one continuous rotation of the cylinder and the embedded microphone. Thus a total of 8,100 samples were recorded for each applied frequency and the results for the various frequencies are shown and compared with the analytical and numerical results in Figs. 6 to 9, respectively. The results shown are normalised relative to the measured sound pressure level at ±180°. For the analytical and numerical results Eqs. (10) and (26) were used.

Table 2 Descriptions of the applied parameters in the experiment.

<table>
<thead>
<tr>
<th>Circumferential length of the cylinder (m)</th>
<th>Applied frequency, f (kHz)</th>
<th>Wavelength, ( \lambda ) (m)</th>
<th>( ka )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3581</td>
<td>0.7</td>
<td>0.49</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.2286</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.1143</td>
<td>3.13</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.0686</td>
<td>5.22</td>
</tr>
</tbody>
</table>

The microphone diameter of 12.5 mm covers an angular sector of about 12.6° on the cylinder (cylinder radius is of 57 mm). Thus each data point for the analytical and numerical results represents an average over approximately the same angular sector as represented by the experimental data. It is interesting to note that this averaging process gave almost identical results to those obtained without averaging. The averaged results are shown in the Figs. 6 to 9.

For the numerical calculations the cylinder surface was divided into 40 elements. Each element had two end nodes and one mid node (this choice has been made to obtain a better accuracy), and there were in total 80 nodes on the surface, as shown in Fig. 10. 40 elements and 80 nodes were considered for all applied frequencies rather than cell number or nodes per wavelength for each applied frequency. This over-discretization was made in order to obtain a better accuracy compared with the analytical results. Refining the mesh density may improve the accuracy even more.

For the analytical calculations 80 data points were considered over the cylinder surface. The total number of terms, \( M \), required for the series (Eq. 10) to converge was determined as the number of terms required for the ratio of the next term and the summation to be less than \( 10^{-7} \). It was found that the coefficient \( A_m \) and phase angle \( \gamma_m \) which determine the total circumferential pressure at the surface of a cylinder, tend to zero as the values of \( m \) increases, as shown in Figs. 11 and 12 for various values of \( ka \). This trend means that after a certain
value of $M$ in Eq. (10) is reached, there will be a negligible contribution from additional terms in the summation, to the total circumferential sound pressure at the surface of a cylinder. The total number of terms, $M$, for Eq. (10) to converge are reported in Table 3 for various values of $ka$.

It is notable that in the experimental findings the source strength was not measured, hence the source amplitude $1 + i \frac{m^2}{s}$ was used for the analytical and numerical calculations, and the normalised analytical and numerical results are shown in Figs. 6 to 9. From these comparisons it can be seen that the analytical and numerical results show very good agreement with the experimental results. For each frequency there is no significant difference between the analytical, numerical and experimental sound pressure patterns. The only difference between them is in the sound pressure magnitudes in some regions at the lower frequencies, especially at the back region of the cylinder. This is because at the lower frequencies a small amount of sound energy is reflected from the walls of the anechoic chamber and is incident on the surface of the cylinder. This sound energy is added with the total amount of reinforcement and cancellation that occurs between the two diffracted waves passing around each side of the cylinder, and results in increased sound pressure amplitudes compared with the analytical and numerical results. For higher frequencies almost no sound energy is reflected by the walls and the experimental results show very good agreement with the analytical and numerical results. The fibreglass wedges that line the anechoic chamber become more absorptive at higher frequencies compared with the lower frequencies, and the effect of the reflection of sound energy from the walls is not observable in the experimental results at higher frequencies, where the agreement between the experimental, analytical and numerical results is excellent. In the experimental arrangement the cylinder was placed at one corner of anechoic chamber, relatively close to the walls, whereas the speaker was placed at the diagonally opposite corner (see Fig. 5a), to provide the maximum distance between the cylinder and speaker. This type of arrangement was chosen so that the sound waves produced by the speaker become plane waves over the distance from the speaker to the cylinder surface, especially at the lower frequencies where the wavefront is large compared with the higher frequencies. Also, the asymmetry of the experimental results about 0°, such as in Fig. 6 at ±50°, is a result of the difference in the distances from the anechoic chamber walls to the cylinder centre line, and the contribution of reflections of sound energy from the chamber walls, which affected the total amount of reinforcement and cancellation that occurs between the two diffracted waves passing around each side of the cylinder. It can be seen that the sound pressure amplitude at 0° at the back of the cylinder, using numerical calculations is very close to the analytical results. It is worth noting that only 80 surface nodes were used for the numerical calculations, which is very small. However, finer discretization may be done to obtain a
better accuracy which will increase the computational time. The average computational time (each calculation was run five times) of the present numerical calculations considering only 80 nodes is shown in Table 3 for a general computer (Pentium (R) dual-core, CPU 2.5GHz, RAM 1GB).

<table>
<thead>
<tr>
<th>$ka$</th>
<th>Total number of terms, $M$, needed to achieve a relative error $&lt; 10^{-7}$</th>
<th>Average computational time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>7</td>
<td>1.925</td>
</tr>
<tr>
<td>1.57</td>
<td>7</td>
<td>2.0936</td>
</tr>
<tr>
<td>3.13</td>
<td>11</td>
<td>2.0654</td>
</tr>
<tr>
<td>5.22</td>
<td>13</td>
<td>2.0186</td>
</tr>
</tbody>
</table>

From these results it is interesting to see that the sound pressure amplitude is relatively smoothly varying at the front face ($\pm 180^\circ$) of the cylinder and varies more aggressively at the back face ($0^\circ$) of the cylinder. The reason for this is a result of positive and/or negative interference of the two diffracted waves travelling around the two sides of the cylinder. For small values of $ka$, that is when the wavelength is bigger than the circumferential length of the cylinder, the sound pressure amplitudes vary less at the back of the cylinder due to a lesser amount of interference of the two diffracted waves travelling around the two sides of the cylinder, as shown in Fig. 6 for $ka = 0.73$ ($f = 700\text{Hz}$). On the other hand, when the value of $ka$ is large, that is when the wavelength is smaller than the circumferential length of the cylinder, the sound pressure amplitudes vary aggressively at the back of the cylinder because the amount of interference of the diffracted waves increases as the value of $ka$ increases, as shown in Figs. 7 to 9 for excitation frequencies of 1.5kHz ($ka = 1.57$), 3kHz ($ka = 3.13$) and 5kHz ($ka = 5.22$) respectively.

The minimum frequency of 700 Hz and higher frequencies of 1.5 kHz, 3 kHz and 5 kHz were chosen in the experiment so that differences in the sound pressure variations at the surface of the experimental cylinder could be observed for a range of wavelengths relative to the cylinder circumference, and the theoretical modelling accuracy verified.
Fig. 6 Comparisons between the experimental, analytical and numerical results for pressure at the surface of the experimental cylinder of radius 0.057m, for $f = 700\text{Hz}$ and $ka = 0.73$.

Fig. 7 Comparisons between the experimental, analytical and numerical results for pressure at the surface of the experimental cylinder of radius 0.057m, for $f = 1.5\text{kHz}$ and $ka = 1.57$. 
Fig. 8 Comparisons between the experimental, analytical and numerical results for pressure at the surface of the experimental cylinder of radius 0.057m, for $f = 3$kHz and $ka = 3.13$.

Fig. 9 Comparisons between the experimental, analytical and numerical results for pressure at the surface of the experimental cylinder of radius 0.057m, for $f = 5$kHz and $ka = 5.22$. 
Fig. 10 BEM surface discretization of a cylinder of radius 0.057m, contains 40 elements and 80 nodes (each element has two end nodes and one mid node) on the surface. $X$ and $Y$ represents the coordinates of the cylinder respectively.

Fig 11 Characteristic behaviour of $\gamma_m$ as a function of $m$ for different values of $ka$. 
V. Application of the Methods to a Boeing Cylinder

The BEM analysis has been conducted in this work, which treats the 3D geometry of a Boeing cylinder to analyse the sound pressure field. The overall dimensions of the cylinder are given in Table 3. For BEM analysis of the Boeing cylinder which has an aspect ratio of 1.13, a similar technique was applied as used for the BEM analysis of the experimental cylinder. The FEA model of the Boeing cylinder is shown in Fig. 13. A quadratic eight node element shape as shown in Fig 14, was used to build the model because the quadratic shape functions represent a curved element better than the linear shape functions. The model had 492 elements and 1440 nodes at the surface as shown more clearly in Fig. 15. The model was imported into MATLAB for numerical analysis.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder Diameter</td>
<td>97° (2.46 m)</td>
</tr>
<tr>
<td>Cylinder Length</td>
<td>110° (2.79 m)</td>
</tr>
</tbody>
</table>
A point source of arbitrary amplitude $1 + i \text{ m}^3/\text{s}$ was placed at $x = -20D$ m ($D =$ diameter of the cylinder), $y = 0$ m and $z = 0$ m (see Fig. 3 for the source geometry). The source position $x = -20D$ m was chosen because it was reported that for a 1,650-pound-thrust solid fuel rocket engine, the source appeared approximately at a distance 20 diameters downstream of the nozzle exit, in the low frequency range$^{31}$. Excitation at a frequency 250 Hz was applied and the results are shown in Figs. 16(a-b).

It can be seen that maximum sound pressure occurs near the ends of the cylinder because there is a greater diffraction of sound waves occurring from the ends of the cylinder. The sound pressure on the circumferential nodes at a height of $z = 1.67$ m (see Fig. 15) of the cylinder was calculated and compared with the analytical result calculated using Eq. (13) and is shown in Fig. 17. The comparison shows good agreement with the analytical result, but there is a slight difference in pressure magnitudes between the results and is a result of diffraction around the ends of the Boeing
cylinder which is taken into account in the BEM analysis but not in the analytical model. The BEM analysis allows determination of the sound pressure near the ends of the cylinder by considering a quarter-point technique \( (r^{2/3}) \) for particle velocity near the edge of the cylinder, where \( r \) is distance from the edge. This approach approximates the condition that the particle velocity tends to infinity as \( r \) tends to zero. This has been explained in detail by Juhl\(^\text{14}\). Unfortunately, it is not yet possible to consider that analytically.

Figs. 16(a-b) Numerical results for the total sound pressure distribution at the surface of the Boeing cylinder due to a point source of 250 Hz placed at \( x = -20D \text{ m}, y = 0 \text{ m} \) and \( z = 0 \text{ m} \) (see Fig. 3 for the source geometry). [Diameter of the cylinder, \( D = 2.46 \text{ m} \) and reference pressure 20\(\mu\)Pa]

Fig. 17 Total sound pressure comparison between the numerical and analytical results calculated on the circumferential nodes (60 nodes) at a height of \( z = 1.67 \text{ m} \) on the Boeing cylinder, due to a point source of 250 Hz placed at \( x = -20D \text{ m}, y = 0 \text{ m} \) and \( z = 0 \text{ m} \) (see Fig. 3 for the source geometry). [Diameter of the cylinder, \( D = 2.46 \text{ m} \), and reference pressure 20\(\mu\)Pa]
VI. Conclusions

For two-dimensional problem geometry the comparisons between the analytical, numerical and experimental results presented in this work are satisfying, except at the lowest presented frequency, for which sound energy reflection from the walls of the anechoic chamber behind the cylinder is not negligible. For a three-dimensional problem geometry, numerical results were only compared to analytical data for a circular cylinder of infinite length. Despite these different geometries, the agreement between both sets of data is satisfying. The comparisons show that the theoretical model used to predict the sound pressure at the surface of a cylinder when the source is positioned at a finite distance from the cylinder is valid.

From the results it has been found that the sound pressure amplitude is relatively smoothly varying at the front face (±180°) of the cylinder and varies more aggressively at the back face (0°) of the cylinder. The reason for this is a result of positive and/or negative interference of the two diffracted waves traveling around the two sides of the cylinder. The sound pressure fluctuation increases as the value of $ka$ increases and decreases as the value of $ka$ decreases. The present work can be extended in the future to consider the following:

1. The theoretical developments conducted in this work represent only an initial investigation, and as such the cylinder and fairing walls were modeled as rigid for the purpose of determining the external acoustic loading. Hence, the effect of secondary radiation due to vibration of the flexible cylinder and Boeing cylinder or absorption of energy due to their damping behavior has not been taken into account. This could be the main focus of an extended investigation.

2. The effects due to the diffraction around the ends of the cylinder, which have been avoided in the analytical modeling work, could be included in a future investigation to develop the full theoretical model of acoustic loading on finite length cylinders such as the Boeing cylinder.

3. The experimental investigation of three dimensional effects using a scaled down version of the Boeing cylinder.

References


