A Sliding Goertzel Algorithm for Adaptive Passive Neutralizers

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Abstract

A common method used in tuning adaptive-passive tuned vibration neutralizers is to adjust its resonance frequency to match the excitation frequency, which has the characteristic that the phase angle between its vibrating mass and its support is $-90$ degrees. A sliding-Goertzel algorithm is presented and demonstrated for extracting the vibration signals at the frequency of interest. The benefit of using the sliding Goertzel algorithm compared to other methods when used in vibration environments with multiple tones is that additional band-pass notch filtering is not required. This algorithm could also be used for adaptive tuned mass dampers, adaptive Helmholtz resonators, and adaptive quarter-wave tubes.

Keywords: vibration, adaptive passive, algorithms, Simulink, neutralizer

1. Introduction

Adaptive tuned passive vibration neutralizers (ATVNs), dampers, and absorbers are secondary devices that are attached to a primary structure with

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the goal of reducing the disturbance in the primary structure. The analogous devices for acoustic systems include adaptive-passive quarter wave tubes, Helmholtz resonators and Herschel-Quincke tubes. An adaptive-passive system comprises the adjustable resonating device attached to a primary structure, sensors that detect the disturbance in the primary system and attached secondary devices, a control system that implements a control algorithm, and actuators that alter the dynamic characteristics of the attached secondary devices. The control system is typically implemented on a digital processor that executes a control algorithm comprising two main parts:

‘cost function’ evaluation where the data from sensors are used to quantify the disturbance, for example acceleration, kinetic energy, energy density, power, etc.

**tuning algorithm** provides signals to actuators that adjust the dynamics of the secondary devices until the ‘cost function’ reaches the target value. A tuning algorithm sometimes implements ‘coarse’ and ‘fine’ tuning algorithms, where large and small changes are made to the control actuators, respectively.

The original contributions from this paper are a method for calculating the cost function for the tuning algorithm, which is accomplished by implementing a sliding Goertzel algorithm, and a state based control algorithm based on the phase angle of the transfer function between the vibration of the attached device and the primary structure. These algorithms were implemented on a two-stage vibration isolation system that incorporated an adaptive-passive tuned vibration neutralizer. Several tuning algorithms were
evaluated through experimental testing. Whilst theoretically all the tuning algorithms should have worked correctly, the small imperfections in the dynamics of the tuned vibration neutralizers resulted in sub-optimal performance in practice. The experimental testing exposed some unforeseen complications with signal-to-noise issues related to (1) the proposed algorithm, (2) the use of multiple neutralizers, and (3) the minimization of vibration at multiple tonal frequencies. This paper describes how these issues were addressed.

The terms tuned vibration absorber, damper, and neutralizer can be differentiated by the mechanism that they operate to reduce the vibration of a primary structure [1]. Tuned vibration dampers or absorbers are attached to a primary structure to reduce a structural resonance, where the device is tuned to a frequency slightly lower than the structural resonance frequency and is constructed with an appropriate amount of damping. A tuned vibration neutralizer is installed to reduce the vibration in a primary structure due to forced excitation at a particular frequency, where the resonance frequency of the device is tuned to the forcing frequency and has low damping to provide the greatest mechanical impedance to the primary structure [2].

2. Mathematical Model

This section contains a description of a mathematical model of a two-stage vibration isolation mount, which can be found in many vibration textbooks. The model is described here to highlight the inherent difficulties with tuning attached secondary devices.

A two-stage vibration isolation system is shown in Figure 1 where an
adaptive tuned vibration neutralizer is attached to the intermediate mass. The rigid upper mass $m_1$ simulates a piece of vibrating machinery, due to an out-of-balance tonal force $F_1$. The upper mass is supported by a set of upper spring and viscous damper elements $k_1$, $c_1$, with a corresponding lower set with subscripts 2. A heavy intermediate mass $m_2$ improves the vibration isolation characteristics. An adaptive-passive tuned vibration neutralizer is attached to the intermediate mass with mass $m_3$, variable stiffness $k_3$, and viscous damping $c_3$. The stiffness of the adaptive tuned vibration neutralizer $k_3$ can be adjusted by a control system. The displacement of the upper mass $m_1$, intermediate mass $m_2$, and mass on the tuned vibration neutralizer $m_3$ are given by $x_1, x_2, x_3$, respectively. The equations of motion for the system

Figure 1: Schematic of a two-stage vibration isolation system with an attached adaptive tuned vibration neutralizer.
can be written in matrix form as

\[-\omega^2 \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \mathbf{x} + j\omega \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1 + c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \]

(1)

where \( \mathbf{x} = [x_1 \ x_2 \ x_3]^T \) is the vector of displacements of the masses, \( \omega \) is the frequency in radians per second. Note that the only driving force on the system acts on mass \( m_1 \). Eq. (1) can be written in matrix form and solved as

\[-\omega^2 \mathbf{M} \mathbf{x} + \mathbf{K} \mathbf{x} + j\omega \mathbf{C} \mathbf{x} = \mathbf{f} \]

(2)

\[\mathbf{x} = [\mathbf{M} + j\omega \mathbf{C}]^{-1} \times \mathbf{f} \]

(3)

Figure 2 shows the phase of the transfer function \( x_3/x_2 \) where the resonance frequency of the TVN is tuned to the forcing frequency (\( \omega_r = \omega_f \)). There is a sharp change in the phase angle from 0 degrees to -180 degrees at the resonance frequency of the TVN, and the slope depends on the amount of damping in the TVN - the less damping in the TVN the steeper the phase change.

A tuned vibration neutralizer presents the greatest mechanical impedance to the primary structure when vibrating at its resonance frequency and its internal damping \( c_3 \) is minimized. For these devices, increasing its damping does not widen its bandwidth, and hence adding damping does not improve
performance robustness to mistuning [3] [2, see Fig. 4(a)]. The only non-adaptive way to improve robustness to mistuning is to increase the mass $m_3$ of the TVN such that even if the TVN is mistuned, it will still provide adequate vibration attenuation [2, see Fig. 4(b)]. However increasing the mass of a TVN is often not a viable option.

Three phase states can be defined where the phase angle is:

- close to 0 degrees = State 1,
- close to -180 degrees = State 3, and
- in the region of the (near) step change between 0 to -180 degrees = State 2.

The goal of an effective tuning algorithm is to alter the dynamics of the TVN so that it is resonant, which occurs when the phase angle is -90 degree (in State 2). The sharp swing in the phase response presents atypical
challenges for designing a stable and practical control system. The response of most systems (plant) involves relatively gradual changes in response due to changes in the input conditions, and textbook control algorithms such as PID, H-∞ and others are suitable. Unfortunately due to the step-response in the cost-function (phase angle) for this problem, many of these textbook control algorithms are not suitable. Researchers have created several algorithms for this control problem and are described below.

3. Previous Methods

Previous researchers have developed several cost-functions and control algorithms for adaptive-passive control systems, and von Flowtow et al. (1994) [3] and Sun et al. (1995) [4] provide good reviews of them.

A common control algorithm used by previous investigators is to use the phase angle between the vibration of the mass on the TVN and the primary structure as the cost-function, and adapt the TVN’s stiffness, thereby altering its resonance frequency, until the phase angle is -90 degrees. The method used to calculate phase angle is the multiplication of the acceleration signals of the primary structure (the intermediate mass $m_2$ in this case) and the mass of the TVN ($m_3$), and calculate a moving time average. For the case where the acceleration of the primary structure and the TVN mass are given by

$$\ddot{x}_2 = \dot{X}_2 \cos(\omega t) \quad (4)$$

$$\ddot{x}_3 = \dot{X}_3 \cos(\omega t - \theta) \quad (5)$$

where $\ddot{X}_2$, $\ddot{X}_3$ are the acceleration amplitudes of the intermediate mass and TVN mass respectively, then the time average product of these two tonal
signals over period $T$ is given by [5]

$$
\bar{x}_2 \bar{x}_3 = \frac{1}{T} \int_{0}^{T} \dot{x}_2 \dot{x}_3 \, dt = \frac{\ddot{X}_2 \ddot{X}_3}{2} \cos(\theta) \tag{6}
$$

This calculation results in a signal that is offset from zero, does not have sinusoid components, and is proportional to the cosine of the phase angle. When the TVN is optimally tuned, the phase angle will be $-90$ degrees, so $\cos(-90) = 0$, and hence the time average product will equal zero. This method has been used by several researchers such as Refs [5, 6, 7].

One of the attractive features of using this method to calculate the phase angle, is that it is computationally simple and hence fast to compute using digital processors, or using analog circuits [7]. However this method does not address some practical issues, namely:

- It cannot be used if the signals contain harmonics of the primary frequency, or two sinusoid components or signal noise, unless the signals are filtered to remove the unwanted components leaving only the vibration signals at the frequency of interest. This requires the use of tracking narrow band-pass filters, that adds complexity to the system. All filters have the potential to introduce amplitude and phase shifts on the original signal and care has to be taken not to unintentionally alter the signals.

- When transient vibration occurs, broadband vibrational energy is injected into the sensors that can be unrelated to the vibration frequency of interest. Again, it is necessary to remove these spurious vibration signals.
Franchek et al. (1993) [8] proposed a feedback tuning algorithm that Ryan et al. (1994) [9] subsequently experimentally tested. They used a proportional gain feedback control system based on the acceleration amplitude of the base structure. In their system, the feedback error sensor was a rectified and low-pass filtered accelerometer signal and was used in an analog feedback circuit to minimize the vibration of the base structure. Their paper describes theoretical and experimental tests that demonstrate that the proposed control system was effective at reducing the vibration of the base structure. Franchek (1996) [10] later used this proportional feedback control algorithm, based on the vibration amplitude of the primary structure, to control the horizontal vibration in an experimental model of a 4 storey building.


A heuristic approach, called a ‘fuzzy’ controller, was proposed by Lai and Wang (1996) [12]. Their controller uses estimates of the total system energy that is to be minimized and provided some theoretical results. They also commented that in general the stability of fuzzy control systems cannot be guaranteed.

Long et al. (1995) [6] and Brennan et al. (1996) [5] used a combination of two control algorithms: a course tuning algorithm based on a lookup table of the required stiffness for a given excitation frequency, and then a fine tuning algorithm is used based on a steepest descent method. The algorithm to
update the stiffness of the absorber is given by [5]

$$k_{\text{new}} = k_{\text{old}} + \mu \ddot{x}_2 \ddot{x}_3$$  \hspace{1cm} (7)

where $\mu$ is the convergence coefficient, and the expression $\ddot{x}_2 \ddot{x}_3$ is calculated using Eq. (6) that provides a measure of the phase angle between the vibrating mass of the system and the base structure. When the device is tuned correctly, the phase angle is -90 degrees and the expression $\ddot{x}_2 \ddot{x}_3$ is close to zero, hence the value of the new stiffness is the same as the old value of stiffness. Readers may note the similarity of Eq. (7) to the update equation for the Least-Mean-Squares algorithm.

Nagata et al. (1999) [13] used a simple control algorithm that measured the vibration of the base structure, altered the resonance frequency of the device, re-measured the vibration amplitude; if the vibration amplitude of the second measurement was smaller than the first, then the device would continue to alter the stiffness in the same sense otherwise the algorithm would change the stiffness of the device in the opposite sense.

Another method that has been devised is to use a feed-forward adaptive filter structure, where a sinusoidal reference signal synchronized with the disturbing harmonic force must be generated [14]. In most rotating machinery, although it is relatively easy to obtain a tachometer pulse signal, the systems utilizing a feed-forward control algorithms it can be necessary to synthesize a sinusoidal signal from the tachometer pulse signal which requires another algorithm and process to operate in real-time in addition to the control system.

Hill et al. [15] opted for a simple control algorithm for an adaptive tuned vibration neutralizer used to attenuate the vibration of an electrical trans-
former. Their algorithm adapted the stiffness in increments based on comparison of the vibration level of the primary structure - if the vibration amplitude increased after the increment in stiffness, then the stiffness was altered in the opposite direction.

Cronje et al. (2005) [16] used a feedback control algorithm using a displacement and velocity signals. Their tunable vibration isolator was unusual in that it used a wax actuator that was capable of exerting a force up to 500N and its properties were altered by a hot-air gun.

4. Proposed Algorithms

The algorithms proposed here include: a new method for evaluating the (cost-function) phase angle response for an adaptive TVN, and three tuning algorithms. The algorithms are described below.

4.1. Cost Function

4.1.1. Goertzel Algorithm

To overcome the practical difficulties with determining the phase angle by the quadrature method in Eq. (6), a new method of determining the phase-angle was developed. The Goertzel algorithm [17] is a computationally fast method of calculating the complex Fourier transform at a single frequency or bin. This method can be used to calculate the complex transfer function (and hence amplitude and phase angle) between the vibration of the mass on the adaptive TVN and the primary structure. The commonly used Fast Fourier Transform (FFT) algorithm requires the calculation of all frequency 'bins' and hence takes longer to calculate than the Goertzel algorithm.
An important feature of using the Goertzel algorithm is that the frequency of interest can be selected precisely, whereas the FFT can only calculate the Fourier transform at frequencies that are multiple of the frequency spacing. The frequency spacing for the FFT method is the sampling frequency divided by half the number of analysis ‘lines’ or bins. The number of bins must be a power of 2, such as $2^9 = 512$ lines. For example, if the sampling frequency is 500 Hz and $(N =) 512$ bins are used, then frequency spacing is $500/(512/2) = 1.95$ Hz. However with the Goertzel algorithm, the number of lines, or frequency resolution, is selected $(N)$, and the frequency of analysis $(k)$ is selected that can be a non-integer within the range from $1-N$. Hence it is possible to calculate the Fourier transform at the exact reference frequency.

4.1.2. Sliding Goertzel Algorithm

A problem was identified during the experimental testing when using the standard Goertzel algorithm that the phase measurements were less accurate when the signal amplitudes were small, or when there was a second tonal signal with a similar frequency. In the experimental testing two adaptive TVNs were used. It was found that if one of the systems had reduced the vibration significantly, but the second system had not, the stability of the first system was compromised. The problem was identified as a limitation with the effective signal-to-noise ratio for the standard Goertzel algorithm. This problem was rectified by replacing the standard Goertzel algorithm with the sliding Goertzel algorithm [18, 19, 20] that does not suffer the same signal-to-noise problems. The ‘sliding’ nature of the algorithm enables the calculation of the Fourier coefficients in less than one signal period, which is advantageous for providing rapid updates of the value of the cost-function.
The sliding Goertzel algorithm implements the z-domain transfer function of \([19, 21]\)

\[
H_{SG}(z) = \frac{1 - \exp^{-j \frac{2\pi k}{N} z^{-1}}(1 - z^{-N})}{1 - 2 \cos\left(\frac{2\pi k}{N}\right)z^{-1} + z^{-2}}
\]  

(8)

where \(N\) is the N-point Discrete Fourier transform (DFT), \(k\) is the frequency variable. Hence the single analysis frequency is \(kf_s/N\) Hz where \(f_s\) is the sampling frequency.

The sliding Goertzel algorithm was implemented in the Matlab-Simulink application and a schematic of the model is shown in Figure 3.

4.2. Control Algorithms

The phase response of the system shown in Figure 2 for states 1 and 3 is markedly different from state 2. Hence, it is appropriate to devise appropriate control algorithms for each state.

The control algorithm developed for states 1 and 3 can be described as a coarse tuning algorithm, where the goal of the algorithm is tune the dynamics of the TVN such that the system is in state 2, which can involve large changes in the stiffness. The control algorithms developed for state 2, where there is a steep change in the phase response, can be described as a fine tuning algorithms, where small changes in stiffness are required.

4.2.1. Coarse Tuning

The coarse control algorithm for states 1 and 3 can be described as follows: if the phase angle of the system is in state 1, the actuator is moved at high speed so that the resonance frequency of the TVN increases and the phase angle approaches state 2. If the phase angle of the system is in state 3, the actuator is moved at high speed in the opposite direction, so that the
Figure 3: Schematic of the sliding Goertzel algorithm implemented in Simulink.
resonance frequency of the TVN decreases and the phase angle approaches state 2.

Some previous researchers have used ‘lookup tables’ that provide a mapping between the measured cost-function (e.g., phase angle, resonance frequency) and the required actuator position. However this requires that the dynamics of the system remain constant over time, so that definitions for the lookup table are appropriate. This is suitable for short-term laboratory testing, however in practical installations the correct mapping cannot be guaranteed and ongoing re-calibration of the lookup table is required.

4.2.2. Fine Tuning

When the system is operating in state 2, a fine tuning algorithm is required. Several algorithms were tested that included:

- Adjusting the phase angle so that it approached a ‘dead-band’ around −90 degrees that was approximately 40 degrees wide. Hence, when the phase angle was within the range from -110 to -70 degrees, the adaptive TVN was deemed to be correctly tuned and adaptation of the stiffness was ceased.

- A proportional controller that attempted to minimize the inverse of the amplitude of the transfer function between the vibration of the TVN mass and the intermediate mass. The effect was that the controller would select a speed for the actuator that was proportional to the amplitude of the ratio of the vibration of the intermediate mass and the TVN mass. When the TVN was tuned, the amplitude of the intermediate mass should be minimized and hence the adaptation should
cease. The transfer function of the TVN was used rather than the vibration amplitude of the base support alone, as it is independent of the driving force, whereas the vibration amplitude of the primary structure (intermediate mass in this experiment) is related to the driving force.

- A stepping algorithm was used that compared the current and previous measurement of the vibration amplitude of the TVN base. If the vibration amplitude of base was decreasing, it was assumed that the actuator must be moving in the correct direction. If the vibration amplitude was not decreasing, then the direction of movement of the actuator was reversed. This algorithm again worked reasonably well. Issues were encountered with the stability of the automatic control system due to transient events. A transient vibration would cause an increase in the vibration measured at the base of the TVN, and hence the control system would start adapting again to find the configuration that causes the minimum vibration amplitude. As there is no defined end-point for the minimum vibration amplitude, the algorithm would continue to hunt. This algorithm worked reasonably well for a laboratory environment, but in a practical implementation, the automatic control system would continue to hunt.

There are variants of these algorithms that will perform satisfactorily. For example, as one reviewer suggested, the fine tuning algorithm could include an integrator in parallel to improve stability for systems that rapidly change. However for the experimental system under investigation here, this is not required.
All of the algorithms described should theoretically work adequately, however there were practical issues with regard to the use of multiple TVNs that limited the robustness of the algorithms.

The first two fine tuning algorithm implicitly require that the TVN will have the highest mechanical impedance, and hence provide the greatest vibration reduction of the intermediate mass, when the phase angle is \(-90\) degrees. Although this is accurate for a theoretical model, it cannot be guaranteed for a practical system. The TVNs used in this work comprise two masses on cantilever arms, so there are two mass-spring-damper systems, and they are tuned so that they should behave as a single TVN with the same resonance frequency, however this could not be guaranteed. The device has to be initially calibrated by locating each mass very accurately on the cantilever arm so that both mass-spring systems have the same resonance frequency. Further, as the masses are moved along the cantilever arms, they must exhibit identical resonance frequencies, so that both masses are always vibrating in-phase thereby providing only translational impedance. This is difficult to achieve in practice. In the small frequency range around the resonance of the TVNs, the vibration of the masses on TVN can be out-of-phase, and hence the variation in phase angle with change in the position of the mass will no longer be monotonic. When this occurs the vibration level of the primary structure (intermediate mass) is not necessarily minimized by tuning the phase angle to \(-90\) degrees. In this situation, the stepping algorithm described above is an alternative that will minimize the vibration amplitude of the primary structure.
5. Experiment

5.1. Test Setup

The control algorithms described above were tested on a laboratory setup of a two-stage vibration isolation system shown in Figure 4 and 5. An electrodynamic was used to simulate vibration from a piece of rotating machinery. The shaker was attached to the upper mass using a thin flexible rod (stinger), which vibrated a force transducer. Two adaptive tuned vibration neutralizers were attached to the intermediate mass. Rubber vibration isolators separated the upper mass, intermediate mass and ground plate. A Bruel
Figure 5: Photograph of the vibration isolation experiment.
and Kjaer Pulse signal analyzer was used to record the vibration spectra from the force transducer and four Bruel and Kjaer accelerometers attached to the intermediate mass. The force exerted by the shaker on the upper mass was used to normalize all the measured results.

Figure 6 shows a schematic of the adaptive-passive tuned vibration neutralizer which has two masses supported on cantilevers. The design is similar to Ref. [15], where the masses translate along cantilever arms by threaded rods that are connected to an electric motor at the base of the neutralizer. The TVN has low damping with a measured quality factor of $Q = 114$. Accelerometers were attached to the end of the cantilever arms and the base support. The base support of the TVN was attached to the intermediate mass, hence by minimizing the vibration at the base support of the TVN also minimized the vibration of the intermediate mass. The control system attempted to position the masses on the cantilever arms such that the phase angle between the vibration of the TVN mass and the base support was $−90$ degrees. Two TVN units were attached to the intermediate mass, one unit on each side.
The hardware used to implement the control system was a DSpace 1104, which implemented the control algorithms that were programmed using the Matlab-Simulink software.

5.2. Results and Discussion

Experiments were conducted to verify that the proposed method of using a sliding Goertzel algorithm to determine the phase angle (cost function), would operate correctly. Several control algorithms were also experimentally tested. The speed of adaptation and magnitude of vibration attenuation are not the focus of the work. To improve the amount of vibration attenuation, the masses on the cantilever arms could be increased, and to improve the speed of convergence, a faster actuator could be used.

Figure 7 shows the normalized acceleration which was calculated as the acceleration at the base of the TVN divided by the force from the excitation shaker, as the two TVNs adapted over time. The control algorithm used for this experiment was the coarse-tuning algorithm described in Section 4.2.1, and the stepping algorithm for fine tuning described in Section 4.2.2. The excitation shaker was initially set to 30Hz and the TVNs adapted to minimize the vibration level of the intermediate mass. The spectrum analyzer was set to record vibration spectra at the target frequency of 40Hz, and hence the initial 10s of results is random. After 10s the excitation shaker was set to 40Hz and the control system commenced adapting. It can be seen that the normalized acceleration initially increases, which is expected, and then decreases as the TVN enters state 2 and remains stable with a vibration reduction of about 35dB after approximately 50 seconds.

The main point from this experiment is confirmation that both the sliding
Figure 7: Change in the normalized vibration level of the intermediate mass with the operation of the adaptive TVN.
Goertzel algorithm for determining the phase angle and the control algorithm can tune the TVN and remains stable. The two other fine-tuning algorithms were also tested and had similar results, but are not presented for sake of brevity.

6. Conclusions

A sliding Goertzel algorithm was presented for use in determining the phase angle in an adaptive tuned vibration neutralizer. This algorithm provides an effective and robust method for extracting tonal vibration signals at the frequency of interest from a complex vibration spectra, whereas other methods require tracking filters. The sliding Goertzel algorithm requires less computational resources than the standard Fast Fourier Transform and can calculate the Fourier coefficients at the exact frequency of interest.

Several tuning algorithms were presented and experimentally tested on a two-stage vibration isolation system with adaptive tuned vibration neutralizers. The results confirmed that the sliding Goertzel and tuning algorithms can be used effectively in adaptive-passive vibration control systems, and could also be used in adaptive-passive acoustic control systems.
References


