Prediction of Acoustic Loads on a Launch Vehicle Fairing During Liftoff

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During liftoff of a launch vehicle, the acoustic pressure fluctuations caused by the engine exhaust gases produce high noise levels inside the fairing cavity and can damage the payload. Work presented in this paper investigates the nature of the external acoustic pressure distribution, in the frequency range from 50Hz to 400Hz, on the fairing of a launch vehicle during liftoff. The acoustic pressure acting on a representative small launch vehicle fairing was estimated from the complex acoustic field generated by the rocket exhaust during liftoff. The estimation procedure involved the use of a unique source allocation technique which considered acoustic sources along the rocket engine exhaust flow. Numerical and analytical results for the acoustic loads on the fairing agree well.

Nomenclature

\( A \) = coefficient matrix

\( a \) = cylinder radius, m

\( b \) = frequency band number

\( C \) = diagonal matrix

\( C(Q) \) = solid angle, rad

\( c \) = speed of sound in the exhaust flow, m/s

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\( D_e \) = nozzle exit diameter, m
\( DI \) = directivity index, dB
\( d \) = projected distance, m
\( E \) = total number of surface elements
\( E(\zeta) \) = element shape function
\( F \) = thrust of rocket engine, N
\( f \) = frequency, Hz
\( \Delta f_b \) = frequency bandwidth, Hz
\( G \) = Green’s function
\( H \) = coefficient matrix
\( H_m(z) \) = Hankel function or Bessel function of the third kind of \( m \)th order and argument, \( z \)
\( I_N \) = identity matrix
\( J_m(z) \) = Bessel function of the first kind of \( m \)th order and argument, \( z \)
\( k \) = wavenumber
\( L_w \) = overall acoustic power level, dB
\( L_{w,b} \) = sound power level for each frequency band, dB
\( L_{P,b,\phi} \) = sound pressure level on the circumference of the vehicle, dB
\( L_{P,OA,\phi} \) = overall sound pressure level, dB
\( M \) = total number of terms
\( M_e \) = Mach number
\( m \) = term number
\( n \) = number of nozzles, and particle outward velocity direction
\( p, p' \) = acoustic pressure, Pa
\( p' \) = incident sound pressure, Pa
\( p'' \) = incident sound pressure amplitude, Pa
\( p''' \) = scattered sound pressure, Pa
\( p''_t \) = total sound pressure at the surface of a cylinder, dB
\( p''' \) = spatially dependent factor
\( p_i \) = constant nodal pressure on \( i^{th} \) element, Pa

\( P \) = field point or observation point

\( Q_s \) = source strength, \( \text{m}^3/\text{s} \)

\( Q \) = projection point, and integration point on the boundary

\( R \) = distance between the integration point and observation point, and resultant oblique distance, m

\( r \) = radial distance, and distance of the source from the nozzle exit, m

\( r_P \) = distance of observation point, m

\( r_Q \) = distance of integration point, m

\( S \) = cylinder surface area, \( \text{m}^2 \)

\( U_e \) = exhaust velocity, m/s

\( u_n \) = outward normal particle velocity, m/s

\( V \) = volume of a problem geometry, \( \text{m}^3 \)

\( V_e \) = exterior volume, \( \text{m}^3 \)

\( V_i \) = interior volume, \( \text{m}^3 \)

\( W_p \) = acoustic power for each frequency band, Watt

\( W(f) \) = sound power per Hz, Watt

\( W_{OA} \) = overall acoustic power, Watt

\( X \) = \( X \) component of the cylindrical coordinate axes

\( x_1 \) = axial distance of the observation point on the vehicle from the nozzle exit, m

\( x_2 \) = vertical distance from the nozzle exit of the sources along the flow axis, m

\( x_3 \) = distance of the source from the vehicle axis along the flow axis, m

\( Y \) = \( Y \) component of the cylindrical coordinate axes

\( Z \) = \( Z \) component of the cylindrical coordinate axes

\( z \) = elevation height, m

\( \theta \) = angle from the horizontal axis of the noise radiation, deg

\( \phi \) = azimuthal angle, deg

\( \beta \) = incline angle between the line joining the observation point to the source location and the normal of the vehicle axis, deg
\( \varepsilon_m \) = constant terms used in the equations
\( \gamma_m \) = phase angle, rad
\( \omega \) = angular frequency, rad/s
\( \rho \) = density of air in the exhaust flow, kg/m\(^3\)
\( \rho_o \) = equilibrium density of the fluid, kg/m\(^3\)
\( \lambda \) = wavelength, m

I. Introduction

To determine an appropriate source model and configuration that will accurately represent the sound field generated by the rocket motor, it is necessary to take into account various complicating parameters such as vehicle geometry, propulsion device configuration, flow configuration and launch pad configuration. The sound fields generated by large propulsion devices mix with the exhaust flow, and are also deflected by the deflector below the vehicle and radiated back towards the vehicle. The reflections and diffractions of the sound fields by the vehicle stand and other objects on or near the launch pad may have a significant effect on the acoustic pressure fluctuations on the vehicle. However, when all of the necessary parameters are known, it should be possible to predict the acoustic loads on the vehicle. Nevertheless, the prediction of the acoustic loading on launch vehicles continues to face tremendous theoretical and technological challenges due to the many complicated parameters and complex geometries involved.

In the past, the prediction of acoustic loads generated by the vehicle propulsion system has been undertaken using semi-empirical analytical methods based on experimental data\(^1\)-\(^4\). The prediction methods for chemical rockets have been outlined in NASA-SP-8072\(^4\), a space-vehicle design criteria document. The recommended methods predict the acoustic loads at a point on the vehicle using two different source allocation methods based on experimental data concerning the sound power spectrum radiated by the propulsion system, the lateral deflection of the jets exhausted by the rocket engines, and allocation of the noise generation sources along the exhaust stream. One of these methods, known as the unique source allocation method, assumes that there is a single equivalent point source location along the exhaust flow axis for each frequency. The other method, the non-unique source allocation method, assumes that there are a number of point source locations making a line source along the exhaust flow axis for each frequency. The scattering effects from the vehicle surface are completely omitted in these methods.
Potter\textsuperscript{2} described an analytical method to implement scattering effects from the surface, which involved the source allocation technique, and was based on the theoretical analyses for a simple cylinder presented by Morse\textsuperscript{5}, Morse & Feshbach\textsuperscript{6} and Morse & Ingard\textsuperscript{7}. Although, Potter’s technique may be partially suitable for analyzing the total sound pressure including the radiation effects from a cylinder, it is not appropriate for dealing with scattering from a hard cylindrical wall.

Malbequi \textit{et al.}\textsuperscript{8} also considered the empirical source allocation technique to analyse the scattering of incident acoustic waves by the surface of the Ariane IV space launcher, using boundary integral methods. However, this analysis examined the sound field in the vicinity of the launcher rather than at the launcher surface and so could not be used to determine the acoustics loads at the surface, which are necessary for predictions of sound transmission into the payload bay.

This paper is concerned with the investigation of the acoustic loads, including the scattering effects, on an RSLVF, using a unique source allocation method as suggested in NASA-SP-8072\textsuperscript{4}. Note that the non-unique source allocation method could be the subject of a future investigation. The unique source allocation method was used here to allocate unique source locations for each one-third-octave frequency band in the range from 50Hz to 400Hz, and the method was extended, using the analytical and numerical tools developed by the authors, to examine the acoustic loads at the surface of the RSLVF. The Boundary Element Method (BEM) software used for the numerical predictions was Open BEM, an open source code mainly developed by the Acoustic Laboratory, Technical University of Denmark\textsuperscript{9}. The Open BEM codes are able to treat 2D, 3D and axisymmetric problems. In the current work, the 3D technique was used to solve the 3D problem geometry. For the purpose of determining the external acoustic loading, the fairing walls were modeled as rigid and thus the blocked loading was determined. As the acoustic medium is air, this was considered to be a good approximation of the exact analysis that would include the acoustic radiation loading as well.

**II. Analytical Modeling**

The theoretical descriptions presented here are for an idealized long cylinder, since the geometry of a launch vehicle fairing is cylindrical at any given cross section. It is assumed that there will be very little effect on the external sound pressure field due to the diffraction of sound waves from the ends of the cylinder. The diffraction of sound waves from both ends of the cylinder is thus neglected in the following analysis. Also the cylinder wall is
considered to be hard so that all of the scattered waves proceed outward from the surface. Fig. 1 shows the geometry of obliquely incident waves impinging on a cylinder from a source located on the $X$ axis. The cylinder axis is aligned with the $Z$ axis and the $Y$ axis is then perpendicular to these two axes. The angle $\phi$ defines the position of the observation point at the surface of the cylinder. Since in the present case the incident waves are oblique to the cylinder, the resultant circumferential sound pressure at a point on the cylinder will depend not only on its location in the $X$-$Y$ plane but also its location along the $Z$ axis.

The approach used here considers a finite portion of the cylinder of length $L$, and $z$ is the elevation within the region of the cylinder where the resultant pressure is to be evaluated due to a point source positioned at a distance $r$ on the $X$ axis as shown in Fig. 1. Let the projection of the point of interest $P$ on the $X$-$Y$ plane be $Q$. Then, the projected distance, $d$, can be calculated as follows

$$d = \sqrt{a^2 + r^2 - 2ar \cos \phi}. \quad (1)$$

The resultant oblique distance can then be calculated as

$$R = \sqrt{d^2 + z^2}. \quad (2)$$

Now a spatially dependent factor can be considered, which can give the solution for the sound pressure produced by a point source as a function of distance between the point source and observation points at the surface of the cylinder. This spatially dependent factor can be determined as$^{10,12}$

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Fig. 1: Geometry of obliquely incident waves for three-dimensional cylinder coordinate axes.

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\[ p' (R, z, t) = -i \omega \rho_0 \frac{Q_s}{4\pi R} e^{i(kR - \omega t)}. \]  

Finally, using this spatially dependent factor, the total external sound pressure at the surface of the cylinder can be derived as the superposition of the incident wave pressure \( p' \) and scattered wave pressure \( p^s \) as follows:

\[
p'^s_{a}(R, k, a, z, \phi, Q_s, t) = p'^i(R, k, a, z, \phi, Q_s, t) + p^s(R, k, a, z, \phi, Q_s, t)
= p'(R, z, t) \sum_{m=0}^{M-1} e_m i^m \cos(m\phi) \left[ J_m(ka) - i e^{-i\gamma_m} \sin \gamma_m H_m(ka) \right],
\]

where the summation is for \( M \) number of terms required in the series calculation, \( e_m = 1 \) if \( m = 0 \) and 2 if \( m > 0 \).

### III. Numerical Modeling

Consider an arbitrary shaped structure of volume \( V_i \) surrounded by surface \( S \) and placed in an acoustic domain of volume \( V_e \), as shown in Fig. 2, where \( Q \) and \( P \) are two points at some distance \( r_Q \) and \( r_P \) respectively, from the centre of the body. In other words, one is the integration point on the boundary and the other is the field point or source point which may be placed in \( V_e \), \( V_i \) or on \( S \). The Helmholtz integral equation can be written in the form \(^9, 13-17\)

\[
C(P) p(P) = \int_S \left[ i \omega \rho_0 u_n(Q) G(R) + p(Q) \frac{\partial G(R)}{\partial n} \right] dS,
\]

where \( R = |r_Q - r_P| \) and the coefficient \( C(P) \) is the solid angle measured from \( V \):

\[
C(P) = 0; \quad P \in V_i
= 4\pi + \int_S \frac{\partial}{\partial n} \left( \frac{1}{R(Q,P)} \right) dS; \quad P \in S
= 4\pi; \quad P \in V_e
\]

Since the aim of the current work is to investigate the scattering problem from a hard wall, only the second term of the integration in Eq. (5) is important, because the first term, which represents the normal velocity on the surface of the body, can be omitted for a rigid surface. Hence, for scattering from a rigid body, Eq. (5) reduces to

\[
C(P) p(P) = \int_S p(Q) \frac{\partial}{\partial n} \left( \frac{e^{-ikR(Q,P)}}{R(Q,P)} \right) dS + 4\pi p^i(P),
\]

where \( G(R) = e^{-ikR(Q,P)}/R(Q,P) \) is the free field Green’s function and \( p^i \) represents the incident sound.
pressure on point $P$. Eq. (7) can be evaluated numerically by discretizing the boundary surface into $E$ surface elements. The discretization of the integral Eq. (7) can be approximated by the sum of integrals over the elements as follows

$$C(P)p(P) = \sum_{i=1}^{E} \left[ \int_{S_i} \left\{ p_i(Q)E_i(\xi) \frac{\partial}{\partial n} \left( e^{-ikR(Q,P)} \right) \right\} dS \right] + 4\pi p^i(P), \quad (8)$$

where $E_i(\xi)$ are the element shape functions. For a surface formulation where $r_Q = r_P$, solving Eq. (7) calculations for all the elements on the surface, Eq. (8) can be written in the matrix form

$$[C][p] = [p][H] + 4\pi p^i, \quad (9)$$

where $[C]$ is equal to $2\pi I_N$ ($I_N$ is an $N \times N$ identity matrix of the $N$ calculation points on the $N$ nodes) for the surface formulation, and $[H]$ is an $H \times H$ global matrix of the $H$ calculation points on the $H$ elements. Here both $[C]$ and $[H]$ are known but the complex vector $\{p\}$ is unknown. Therefore to find $\{p\}$, Eq. (9) reduces to

$$[A][p] = 4\pi p^i, \quad (10)$$

where $[A] = [H - C]$. On each node of the surface the incident sound pressure $p^i$ of Eq. (10) can be calculated for a point source using Eq. (3).
IV. Estimation of the Rocket Engine Exhaust Noise

High sound levels produced during the lift-off of a launch vehicle are related to the various physical and operating conditions of the rocket engine and the exit conditions of the exhaust flow. Therefore, for prediction purposes, several parameters need to be taken into account. Operating and exhaust parameters for various chemical rocket engines are listed in Table 1, where engine ‘A’ has multiple nozzles (eight nozzles) and the remainder have a single nozzle.

Table 1 Rocket engine operating and exhaust parameters. [Data taken from Mayes et al.]

<table>
<thead>
<tr>
<th>Engine</th>
<th>Average Thrust, ( F \times 10^5 ) (N)</th>
<th>Exit Diameter, ( D_e ) (m)</th>
<th>Exhaust Velocity, ( U_e \times 10^3 ) (m/sec)</th>
<th>Mach Number, ( M_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.1777</td>
<td>0.0897</td>
<td>2.2128</td>
<td>2.65</td>
</tr>
<tr>
<td>B</td>
<td>0.3292</td>
<td>0.1003</td>
<td>2.2845</td>
<td>2.82</td>
</tr>
<tr>
<td>C</td>
<td>0.2994</td>
<td>0.1516</td>
<td>2.4582</td>
<td>3.43</td>
</tr>
<tr>
<td>D</td>
<td>0.3614</td>
<td>0.1768</td>
<td>2.5466</td>
<td>3.67</td>
</tr>
<tr>
<td>E</td>
<td>0.3180</td>
<td>0.2304</td>
<td>2.6700</td>
<td>4.07</td>
</tr>
<tr>
<td>F</td>
<td>0.0734</td>
<td>0.0732</td>
<td>2.0117</td>
<td>2.59</td>
</tr>
</tbody>
</table>

The acoustic power generated by the rocket engine is dependent on the mechanical power of the jet stream. This can be expressed using the following empirical equation as suggested in NASA-SP-8072, a space-vehicle design criteria document:

\[
W_{OA} = 0.005n F U_e. \tag{11}
\]

The overall acoustic power level is

\[
L_w = 10 \log_{10} (0.005n F U_e) + 120, \text{ dB (re } 10^{-12} \text{ watts).} \tag{12}
\]

In Eq. (12), it is assumed that 0.5% of stream power is converted to acoustic power. The overall acoustic power levels for various engines are shown in Fig. 3, where it can be seen that engine ‘A’ generates the highest and engine ‘F’ generates the lowest, while the remaining engines generate almost the same overall acoustic power level. It should be noted that there is a difference in the magnitudes of the results presented here and the previous results presented by Morshed, which mistakenly neglected to add 120dB as required in Eq. (12). Therefore, the corrected results presented here differ only in magnitude compared to the results presented by Morshed. For the current work, engine ‘E’ was chosen for the prediction of acoustic loads on the rocket fairing because it has the greatest nozzle exit diameter, exhaust velocity and mach number.
V. Geometry Under Consideration to Predict the Acoustic Loads

For the prediction of external sound pressure loading on a fairing structure, a Representative Small Launch Vehicle Fairing (RSLVF), which has been used in previous work\textsuperscript{20,21} was used. The overall dimensions of the RSLVF are given in Table 2. The geometry of the RSLVF was represented in ANSYS using quadratic eight node elements, and a ‘mesh-only’ element, MESH200, was used to generate surface elements and nodes, as shown in Fig. 4. The model consisted of 726 elements and 2120 nodes at the surface, which was imported into MATLAB for numerical analysis.

For the current work, the geometry chosen for the acoustic loading calculations is shown in Fig. 5. The parameters shown in Fig. 5 are defined in the nomenclature. The rocket is assumed to be launched vertically, with a 90° bucket deflector turning the exhaust stream horizontally, resulting in the flow axis being perpendicular to the vehicle axis. For this particular case, it was assumed that all the sources are situated below the fairing along the exhaust flow axis so that all the incoming waves impinge obliquely on the fairing. The interaction of the supersonic jet with the deflector is beyond the scope of the current investigation.

<table>
<thead>
<tr>
<th>Table 2 RSLVF physical dimensions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td>Maximum Diameter</td>
</tr>
<tr>
<td>Overall Length</td>
</tr>
</tbody>
</table>
Fig. 4  RSLVF surface elements and nodes. The circles show the circumferential nodes at a height of $z = 2.17\text{m}$ on the RSLVF.

Fig. 5  Geometry of source locations relative to the vehicle and exhaust flow axis.
VI. Prediction of Acoustic Loads Using the Unique Source Allocation Method

The methodology used here for the prediction of acoustic loads on the vehicle is outlined in NASA-SP-8072. This technique assumes that the rocket exhaust noise can be modelled as originating from a single unique effective source location along the flow axis in each frequency band of interest. For the current analysis, one-third-octave band centre frequencies from 50Hz to 400Hz are used.

A. Prediction Formulations

In the NASA-SP-8072 document, the normalized relative sound power spectrum levels for various chemical rockets with single nozzles, within a thrust range of 1.56 kN to 31,100 kN, are given in terms of Strouhal number, \( (fD_e/U_e) \). The apparent axial distance of the source from the nozzle plane is also given in terms of Strouhal number. Both of these results were used to estimate the sound power level and effective unique source location for each frequency band of interest. The estimated results for the normalized relative sound power level and apparent source location for each one-third octave band centre frequency of engine ‘E’ are given in Figs. 6 and 7, respectively, as a function of Strouhal number. Fig. 6 shows that in the low frequency range the sources are located much further downstream than at higher frequencies. However, the normalized relative sound power spectrum shown in Fig. 6 can be converted to any acoustic bandwidth as follows

\[
L_{w,b} = 10 \log_{10} \left( \frac{W(f)}{W_{OA}} \right) + L_w - 10 \log_{10} \left( \frac{U_e}{D_e} \right) + 10 \log_{10} \Delta f_b. \tag{13}
\]

The bandwidth, \( \Delta f_b \), was calculated for each one-third octave band centre frequency from 50Hz to 400Hz using Bies & Hansen. Fig. 8 shows the calculated sound power level for each one-third octave bandwidth for the current analysis.

The directional characteristic of the sound radiated back to the vehicle depends on Strouhal number, exhaust flow direction, effective source location of the rocket exhaust noise and observation position on the vehicle. Fig. 9 shows the far-field directivity curve for chemical rockets, which was estimated from NASA-SP-8072. In the referenced document, there are many directivity curves presented in terms of Strouhal number, \( (fD_e/U_e) \). To simplify the analysis while demonstrating the principles involved, only one directivity curve was chosen for engine ‘E’ for use with all the frequencies and source locations. To find the directivity index from Fig. 9, the directivity angle \( \theta \) relative to the exhaust axis needs to be calculated using the equivalent source position for each frequency.
and observation point on the vehicle.

Fig. 6  Estimated relative sound power levels for each one-third octave band centre frequency from 50Hz to 400Hz. [Data estimated from Fig. 5, presented in NASA-SP-8072^4]

Fig. 7  Estimated source locations for each one-third octave band centre frequency from 50Hz to 400Hz. [Data estimated from Fig. 14, presented in NASA-SP-8072^4]
Fig. 8 Calculated sound power levels for each one-third octave band from 50Hz to 400Hz.

Fig. 9 Simple directivity curve used in the calculations for all one-third octave bands analysed. [Data estimated from Fig. 10, presented in NASA-SP-8072\textsuperscript{4}]
The acoustic power of each source for each frequency band can be determined from the acoustic power level, \( L_{w,b} \), presented in Eq. (13), as follows

\[
W_b = 10^{\left( \frac{L_{w,b}}{10} \right)} \text{ Watts. (14)}
\]

The strength of each source in each frequency band can be determined from

\[
Q_{s,b} = \sqrt{\frac{4 \pi W_b}{k^2 \rho c}} \text{ m}^3/\text{s} . \quad (15)
\]

The sound pressure level on the circumference around the vehicle, including the effects due to the reflecting surface of the vehicle, for a band centred on any frequency can be expressed as

\[
L_{p,b,\theta} = 10 \log_{10} \left\{ p_a^f \left( R, k, a, z, \phi, Q_{s,b}, \theta \right) \right\} + \text{DI}(\theta) - 3 \text{ (dB re 20 \mu Pa)}
\]

where the pressure quantity, \( p_a^f \), represents the total sound pressure amplitude on the vehicle due to incident and scattered waves, and is a function of the distance \( R \) from the source to the observation point on the vehicle, wave number \( k \), radius of the vehicle \( a \), elevation height \( z \), azimuthal angle \( \phi \) and source strength \( Q_{s,b} \), and the term \( \text{DI}(\theta) \) is the directivity index determined from Fig. 9. This pressure quantity can be determined using Eqs. (4) and (10) for the analytical and numerical calculations respectively. For all the sources, \( L_{p,b,\theta} \) must be assumed over all one-third octave frequency bands to give

\[
L_{p,OA,\phi} = 10 \log_{10} \sum_{\text{All } b} 10^{\frac{L_{p,b,\theta}}{10}} \text{ (dB re 20 \mu Pa). (17)}
\]

In Eqs. (16) and (17), the sound pressure due to scattering from the reflecting surface of the vehicle as well as the spreading due to distance are included. The effect of the scattered sound field on the sound pressure on the surface of the vehicle was not considered in the previous analysis presented by NASA-SP-8072. It is also noticeable that in the preceding equations the 4\pi term has been used because the directivity index, \( \text{DI}(\theta) \), of the sound radiation relative to the exhaust axis includes the effect of the sources being located on a hard surface.
B. Acoustic Loading on the Fairing

In this section, the unique source allocation technique, which approximates the sound generation using an equivalent single point source for each frequency band of interest along the flow axis, was used to predict the acoustic loads on the RSLVF both analytically and numerically. Both calculations were conducted for each one-third octave band centre frequency, from 50Hz to 400Hz. There are in total 10 equivalent point sources; one for one-third octave band. It was assumed that the fairing structure and the flow axis are situated at $x_1 = 15D_e$ upstream and $x_2 = 5D_e$ downstream of the vehicle respectively. A reference point was chosen at $\phi = 0^\circ$, which is the front point on the vehicle facing the exhaust flow. Hence $\phi = 180^\circ$ is the rear point on the vehicle. For simplicity, it was assumed that the temperature along the flow axis is $T = 1000^\circ$C. At that temperature, the speed of sound and density in air is 715.49 m/s and 0.28 kg/m$^3$ respectively.

For comparison, both the analytical and numerical calculations were conducted around the circumference at a height of $z = 2.17m$ from the bottom face of the RSLVF (see Fig. 4). This height was chosen because at that height there are sixty circumferential nodes in the BEM model, which are sufficient to achieve an accurate numerical estimation of the sound pressure pattern for comparison with the analytical results. The estimated and calculated values of all the parameters that were used in the calculations are given in Table 3 for each one-third octave band centre frequency, and the directivity index was assumed to be same for all the circumferential positions at the observation height on the fairing.

Table 3  Details of the 10 sources. (All the data given here for engine ‘E’: overall acoustic power level, $L_w = 176.28$dB; speed of sound and density in air at $T = 1000^\circ$C is 715.49 m/s and 0.28 kg/m$^3$ respectively.)

<table>
<thead>
<tr>
<th>Source Number</th>
<th>One-Third Octave Band Centre Frequency (Hz)</th>
<th>Estimated Source Distance, $r$ (m)</th>
<th>Source Distance From the Vehicle Axis, $x_1$(m)</th>
<th>Elevation Angle, $\beta$ (Degrees)</th>
<th>Directivity Angle, $\theta$ (Degrees)</th>
<th>Estimated Directivity Index, DI($\theta$) (dB)</th>
<th>Calculated Acoustic Power Level in Each Band, $L_{w,b}$ (dB)</th>
<th>Source Strength, $Q_s$ × $10^7$ (m$^3$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>9.20</td>
<td>9.13</td>
<td>26.74</td>
<td>153.26</td>
<td>-14.00</td>
<td>151.47</td>
<td>2.136</td>
</tr>
<tr>
<td>2</td>
<td>63</td>
<td>9.18</td>
<td>9.11</td>
<td>26.79</td>
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<td>80</td>
<td>9.16</td>
<td>9.09</td>
<td>26.84</td>
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<td>-13.90</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
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<td>152.88</td>
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VII. Results and Discussions

A. Analytical Results

For the analytically calculated acoustic loading on the RSLVF using the unique source allocation technique, which approximates the sound generation using an equivalent single point source for each frequency band of interest, the maximum diameter of the RSLVF (see Table 2) was considered as a candidate for a representative evaluation of the surface sound pressure levels as a function of azimuthal angle $\phi$. The scattering of sound from the surface was considered in the calculations as described in Eqs. (4), (16) and (17). The parameters used for the analytical calculations are given in Table 3. Figs. 10 to 12 show the sound pressure levels at the surface of the RSLVF at each one-third octave band centre frequency from 50 Hz to 400Hz, calculated using Eqs. (4) and (16) at a height of $z = 2.17$ m from the bottom face of the RSLVF. The overall sound pressure level (OSPL) for the array of ten equivalent point sources, corresponding to one-third octave bands from 50Hz to 400Hz, calculated using Eqs. (4) and (17) at a height of $z = 2.17$ m from the bottom face of the RSLVF, is shown in Fig. 18. The overall sound pressure levels (OSPL) calculated using Eqs. (4) and (17) at the forward ($\phi = 0^\circ$) and backward ($\phi = 180^\circ$) point of the RSLVF are 147.3 dB and 136.9 dB respectively. From these results, it can be seen that the sound pressure amplitude varies relatively smoothly at the front of the RSLVF facing the exhaust flow and varies more aggressively at the back of the RSLVF, due to positive and/or negative interference of the two diffracted waves travelling around the two sides of the RSLVF. The sound pressure fluctuation increases at the back of the RSLVF as the frequency increases because the level of interference of the two diffracted waves travelling around the two sides of the RSLVF increases.
Fig. 10 Analytically calculated sound pressure levels at the RSLVF surface, for an equivalent single point source for each one-third octave band from 50Hz to 100Hz. \(z = 2.17 \text{m}, x_1 = 15D_e, x_2 = 5D_e\) and reference pressure 20\(\mu\text{Pa}\)

Fig. 11 Analytically calculated sound pressure levels at the RSLVF surface, for an equivalent single point source for each one-third octave band from 125Hz to 200Hz. \(z = 2.17 \text{m}, x_1 = 15D_e, x_2 = 5D_e\) and reference pressure 20\(\mu\text{Pa}\)
Fig. 12 Analytically calculated sound pressure levels at the RSLVF surface, for an equivalent single point source for each one-third octave band from 250Hz to 400Hz. \([z = 2.17m, x_1 = 15D, x_2 = 5D, \text{ and reference pressure } 20\mu Pa]\)

B. Numerical Results

For the numerical analysis the Boundary Element Method (BEM) was used. The parameters used for the numerical calculations are given in Table 3, for each equivalent point source at each one-third octave band centre frequency from 50Hz to 400Hz. The numerical results for each one-third octave band centre frequency are presented in Figs. 13 (a-t), calculated using Eqs. (10) and (16). The results show that for very low frequencies (50Hz to 100Hz), the highest acoustic pressures mostly occur on the surface of the lower portions of the RSLVF, as shown in Figs. 13(a-h). The reasons for this are: (a) the sources are located below the RSLVF, along the exhaust flow axis; and (b) the RSLVF dimension is relatively small compared to the wavelength of the very low frequency noise. As the frequency increases, the wavelength becomes shorter compared to the RSLVF length and the sound field becomes more uniformly distributed, as shown in Figs. 13(i-t). The numerical results, calculated using Eqs. (10) and (17), for the overall sound pressure distribution for the frequency range from 50Hz to 400Hz, are shown in Figs. 14 (a-b). It appears that the overall sound pressure level reaches around 148dB for ten equivalent single point sources situated close to the vehicle, along the exhaust flow, as shown in Figs. 14(a-b).
The sound pressure levels as a function of RSLVF circumferential location at a height of $z = 2.17$ m from the bottom face of the RSLVF, calculated using BEM, are shown in Figs. 15 to 17. The results show similar pressure fluctuations, at each band centre frequency, to the fluctuations in the analytical results shown in Figs. 10 to 12, which were obtained using the unique source allocation technique. There are slight differences between the overall sound pressure level amplitudes obtained using the numerical and analytical calculations, as shown in Fig. 18, over the frequency range from 50Hz to 400Hz. The reason for the small differences is that the diffraction of sound waves around the ends of the RSLVF was not considered in the analytical source allocation calculation, but was included in the BEM analysis, which allows determination of the sound pressure near the ends of the cylinder by considering a quarter-point technique ($r^{2/3}$) for particle velocity near the edge of the cylinder, where $r$ is distance from the edge. This approach approximates the condition that the particle velocity tends to infinity as $r$ tends to zero, as explained in detail by Juhl.

It has been observed that the unique source allocation method is capable of predicting the acoustic loads on the surface of the RSLVF in the low frequency range from 50Hz to 400Hz, but it approximates the sound generation using an equivalent single point source for each frequency of interest only. In future work, a non-unique source allocation method will be of interest in the low frequency range from 50Hz to 400Hz, which assumes that there are a number of point source locations making an array of point sources along the exhaust flow axis for each one-third octave frequency band of interest. This approach should yield more accurate results than the unique source allocation method discussed in the current work.

(a) Front face, 50Hz. (b) Rear face, 50Hz.
(c) Front face, 63Hz.
(d) Rear face, 63Hz.
(e) Front face, 80Hz.
(f) Rear face, 80Hz.
(g) Front face, 100Hz.
(h) Rear face, 100Hz.
(i) Front face, 125Hz.

(j) Rear face, 125Hz.

(k) Front face, 160Hz.

(l) Rear face, 160Hz.

(m) Front face, 200Hz.

(n) Rear face, 200Hz.
Figs. 13 (a-t) Numerically calculated sound pressure excitation at the surface of the RSLVF, for various one-third octave band centre frequencies from 50Hz to 400Hz. [$x_1 = 15D_e$, $x_2 = 5D_e$ and reference pressure 20µPa]
Figs. 14(a-b) Numerically calculated overall sound pressure excitation at the surface of the RSLVF, for the entire spectrum of one-third octave band centre frequency range from 50Hz to 400Hz. \[ x_1 = 15D_e, \ x_2 = 5D_e \text{ and reference pressure } 20\mu\text{Pa} \]

Fig. 15 Numerically calculated sound pressure levels as a function of RSLVF circumferential location, for various one-third octave band centre frequencies from 50Hz to 100Hz. \[ z = 2.17m, \ x_1 = 15D_e, \ x_2 = 5D_e \text{ and reference pressure } 20\mu\text{Pa} \]
Fig. 16 Numerically calculated sound pressure levels as a function of RSLVF circumferential location, for various one-third octave band centre frequencies from 125Hz to 200Hz. [$z = 2.17\, \text{m}, x_1 = 15D_e, x_2 = 5D_e$ and reference pressure $20\, \mu\text{Pa}$]

Fig. 17 Numerically calculated sound pressure levels as a function of RSLVF circumferential location, for various one-third octave band centre frequencies from 250Hz to 400Hz. [$z = 2.17\, \text{m}, x_1 = 15D_e, x_2 = 5D_e$ and reference pressure $20\, \mu\text{Pa}$]
Fig. 18 Comparison of the overall sound pressure level between the numerical and analytical results, for the entire spectrum of one-third octave band centre frequency range from 50Hz to 400Hz. \[z = 2.17m, x_1 = 15D_e, x_2 = 5D_e\text{ and reference pressure }20\mu Pa\]

VIII. Conclusions

The acoustic field generated on a launch vehicle fairing during launch has been estimated for a chemical rocket engine ‘E’. The engine exhaust noise has been assumed to be originating along the exhaust flow, perpendicular to the vehicle axis, and has been modelled using the unique source allocation method to determine the acoustic pressure loading at the surface of the Representative Small Launch Vehicle Fairing (RSLVF) in the frequency range from 50Hz to 400Hz. The unique source allocation method assumes that the rocket noise can be modelled using a single unique source location along the flow axis for each one-third octave frequency band. The analytical and numerical results obtained using this method showed that the overall sound pressure level (OSPL) reaches an OSPL of around 148 dB for ten equivalent single point sources in an array along the exhaust flow, over the frequency range from 50Hz to 400Hz.

The method described here allows analysis of the acoustic field at the surface of a launch vehicle fairing for incident oblique waves, and the theoretical analysis showed good agreement with the numerical results. However, the diffraction of sound waves around the ends of the fairing was not taken into account in the analytical calculations in contrast to the results calculated using the Boundary Element Method (BEM), resulting in a small variation.
between the analytical and numerical results. It is intended to extend the present work in the future by consideration of the following: 1) the effects due to the diffraction around the ends of the cylinder, which have been avoided in the analytical modeling work, could be included in a future investigation to develop a full analytical model of acoustic loading on finite length cylinders such as cylindrical launch vehicles; and 2) a non-unique source allocation method can be investigated in the low frequency range from 50Hz to 400Hz, which assumes that there are a number of point source locations making an array of point sources along the exhaust flow for each frequency of interest.

References


