A nonlinear dynamic vibration model of defective bearings: The importance of modelling the finite size of rolling elements

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ABSTRACT

This paper presents an improved nonlinear dynamic model of the contact forces and vibration response generated in rolling element bearings with raceway defects of varying length, depth, and surface roughness. The improvement comes about by considering the finite size of the rolling elements, which overcomes the limitations exhibited by previous models caused by the modelling of rolling elements as point masses. The improvement is demonstrated by comparing simulated and experimentally measured vibration responses for a bearing with a rectangular shaped, sharp edged outer raceway defect. For such a defect, a low frequency event occurs in the measured vibration response when a rolling element enters the defect. Previous models were not able to accurately predict this low frequency event without making the simulated and actual defect geometries significantly different. This limitation is overcome in the proposed model by consideration of the finite size of the rolling elements. Comparisons between the proposed model and previous models are carried out to show that the low and high frequency events which occur when a rolling element passes through the defect, as observed in the experimental results, are more accurately predicted. This demonstrates the benefits over the previous models that do not consider the finite size of the rolling elements. The benefits are further illustrated by a detailed analysis of the variation in the contact forces that occur as a rolling element passes through the defect. This analysis identifies and explains the mechanisms leading to inaccuracy of the previous models in predicting the time-domain vibration response of defective bearings. The model developed here can be used to simulate the vibration response of bearings with a raceway defect to aid in the development of new diagnostic algorithms.

Keywords: Rolling element bearing, spall, vibration model, defect size, contact forces

1. Introduction

Rolling element bearings are widely used in rotary machinery and the failure of bearings are the most common reason for machine breakdowns. Defects in bearings are commonly categorised into localized and distributed defects. Distributed defects, such as waviness, surface roughness, or off-size rolling elements, are usually the result of manufacturing errors [1, 2]. Localized defects are often initiated by insufficient lubrication film between the surfaces in contact. This causes metal-to-metal contact between the rolling elements and the raceways which generates stress waves which in time form sub-surface cracks. The large forces in bearings cause the sub-surface cracks to grow into surface defects. This phenomenon is called pitting or spalling [3]. This paper considers the vibrations and contact forces generated in bearings with raceway spalls.

The study of the bearing vibration response generated by raceway spalls is useful for quality inspection and bearing condition monitoring. Numerical models are often used to develop and test diagnostic algorithms for condition monitoring purposes. Developing an understanding of the dynamic behaviour of defective bearings has received a great deal of attention and has led to the development of a number of numerical models. The previous models developed for distributed defects, such as raceway waviness, are not suitable for bearings with raceway spalls because they assume that the rolling elements are always under
compression while in the load zone, which is not necessarily the case when a raceway defect is present. Numerous models have been developed that consider localized defects. The majority of these models [4-10] are designed to model bearings with line spall defects which occur at the early stages of bearing failure. The extended spall defects that occur at later stages due to successive rolling element passes over the defect have received very little attention [11]. In many models, the rolling elements are not included and hence the path of the rolling elements in the defect zone cannot be predicted [4-9]. In other models such as Ref. [10], the rolling elements are modelled as point masses, and although there is some improvement compared to models that do not consider rolling element inertia, the vibration response is not accurately predicted because the finite size of the rolling elements is neglected. In these models, the results from simulations involving a line spall defect were shown to be reasonably consistent with experimental observation but only if the shape of the defect was altered based on specific assumptions made for the path of the rolling elements while traversing the defect zone. In summary, no model is currently available that has the capability to accurately predict the vibration response of a bearing with a raceway defect without making specific assumptions on the paths taken by the rolling elements as they traverse the defect zone.

This paper presents a nonlinear multi-body dynamic model that can be used to predict the contact forces and time-domain vibration response of radially loaded rolling element bearings due to surface defects on a raceway. The presented model is based on the multi-body nonlinear dynamic models previously developed by Harsha and Randall [6, 12]. Its main contribution is that it considers the finite size of the rolling elements. The advantages of this are that no rolling element path assumptions need to be made and, importantly, the vibration response is predicted more accurately compared to the previous models that modelled the rolling elements as point masses. Therefore, there is no need to assume a defect shape in the model that is different to the real defect geometry, as required in previous models [6, 7].

A simulation and experimental study was conducted involving a double row rolling element bearing with an outer raceway spall. The size of the spall was such that the rolling elements strike the bottom of the defect on the raceway. This type of defect is specifically chosen here to demonstrate the capabilities of the proposed model in correctly predicting the characteristics of the vibration response of an experimentally tested defective bearing. The aim is to demonstrate that the proposed model is capable of predicting typical vibration characteristics of defective bearings with rectangular shaped, sharp edged raceway defects while previous models are either incapable of or limited use for this purpose. The proposed model is also compared with an improved version of the models developed previously by Harsha and Randall [6, 12] in which the rolling elements are modelled as point masses. The characteristics of the resultant vibration signal are discussed and compared to the proposed model to highlight the benefits of considering the finite size of the rolling elements. Moreover, the mechanisms leading to inaccuracy or incapability of the previous models in predicting the time-domain vibration response of defective bearings are identified and explained. The proposed model can be used to simulate the vibration signal generated by bearings with different types of defects, understand the dynamic behaviour of bearings, and to develop new diagnostic algorithms including algorithms to determine the size of a defect.

2. Review of vibration characteristics and models of defective bearings

2.1. Vibration signature for a rectangular shaped, sharp edged raceway defect

Figure 1(a) illustrates a diagram of a defective rolling element bearing with a rectangular shaped, sharp edged defect on the outer raceway. A typical measured vibration signal due to such a defect [13] is shown in Figure 1(b). It has been shown by previous experimental studies that the entry of a rolling element into a sharp edged defect produces a vibration signal with low frequency content [13, 14]. Moreover, the exit of the rolling element excites a much broader range of frequencies including the high frequency bearing resonances. These resonances are excited by the impact of the rolling element mass on the sharp edged defect exit, as well as the parametric excitations caused by rapid changes in the bearing stiffness which occur when the rolling element restresses between the raceways [11]. The high frequency event observed in experimental results [13, 15] often appears to have been caused by multiple impacts rather than a single impact. Simulation results of defective bearings presented by Singh [16] indicate that the multiple impacts occur when the rolling element successively impacts on the inner and outer raceways at it restresses at the exit point.
2.2. Vibration models of defective bearings

Numerous vibration generation mechanisms in rolling element bearings have been investigated by researchers and several models have been developed for predicting the vibration response due to these mechanisms [1, 2, 11, 17-27]. These mechanisms include varying compliance due to the existence of different number of rolling elements in the load zone, distributed defects such as waviness due to manufacturing errors, and localized defects such as cracks, pits, line spalls and extended spalls caused by fatigue [28-32]. A limitation of all previous dynamic models is that the correct path of the rolling elements cannot be predicted for a wide range of defects. Prediction of the path of the rolling element in the defect zone (at the entry, mid-way through and exit from the defect) is essential to overcome the limitations of the previous models and makes developing a more generally applicable dynamic model that is not limited to certain types of defects possible. In the model developed here, the path of a rolling element is predicted by taking into account its mass and finite size, and the actual defect geometry is used in the model. The proposed model does not make assumptions about the path of the rolling elements or artificially modify the simulated defect geometry in order to obtain a more accurate prediction of the measured vibration response.

Simplified bearing models [17-20] have been developed which model the raceways as circular rings whose resonance modes are excited by a train of force impulses [33]. The vibration signature is predicted by considering the applied load, the exponential decay of a resonant mode and the characteristic defect frequency. These models do not take the shape and size of a localized defect into account and are limited to bearing defects that produce only impulsive vibration signals. Hence, these simple models cannot be employed to analyse the vibration response of bearings with defects of varying shape and size.

Dynamic models have been developed for distributed defect geometries such as waviness and surface roughness [1, 2, 21-27]. In these models, the bearing contact forces are related to the displacement of bearing components using Hertzian contact theory [6, 34]. The vibration response prediction in these models is based on the assumption that the rolling elements in the load zone are always in contact with both raceways, and they do not allow for inclusion of localized spalls in modelling. Therefore, these models cannot be used for predicting the vibration response of localized defects for which rolling elements may become unloaded when traversing the defect zone.

Numerous multi-body dynamic models have been developed for modelling line spall defects [4-10] which do not consider the mass and finite size of the rolling element. In these models, the path of a rolling element is modelled such that its centre follows the geometry of the defect. For the case of rectangular shaped, sharp edged defects, this produces very large impulsive forces at the entry point into the defect, which results in large amplitude, high frequency accelerations that are not observed in experimental results. In order to avoid such incorrectly predicted large impulsive forces at the defect entrance point, Sawalhi and Randall [6] modified the shape of the modelled defect to resemble an assumed path travelled by rolling elements in the defect zone. They only modelled and experimentally investigated line spall defects for which it was assumed that the exit impact occurs mid-way through the defect. In their model, the path of the rolling element is assumed such that the high frequency event observed in measurements occurs when the centre of the rolling element is halfway through the line spall [6, 14]. However, they have not suggested any
modifications for extended defects in which the high frequency event would not occur when the rolling element is half way through the defect. Extended defects that are sufficiently deep to make a rolling element unload momentarily could be modelled using these models by increasing the length of the modelled defect. However, the path of the rolling elements and, consequently, the force impacts that are generated as the unloaded rolling elements strike the bottom of extended defects cannot be predicted as these models do not consider the mass and finite size of the rolling elements. Moreover, since rolling element impacts on the bottom of extended defects have not been previously modelled or experimentally investigated, the assumption of the rolling element’s path makes these models incapable of predicting such events.

Recently, a more comprehensive multi-body dynamic model was developed by Harsha [12, 25, 26, 35]. This model was initially developed to predict the vibration response of defective bearings with distributed defects [34]. Later on the model was improved to include the mass of the rolling elements in order to predict the nonlinear dynamic behaviour of a rolling element bearing due to waviness and unbalanced rotor support [12, 25, 26, 35]. The improved version of the model was further modified by Tadina [10] to predict the vibration response of bearings with localized spall defects on raceways. All of the aforementioned models are designed for defects with curvatures larger than the curvature of the rolling element which maintains the contact between the raceways in the load zone. Therefore, Harsha’s model is not suitable for modelling rectangular shaped, sharp-edged defects that are large compared to the size of the rolling elements. Furthermore, Harsha’s model and subsequent improved versions of it do not include mass-spring-dampers to represent a measured high frequency resonant response of the bearing, as included in the model proposed by Sawalhi & Randall [6]. None of these models consider damping between the rolling elements and the raceways due to lubrication, as was done by Soppanen & Mikkola [4, 5]. Additionally, Harsha’s model has never been compared or validated against experimental measurements. Table 1 lists and compares the features of the models developed by Sawalhi & Randall [6, 14] and Harsha [12, 25, 26, 35] to those of the model proposed in this paper.

Table 1: Summary of the features of previously developed and proposed bearing models.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Model name</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Randall &amp; Sawalhi</td>
</tr>
<tr>
<td>Mass of rolling elements</td>
<td>✗</td>
</tr>
<tr>
<td>High-frequency resonant mode</td>
<td>✓</td>
</tr>
<tr>
<td>Finite size of rolling elements</td>
<td>✗</td>
</tr>
<tr>
<td>Contact damping due to lubricant film</td>
<td>✗</td>
</tr>
<tr>
<td>No assumption on path of rolling element centre</td>
<td>✗</td>
</tr>
<tr>
<td>Independent coordinate system for all bearing components</td>
<td>✗</td>
</tr>
</tbody>
</table>

The only model which considers the finite size of the rolling elements is the model developed by Epps [13]. Epps calculated the contact forces acting on the rolling elements but the results were only valid for a particular case of a rectangular shaped defect and with the inner raceway constraint to move only in the direction of the vertically applied static load. Therefore, the contact positions between a rolling element and the raceways for defect geometries other than rectangular shaped ones cannot be calculated using this method. In bearings with higher radial clearance, lower radial loads, or non-vertical radial loads in which the inner raceway have movements in both x and y directions, Epps’ model cannot be used either [13]. The model proposed in this paper does not make the assumptions made by Epps [13] which makes it a more comprehensive model that can accept a wider range of defect geometries.

3. Bearing test rig and the test bearing

Figure 2 shows a photo of the test rig used to conduct experiments on rolling element bearings. The test rig has a 15 kW three-phase induction motor, a steel structure and a hydraulic piston for applying a vertical load to the bearing under test, as well as a variable frequency drive to control the rotational speed of
the motor. Four V-belts and pulleys couple the electric motor to the shaft. The test rig is capable of applying vertical loads up to 100 kN and run speeds up to 1200 rpm. The bearing to be tested is pressed onto the shaft and located at the loading-end of the shaft and away from the motor (drive-end). The bearing installed at the drive-end of the shaft is a defect-free bearing. The force from the hydraulic pistons is applied to the shaft using two rollers via the loading structural arrangement.

![Figure 2: Photo of the test rig with a test bearing installed on the loading-end of the shaft, and a defect-free bearing on the drive-end.](image)

An accelerometer is mounted onto the outer shell of the bearing in order to measure the defect-induced vibrations. The accelerometer is screwed to an aluminium base and the bases are attached to the bearing using superglue. The bases are specifically made to match the curvature of the outer bearing shell. A tachometer is mounted near the shaft in order to measure its rotational speed. The data acquisition system consisted of a National Instruments (NI) CompactDAQ system with NI 9234 modules. Data was acquired using the NI Labview Sound and Vibration toolbox and post-processing was done using MATLAB. The vibration signals were acquired with a sampling frequency of 102.4 kHz. The test bearing is a double row roller element bearing which has a nominal pitch diameter $D_p = 180.2$ mm, a roller diameter $D_b = 17.9$ mm, clearance $cl = 0.015$ mm and $N_b = 24$ rolling elements on each row.

A rectangular shaped, sharp edged defect was machined into the surface of one of the outer raceways using an electric discharge method. The circumferential length of the bearing defect was chosen as 10 mm and the depth as 0.2 mm, so that a rolling element loses becomes unloaded as it traverses through the defect. The test bearing was subjected to a static vertical load of $W = 50$ kN and the vibration measurements were recorded at a shaft rotational speed of 250 rpm. The test bearing was orientated such that the defect was located in the load zone.

4. **Nonlinear multi-body dynamic modelling of a defective bearing**

4.1. **Diagram of the model**

Figure 3 shows a diagram of the multi-body nonlinear dynamic model used to simulate the vibration response of a defective rolling element bearing. The model includes the masses of the outer raceway plus the support structure ($m_o$), inner raceways plus the shaft ($m_i$), and rolling elements ($m_b$). The static load $W$ is applied to the shaft in the $y$ direction.
In the proposed model, the rows $r = 1, 2$ of the double row bearing share the same outer bearing shell ($m_o$). The inner raceways of the two rows are press-fitted onto a common shaft. Therefore $(x_i, y_i)$ and $(x_o, y_o)$ represent the centre line of the two inner raceways (rows) and the two outer raceways, that encompasses the two rows, respectively. The model has $2N_b + 6$ degrees of freedom, where $N_b$ is the number of rolling elements, but note that Figure 3 only shows one of the rows. The additional six degrees of freedom are the displacement of the inner raceway ($x_i, y_i$), displacement of outer raceway ($x_o, y_o$), and the measured vibration response of the outer raceway ($x_r, y_r$), which includes a high frequency bearing resonant mode.

The nonlinear contact stiffness and damping between the $j$th rolling element on row $r$ and the raceways are represented by $k_{\text{out } j,r}$, $c_{\text{out } j,r}$, $k_{\text{in } j,r}$ and $c_{\text{in } j,r}$. The stiffness and damping of the bearing support are represented by $k_{s x}$, $k_{s y}$, $c_{s x}$, and $c_{s y}$. Using the method explained in [11], these parameters can be adjusted to reasonably match the characteristics of the low frequency event observed in experimental results. A typical high frequency resonant response of the bearing is modelled by including the spring-mass-damping systems ($k_r, m_r, c_r$) in both $x$ and $y$ directions, as shown in Figure 3. These parameters are adjusted to match one of the high frequency resonant responses observed in the measured experimental vibration response.

4.2. Kinematics of the rolling elements

The role of the cage in rolling element bearings, which is not shown in Figure 3, is to maintain constant distance and zero relative speed between the rolling elements. Assuming that there is no slippage between the shaft and the two inner raceways, the nominal rotational speed of the cage is given by
\[
\omega_c = \omega_s \left(1 - \frac{D_b \cos \alpha}{D_p}\right)
\]  

where \( \omega_c \) is the shaft run speed, \( D_b \) the diameter of a rolling element, \( D_p \) the pitch diameter, and \( \alpha \) the contact angle. The angular position of the \( j \)th rolling element of the \( r \)th row is

\[
\phi_{j,r}(t) = \phi_c(t) + \frac{2\pi(j - 1)}{N_b} + \varphi_{rnd}(r - 1) \; ; \; j = 1, 2, ..., N_b; r = 1, 2
\]

with the cage position \( \phi_c(t) \) given by

\[
\phi_{j,r}(t) = \phi_c(t) + \omega_c dt + \nu(t)
\]

where \( \nu(t) \) is a random process uniformly distributed between \([-\varphi_{slip}, \varphi_{slip}]\) that accounts for slippage of the rolling elements. Typical values for the maximum phase variation \( \varphi_{slip} \) are of the order 0.01 – 0.02 radians [6]. In Eq. (2), \( \varphi_{rnd} \) is a number between \([0, \pi/N_b]\) and defines the relative positions of the two rows to take into account that they do not share the same cage. The simulated results presented in this paper are for one fixed angle \( \varphi_{rnd} \), since the effects of varying \( \varphi_{rnd} \) was found to be negligible when a defect is present on one of the rows.

### 4.3. Contact deformation for defect-free bearings

The contact deformations between a rolling element and the raceways are calculated from their relative positions described by the dynamic motion of the bearing components. Therefore, the nonlinear contact stiffness parameters are related to the dynamics of the system.

Figure 4 presents a diagram of the relative positions of the components of a rolling element bearing. A set of independent generalised Cartesian coordinates for the inner raceway \((x_{in}, y_{in})\), the outer raceway \((x_{out}, y_{out})\), and an independent a set of polar generalised coordinates for the rolling elements \((\rho_j, \gamma_j; j=1, 2, 3, ..., N_b; r = 1, 2)\) are defined. Compared with Harsha’s model, in which the general coordinate \( \rho_j \) is placed on the general coordinate defined for the centre of the outer raceway \([12, 25, 26, 35]\), in the proposed model, the general coordinates defined for the centre of the outer raceway and the cage are independent.
Figure 4: Diagram of the relative position of the components of a rolling element bearing, with \( \delta_{\text{out},j,r} \) the contact deformation between the \( j^{\text{th}} \) rolling element and the outer raceway, \( \delta_{\text{in},j,r} \) the contact deformation between the \( j^{\text{th}} \) rolling element and the outer raceway, \( X_{j,r} \) and \( Z_{j,r} \) are the location vectors of the \( j^{\text{th}} \) rolling element with respect to the centre of the raceways, \( \rho_{j,r} \) independent polar generalised coordinate for the \( j^{\text{th}} \) rolling element, \( \theta_{\text{out},j,r} \) the angle of the vector \( Z_{j,r} \) with respect to the centre of the outer raceway, \( \theta_{\text{in},j,r} \) the angle of the vector \( X_{j,r} \) with respect to the centre of the inner raceway and independent Cartesian coordinates for the inner raceway \( (x_{\text{in}}, y_{\text{in}}) \), the outer raceway \( (x_{\text{out}}, y_{\text{out}}) \).

The contact deformations \( \delta_{\text{in},j,r} \) and \( \delta_{\text{out},j,r} \) between the \( j^{\text{th}} \) rolling element on row \( r \) and both raceways with no defect present are:

\[
\delta_{\text{in},j,r} = \frac{d + D_b + cl}{2} - X_{j,r} \tag{4}
\]

\[
\delta_{\text{out},j,r} = \frac{-D + D_b - cl}{2} + Z_{j,r} \tag{5}
\]

where \( d, D \) and \( D_b \) are the diameters of the inner raceway, outer raceway and a rolling element respectively, and \( cl \) is the radial clearance of the bearing. The displacement vectors \( X_{j,r} \) and \( Z_{j,r} \), which specify the location of the \( j^{\text{th}} \) rolling element on row \( r \) with respect to the centre of the inner and outer raceways, can be related to the parameter \( \rho_{j,r} \) by solving the following relations.

\[
x_{\text{in}} + X_{j,r} \cos \theta_{\text{in},j,r} = \rho_{j,r} \cos \phi_{j,r} = x_{\text{out}} + Z_{j,r} \cos \theta_{\text{out},j,r} \tag{6}
\]

\[
y_{\text{in}} + X_{j,r} \sin \theta_{\text{in},j,r} = \rho_{j,r} \sin \phi_{j,r} = y_{\text{out}} + Z_{j,r} \sin \theta_{\text{out},j,r} \tag{7}
\]

Therefore:

\[
X_{j,r} = \left[ \rho_{j,r}^2 + x_{\text{in}}^2 + y_{\text{in}}^2 - 2 \rho_{j,r} \left( x_{\text{in}} \cos \phi_{j,r} + y_{\text{in}} \sin \phi_{j,r} \right) \right]^{1/2} \tag{8}
\]
where \( \phi_{j,r} \) is the angular position of the \( j \)th rolling element with respect to the polar generalised coordinates of the row \( r \) illustrated in Figure 4 and defined by Eq. (2).

4.4. Contact deformation for defective bearings including finite size of rolling elements

In order to model the geometry of a defect on one of the raceways, \( r = 1 \), the defect shape function \( \gamma(\phi) \) is introduced, which is a function of the angle \( \phi \). This function can be adapted to include defects of any geometry as explained in [7, 11]. A rectangular shaped, sharp edged bearing defect on the outer raceway can be modelled as:

\[
\gamma(\phi) = \begin{cases} 
\lambda \phi_{en} < \phi_{j,r} < \phi_{ex} \\
0 & \text{otherwise}
\end{cases}, \quad r = 1
\]  

where \( \phi_{en} \) and \( \phi_{ex} \) are the angular positions of the defect entry and exit, and \( \lambda \) is the depth of the defect. Therefore, the geometry function of the outer raceway for a given angle \( \varphi \) with reference to the centre of the outer raceway is given by:

\[
R(\varphi) = \frac{(D + c)}{2} + \gamma(\varphi)
\]  

In contrast to the previous models [6, 12], where the rolling element is considered as a point mass to calculate the contact deformation between each rolling element and the raceways, the model proposed in this paper takes into account a finite number of points on the circumference of a rolling element. In other words, the dimensions of the rolling element are taking into account rather than modelling it as point mass.

Figure 5 shows a diagram of a rolling element in the defect zone. On the outer raceway, as a rolling element at a given angular position \( \phi_{j,r} \) contacts the edges of the defect, the deformation \( \delta_{b, j,r} \) occurs at the angle \( \beta_{j,r} \) perpendicular to the \( j \)th rolling element, as illustrated in Figure 5.
deformation perpendicular to the \( j \)th rolling element, \( \delta_{\text{out},j,r} \) deformation toward the centre of the outer raceway, and \( \psi_{j,r} \) the angle between the point of the maximum deformation and the vector \( Z_{j,r} \). \( \theta_{\text{out},j,r} \) the angle of the vector \( Z_{j,r} \) with respect to the centre of the outer raceway.

For the example used in this paper to demonstrate the capabilities of the proposed model, the defect is modelled on the outer raceway. The deformations between the rolling elements and the inner raceway always happen at a point perpendicular to the inner raceway. However, the deformation between a rolling element and the outer raceway in the defect zone happens at a point which is not necessarily at the angular position \( \beta_{j,r} = 0 \) as assumed in previous models [12]. This limitation is overcome by considering a finite number of points on each rolling element to calculate the angular position of the maximum contact deformation between a rolling element and the defect on the outer raceway at a given angular position \( \theta_{\text{out},j,r} \). This angle can be related to the angular position \( \phi_{j,r} \) of the rolling element using Eq. (6), such that:

\[
\theta_{\text{out},j,r} = \cos^{-1}\left(\frac{\rho_j \cos \phi_{j,r} - x_{\text{out}}}{Z_{j,r}}\right)
\]  

(12)

The contact deformation \( \delta_{\text{out},j,r,\beta} \) perpendicular to the outer raceway with a defect profile \( \gamma(\phi) \) for every point at an angle \( \beta_j \) on the rolling element is given by:

\[
\delta_{\text{out},j,r,\beta}(\beta) = \frac{2Z_{j,r} + D_b \cos \beta_{j,r}}{2 \cos \psi_{j,r}} - R(\theta_{\text{out},j,r} \pm \psi_{j,r})
\]

(13)

where \( R(\theta_{\text{out},j,r} \pm \psi_{j,r}) \) is the polar function of the outer raceway defined by Eq. (11). The angle \( \psi_{j,r} \), illustrated in Figure 5, is the angle between the point of the maximum deformation on a rolling element and the displacement vector \( Z_{j,r} \) of the rolling element and is given by:

\[
\psi_{j,r} = \tan^{-1}\left(\frac{D_b \sin \beta_{j,r}}{2Z_{j,r} + D_b \cos \beta_{j,r}}\right)
\]

(14)

Therefore the contact deformation \( \delta_{\text{out},j,r} \) perpendicular to the outer raceway with a defect profile \( \gamma(\phi) \) is:

\[
\delta_{\text{out},j,r} = \max\left[\delta_{\text{out},j,r,\beta}\right] \text{; } -\frac{\pi}{2} < \beta_{i,r} < \frac{\pi}{2}
\]

(15)

Considering the finite size of a rolling element provides a more realistic prediction of its path as it traverses a defect, especially when the defect has a sharp edged entry and exit. This improves the accuracy of the simulated vibration response compared to previous models which include the rolling elements as point masses.

4.5. Hertzian contact model

The contact forces are related to the elastic contact deformations, defined in the previous section, using the Hertzian elastic contact theory [36]. Since the Hertzian contact force arises only when there is contact between a rolling element and a raceway, the respective contact force is set to zero when the contact deformation is equal or smaller than zero. This is indicated by subscript “+” throughout this paper. The radial contact forces \( Q_{\text{in},j,r} \) and \( Q_{\text{out},j,r} \) between rolling element \( j \) on row \( r \) and the inner and outer raceways are calculated using the load-deflection relationship

\[
\begin{bmatrix}
Q_{\text{in},j,r} \\
Q_{\text{out},j,r}
\end{bmatrix} =
\begin{bmatrix}
K_{\text{in}} [\delta_{\text{in},j,r}]^n \\
K_{\text{out}} [\delta_{\text{out},j,r}]^n
\end{bmatrix}
\]

(16)

where \( \delta_{\text{in},j,r} \) and \( \delta_{\text{out},j,r} \) are calculated using Eqs. (4) and (5). The load-deflection factors \( K_{\text{in}} \) and \( K_{\text{out}} \) depend on the curvature of the rolling elements and raceways, and \( n = 10/9 \) for roller bearings and \( 3/2 \) for
ball bearings [3]. Using Eq. (16), the nonlinear stiffnesses \( k_{in,j,r} \) and \( k_{out,j,r} \) of the springs shown in Figure 3 are defined as

\[
\begin{bmatrix}
  k_{in,j,r} \\
  k_{out,j,r}
\end{bmatrix} = \begin{bmatrix}
  \frac{\partial Q_{in,j,r}}{\partial \delta_{in,j,r}} \\
  \frac{\partial Q_{out,j,r}}{\partial \delta_{out,j,r}}
\end{bmatrix} = \begin{bmatrix}
  K_{in}\delta_{in,j,r}^{n-1} \\
  K_{out}\delta_{out,j,r}^{n-1}
\end{bmatrix}
\]  

which shows that the nonlinear contact stiffnesses are functions of the contact deformations \( \delta_{in,j,r} \) and \( \delta_{out,j,r} \). Using Eqs. (6), (7) and (16), the total contact forces in the \( x \) and \( y \) directions acting on the inner raceway are obtained by summing over the \( N_b \) rolling elements on each row, such that

\[
\begin{bmatrix}
  F_{in,x} \\
  F_{in,y}
\end{bmatrix} = \sum_{r=1}^{2} \sum_{j=1}^{N_b} K_{in} \left[ \delta_{in,j,r} \right]_+ \begin{bmatrix}
  (\rho_{j,r} \cos \phi_{j,r} - x_{in})/X_{j,r} \\
  (\rho_{j,r} \sin \phi_{j,r} - y_{in})/X_{j,r}
\end{bmatrix}
\]

Similarly, the total contact forces acting on the outer raceway are defined as

\[
\begin{bmatrix}
  F_{out,x} \\
  F_{out,y}
\end{bmatrix} = \sum_{r=1}^{2} \sum_{j=1}^{N_b} K_{out} \left[ \delta_{out,j,r} \right]_+ \begin{bmatrix}
  (x_{out} - \rho_{j,r} \cos \phi_{j,r})/Z_{j,r} \\
  (y_{out} - \rho_{j,r} \sin \phi_{j,r})/Z_{j,r}
\end{bmatrix}
\]

4.6. Contact damping

Damping in the contacts due to the lubricant film between the rolling elements and raceways is taken into account by including linear dampers \( c_{in,j,r} \) and \( c_{out,j,r} \) as shown in Figure 3. The radial contact damping forces associated with the rolling element \( j \) on row \( r \) are given by

\[
\begin{bmatrix}
  Q_{d in,j,r} \\
  Q_{d out,j,r}
\end{bmatrix} = c \begin{bmatrix}
  \delta_{in,j,r} \\
  \delta_{out,j,r}
\end{bmatrix}_+
\]

where \( c \) is the viscous damping constant. The total contact damping forces acting on the inner and outer raceways in the \( x \) and \( y \) directions are now given by:

\[
\begin{bmatrix}
  F_{d in,x} \\
  F_{d in,y}
\end{bmatrix} = \sum_{r=1}^{2} \sum_{j=1}^{N_b} Q_{d in,j,r} \begin{bmatrix}
  (\rho_{j,r} \cos \phi_{j,r} - x_{in})/X_{j,r} \\
  (\rho_{j,r} \sin \phi_{j,r} - y_{in})/X_{j,r}
\end{bmatrix}
\]

\[
\begin{bmatrix}
  F_{d out,x} \\
  F_{d out,y}
\end{bmatrix} = \sum_{r=1}^{2} \sum_{j=1}^{N_b} Q_{d out,j,r} \begin{bmatrix}
  (x_{out} - \rho_{j,r} \cos \phi_{j,r})/Z_{j,r} \\
  (y_{out} - \rho_{j,r} \sin \phi_{j,r})/Z_{j,r}
\end{bmatrix}
\]

The damping in a bearing assembly is typically in the order of 0.25 \( \times \) \( 10^{-5} \) times the linearised stiffness of the bearing assembly [4, 5]. The viscous damping constant \( c \) in Eq. (20) is adjusted to achieve damping within this range [11].

4.7. Nonlinear equations of motion

The equations of motion for the inner raceway, outer raceway and the measured response which includes the high-frequency resonant mode are now given by:

\[
\begin{bmatrix}
  m_{in} \ddot{x}_{in} \\
  m_{in} \ddot{y}_{in}
\end{bmatrix} + \begin{bmatrix}
  F_{in,x} + F_{d in,x} \\
  F_{in,y} + F_{d in,y}
\end{bmatrix} = \begin{bmatrix}
  0 \\
  -W
\end{bmatrix}
\]
\[
\begin{align*}
\mathbf{m}_{\text{out}} \begin{bmatrix}
\ddot{x}_{\text{out}} \\
\ddot{y}_{\text{out}} \\
\end{bmatrix} + \begin{bmatrix}
c_{s, x} x_{\text{out}} + k_{s, x} x_{\text{out}} + F_{\text{out}, x} + F_{\text{d, out}, x} \\
c_{s, y} y_{\text{out}} + k_{s, y} y_{\text{out}} + F_{\text{out}, y} + F_{\text{d, out}, y}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\end{align*}
\] (24)

\[
\begin{align*}
m_r \ddot{y}_r + k_r (y_r - y_{\text{out}}) + c_r (y_r - y_{\text{out}}) &= 0 \\
m_r \ddot{x}_r + k_r (x_r - x_{\text{out}}) + c_r (x_r - x_{\text{out}}) &= 0
\end{align*}
\] (25)

where \( g = 9.81 \, \text{m/s}^2 \), and the contact forces are calculated as described in Sections 4.5 and 4.6. In order to derive the equations of motion for the rolling elements on row \( r \), Lagrange’s equation for the set of generalised coordinates \( \rho_{j, r} \) are used

\[
\frac{d}{dt} \frac{\partial T_{r}}{\partial \dot{\rho}_{j, r}} - \frac{\partial T_{r}}{\partial \rho_{j, r}} + \frac{\partial V_{r}}{\partial \rho_{j, r}} = \{ f_r \}
\] (26)

where \( T_r \) is the kinetic energy, \( V_r \) is the potential energy, \( \rho_{j, r} \) is a vector of the generalised coordinates defined for the rolling elements, and \( \{ f_r \} \) is the vector with generalised contact forces. The total kinetic and potential energy of the rolling elements on row \( r \) is:

\[
V_r = \sum_{j=1}^{N_b} m_b \rho_j \sin \phi_{j, r}
\] (27)

\[
T_r = \sum_{j=1}^{N_b} 0.5 m_b (\dot{\rho}_{j, r} \cdot \dot{\rho}_{j, r}) + 0.5 I \dot{\phi}_{j, r}^2
\] (28)

where \( \dot{\rho}_{j, r} \) is given by:

\[
\dot{\rho}_{j, r} = \left( \rho_{j, r} \cos \phi_{j, r} \right) \dot{i} + \left( \rho_{j, r} \sin \phi_{j, r} \right) \dot{j}
\] (29)

The equations can be re-arranged to

\[
\dot{\rho}_{j, r} \cdot \dot{\rho}_{j, r} = \rho_{j, r}^2 + \rho_{j, r}^2 \phi_{j, r}^2
\] (30)

By substituting equation Eq. (30) into (28) the total kinetic energy of each row is:

\[
T_r = \sum_{j=1}^{N_b} 0.5 m_b \left( \dot{\rho}_{j, r}^2 + \rho_{j, r}^2 \phi_{j, r}^2 \right) + 0.5 I \phi_{j, r}^2 \left( 1 + \frac{r_o}{r_r} \right)^2
\] (31)

where \( I \) is the moment of inertia of a rolling element and \( r_o = D/2 \) is the outer raceway radius. The terms in Eq. (26) can be evaluated individually as

\[
\frac{\partial V_{j, r}}{\partial \rho_{j, r}} = m_b g \sin \phi_{j, r}
\] (32)

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{\rho}_{j, r}} - \frac{\partial T}{\partial \rho_{j, r}} = m_b \ddot{\rho}_{j, r} + m_b \rho_{j, r} \phi_{j, r}^2
\] (33)
The vector of generalised contact forces \( \{ f_r \} \) in Eq. (26) is the sum of the radial contact and damping forces acting on each rolling element with respect to the generalised coordinates \( \rho_{j,r} \) which can be calculated by differentiating Eq. (16) and (20) with respect to the generalised coordinates \( \rho_j \), such that

\[
\{ f_r \} = \frac{\partial (Q_{\text{in},j,r} + Q_{\text{out},j,r} + Q_{\text{d,in},j,r} + Q_{\text{d,out},j,r})}{\partial \rho_{j,r}} = \left( K_{\text{in}}[\delta_{\text{in},j,r}]_n^+ + c[\delta_{\text{in},j,r}]_n^+ \right) \frac{\partial x_{j,r}}{\partial \rho_{j,r}} + \left( K_{\text{out}}[\delta_{\text{out},j,r}]_n^+ + c[\delta_{\text{out},j,r}]_n^+ \right) \frac{\partial z_{j,r}}{\partial \rho_{j,r}}
\]  

(34)

where the partial derivatives of \( x_{j,r} \) and \( z_{j,r} \) with respect to \( \rho_{j,r} \) are defined as

\[
\frac{\partial x_{j,r}}{\partial \rho_{j,r}} = \frac{\rho_{j,r} - x_{\text{in}} \cos \phi_{j,r} - y_{\text{in}} \sin \phi_{j,r}}{x_{j,r}}
\]

(35)

\[
\frac{\partial z_{j,r}}{\partial \rho_{j,r}} = \frac{\rho_{j,r} - x_{\text{out}} \cos \phi_{j,r} - y_{\text{out}} \sin \phi_{j,r}}{z_{j,r}}
\]

(36)

Substituting Eqs. (32), (33) and (34) into Eq. (26) gives the following equations of motion for the rolling elements on row \( r \):

\[
m_b \ddot{x}_{j,r} + m_b \rho_{j,r} \omega_c^2 + m_{j,r} g \sin \phi_{j,r} - \{ f_r \} = 0
\]

(37)

Eqs. (23) to (25) and Eq. (37) form a system of coupled, second order, nonlinear, ordinary differential equations. The dynamic system is excited by variations in the stiffnesses of the nonlinear contact springs (parametric excitations [11]) as well as the impacts that occur when a rolling element mass traverses though a defect.

5. Simulation and experimental results

5.1. Simulation and model parameters

This section presents a comparison between simulated and experimental vibration results, as well as a validation of the static load distribution predicted by the model. The test bearing described in Section 3 was modelled using the proposed model presented in the Section 4. The simulations were undertaken using Matlab® and Simulink® and the equations of motion were solved using the ‘ode45’ continues differential equation solver. The high frequency resonant mode was modelled to have a resonance frequency of 10 kHz and damping ratio of 3%, which corresponds to one of the experimentally measured bearing resonant modes. The model parameter values used in the simulations are given in Table 2.

<table>
<thead>
<tr>
<th>Hertzian contacts</th>
<th>Mass</th>
<th>Stiffness</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{\text{in}} = 4576 \text{ MN/m} )</td>
<td>( m_0 = 200 \text{ kg} )</td>
<td>( k_r = 11939 \text{ MN/m} )</td>
<td>( c_r = 18850 \text{ Ns/m} )</td>
</tr>
<tr>
<td>( K_{\text{out}} = 4576 \text{ MN/m} )</td>
<td>( m_1 = 480 \text{ kg} )</td>
<td>( k_{s,x} = 457.6 \text{ MN/m} )</td>
<td>( c_{s,x} = 1.46 \text{ MNs/m} )</td>
</tr>
<tr>
<td>( c = 9000 \text{ Ns/m} )</td>
<td>( m_r = 5 \text{ kg} )</td>
<td>( k_{s,y} = 457.6 \text{ MN/m} )</td>
<td>( c_{s,y} = 1.46 \text{ MNs/m} )</td>
</tr>
</tbody>
</table>

The aim of the simulations is to show that the developed model can predict characteristics observed in the measured vibration response, whilst modelling the actual shape of the defect. This is in contrast with previous models in which the modelled defect shape was altered from the actual shape in order to achieve better agreement between modelled and measured results.

5.2. Load distribution validation

This section compares the static load distribution results of the proposed model and the well-known Strubeck results [3, 11]. The static load distribution for a defect-free bearing is defined by [3]:

\[
T = \frac{1}{2} m \omega^2 r^2
\]
where \( n = 10/9 \) for roller bearings and \( \varepsilon = 0.5 \) for zero clearance. The maximum load \( F_{\text{max}} \) for double row roller bearing with zero clearance is given by Harris [3]:

\[
F_{\text{max}} = \frac{4.37W}{2N_b \cos \alpha}
\]

The static load distribution as a function of the cage position is obtained by modelling a defect-free double row roller bearing using the proposed model while the cage is stationary. Figure 6 compares the simulated and analytical static load distribution for cage angular positions \( \phi_{j,r} \) of 0° and 7.5°. The results show that the simulated load distribution accurately matches the analytical solution.

![Graph](image)

Figure 6: Static load distribution validation for cage angular positions of 0° and 7.5°

### 5.3. Comparison of simulated and measured results

Figure 7(a) shows 0.85 seconds of the acceleration that was experimentally measured on the test bearing at a speed of 250 rpm and 50 kN radial load. The simulated acceleration response \( \ddot{y}_j \) obtained with the proposed model is included in Figure 7(b). The experimental data was low-pass filtered at 12 kHz as the bearing resonances at higher frequencies are not considered in the model.
Figure 7 shows that the amplitude of the high frequency events is fluctuating in time, and that these fluctuations are predicted by the proposed model. This is not the case for the previous models [1, 2, 11, 17-27] which predict near-constant amplitudes. In order to compare the simulated and measured fluctuations in the amplitude of the high-frequency events, the variances of the envelopes were calculated after band-pass filtering both signals between 9.6 and 10.8 kHz. This resulted in variances of 17.1 and 17.8 for the simulated and measured vibration responses, which shows that the proposed model accurately predicts the fluctuations in amplitude of the high-frequency event.

Figure 8 compares the acceleration squared envelope spectra of the measured and simulated vibration response. The envelope spectra were calculated after band-pass filtering the acceleration signals between 9.6 and 10.8 kHz. The defect frequency components at $f_{dpo} = 45.4$ Hz and its harmonics are clearly visible, and very good agreement is achieved between the measured and simulated results.
Figure 8: Comparison of the measured and modelled acceleration squared envelope spectra. The signals were band-pass filtered between 9.6 and 10.8kHz before calculating the envelope spectra. The markers indicate the defect frequency $f_{bpo} = 45.4$ Hz and its harmonics.

Figure 9 (a) and (b) show zoomed-in sections of the signals presented in Figure 7, where the zoomed-in section corresponds to the period of time in which roller $j = 5$ on the defective raceway $r = 1$ approaches and leaves the defect. Figure 9(c) shows the path of the radial position of the 5th rolling element on row $r = 1$ ($Z_{j,k}$). In Figure 9(c), the small difference between the roller path and the defect geometry, outside the defect zone, is the roller contact deformation. Figure 9(a) and (b) show that at approximately 0.0394s, when the roller enters the defect, low frequency vibration is generated in both the experimentally measured and predicted vibration. Similarly experimentally measured and predicted vibration signals indicate that impacts occur when the rolling element hits the bottom of the defect and upon exiting the defect. The key vibration characteristics observed in the simulated and measured results are analysed as follows.
Figure 9: (a) Measurement vibration response of the test bearing at 252 rpm and under 50kN vertical static load (b) Simulated vibration response (c) Simulated path $\left(Z_{5,1} - D_p - D_b + \delta_{\text{out},5,1}\right)$ of the rolling element $j = 5^{th}$ as it travels through the defect zone (d) Simulated inner and outer contact forces acting on the $5^{th}$ rolling element.
Rolling element entry: The entry of the rolling element into the defect generates predominantly low frequency content in the vibration response. This low frequency vibration is due to the gradual de-stressing of the rolling element, which starts when it is positioned at an angle of \( \phi_{en} \) and ends when it loses its load carrying capacity. The gradual de-stressing of the rolling element as it enters the defect can be seen in the gradual decay in the contact force in Figure 9(d). During this stage, the load has to be redistributed amongst the other rolling elements in the load zone. Therefore the contact forces of the leading and lagging rolling elements in the load zone gradually increase. Consequently, as the number of the load carrying rolling elements decreases, the bearing assembly stiffness decreases. As this reduction is gradual, only low frequency modes of the system will be excited which results in the low frequency event in the vibration signal. The low frequency event can also be explained by examining the rolling element path in Figure 9(c). The change of the acceleration at the entry is due to the trace of the rolling element following an arc path and gradually losing contact with both raceways.

Impact of the rolling element and outer raceway: After losing contact with the raceways, the rolling element travels through the defect and impacts the bottom of the outer raceway defect, as shown in Figure 9(b-d) at approximately 0.043 s. It follows a curved path, the exact pattern of which depends on the centrifugal and inertia forces of the rolling element, until it hits the bottom of the defect. Depending on the dynamics of the system, the rolling element bounces a couple of times between the raceways before it hits the exit point of the defect. The time from when the 5\(^{th}\) rolling element is at the defect entry to the time that it strikes the bottom of the outer raceway, matches the timing when a high frequency excitation of relatively low amplitude is observable at time 0.043 s in the experimental results, see Figure 9(a). This event is predicted by the proposed model as shown in Figure 9(b) whereas previous models do not correctly predict this event.

Rolling element exit: The high frequency event in the vibration response observed in Figure 9(b) at approximately 0.048 s is associated with the rolling element exiting the defect. The rolling element has to change direction suddenly and restresses back to its normal load carrying capacity [23]. This abrupt change in the direction of movement causes a step change in velocity and generates an impulse in the acceleration signal, which excites the high frequency resonance mode of the bearing assembly that is included in the proposed model. As the rolling element restresses between the raceways at the defect exit, multiple impacts occur as it alternatively strikes the outer and inner raceway. The multiple impacts at the defect exit can be seen in the experimental results in Figure 9(a) at approximately 0.048 s, and have also been observed in experiments presented in previous studies [16]. The proposed model is able to predict and explain the mechanism of multiple impact events at the exit which cannot be predicted by the previous models [4-7].

6. Comparing results to the previous models and discussion

This section demonstrates the importance of considering the finite size of the rolling elements. This is achieved by comparing the simulated results of the previous section against those predicted when not including the finite size algorithm introduced in Section 4 into the proposed model. The resulting model will be referred to as the ‘point mass model’ and is representative of the previous models which included the rolling elements as point masses. The mechanisms that lead to inaccurate vibration response predictions in these previous models are identified and explained.

Figure 10 shows a comparison of the simulated vibration responses of the test bearing using the point mass and proposed models. Figure 11 shows a comparison of the path of the rolling element in the defect zone and the corresponding dynamic contact forces calculated using the proposed model and the point mass model. The following paragraphs describe observations of these simulation results by studying the significant events in the responses.

Entry event: Figure 10(c) shows that as the rolling element enters the defect zone at time \( t_1 \) (marked on the graph), the low frequency event is observable in the vibration response. The frequency characteristics of this event match closely with the experimentally measured response shown in Figure 9(a). In contrast to the proposed model, the simulation result of the point mass model exhibits a high frequency event superimposed on the low frequency event at time \( t_1 \) that is not consistent with the experimental results shown in Figure 9(a). The high frequency event modelled by the point mass model is the result of a near step change in contact forces, as shown in Figure 11(e) and (f) at time \( t_1 \). Contact forces in the previous models are predicted for only one point on the rolling element and any step change in the defect profile results in a step change in the contact forces. By modelling rectangular shaped defects using the previous models, as the rolling element reaches the entry point of the defect (the time when the radius of the outer raceway is \( D/2 \)),
both inner and outer contact forces are non-zero. The value of these contact forces suddenly changes to zero when the radius of the outer raceway changes to $D/2 + \lambda$, as is shown in Figure 11(c). However, the proposed model considers the finite size of the rolling elements, which results in a gradual decay of the contact forces, as opposed to a step change, as seen in Figure 11(c).

![Diagram](image-url)

Figure 10: The effect of including the finite size of rolling elements (a) Defect profile. (b) Simulated vibration response using the proposed model. (c) Simulated vibration response using the point mass model.
**Mid-events:** The next major event is the rolling element striking the bottom of the defect at time \( t_2 \), shown in Figure 11(a), which excites the high-frequency natural mode of the system (shown in Figure 10(b) at time \( t_2 \), and matches the timing of the high frequency event of the measured response shown in Figure 9(a) at time \( t_2 \). Figure 11(a) shows the path of the rolling element in the defect zone \( (Z_{5,1} - D_p - D_b + \delta_{out,5,1}) \) and the corresponding contact forces are shown in the Figure 11(b). The rolling element falls into the defect once both the inner and outer raceway contact forces acting on the rolling element have gradually decayed to zero. This results in the rolling element falling and striking the bottom of the defect at time \( t_2 \), which matches the vibration signal at time \( t_2 \) in the experimental results shown in Figure 9(a). The point mass model incorrectly predicts a faster drop of the rolling element to the bottom of the defect, which in turn results in a higher contact force estimation than the proposed model, shown in Figure 11(d), (e) and (f).
prediction of the sudden fall of the rolling element into the defect by point mass models [12, 25, 26, 35] is due to the step change of the defect profile. In these point mass models, the contact forces do not gradually decay to zero before losing contact with the entry point as predicted by the proposed model. Rather, the outer raceway contact force suddenly becomes zero when the inner raceway contact force still has a non-zero value, which results in an unrealistic rapid fall of the rolling element into the defect. It can be seen from the path of the rolling element in the point mass simulation, illustrated in Figure 11(d), that the first strike to the bottom of the defect occurs at time $t_5$. This is earlier than the more accurate prediction provided by the proposed model, as shown in Figure 11(a) at time $t_5$. Further, the point mass model predicts that the roller strikes the bottom of the defect three times as it traverses through the defect, whereas the proposed model predicts only a single strike. The experimental results in Figure 9(a) suggest that there is only a single strike, demonstrating the improvements provided by considering the finite size of the rolling element.

**Exit events:** The point mass model incorrectly predicts only one impact at the exit point as shown in Figure 10(c) and Figure 11(d) and (e). In contrast, the proposed model has the capacity to predict the multiple impacts observed in the experimental results, as seen in Figure 10(b) and Figure 11 (a) and (b). The simulation results for the point mass model show that the impact at the exit point occurs at time $t_5$ in Figure 11(c) and (d), whereas the first exit event (of the multiple exit events) predicted by the proposed model occurs at time $t_5$ in Figure 11(a) and (b), which is similar to the measured response shown in Figure 9(a). The proposed model is able to correctly predict the timing of this event because the finite size of the rolling elements is included in the model. In the proposed model, the circumference of the rolling element can strike the defect at any point before the centre of the rolling element reaches the exit point (i.e. before $\phi_{j,1} = \phi_{en}$). However, in the point mass model, the exit impact event incorrectly occurs when the rolling element centre is positioned at the exit point ($\phi_{j,1} = \phi_{en}$). Moreover, the step changes in the contact forces at the exit predicted by the point mass model result in unrealistically large contact forces and accelerations. Therefore, the exit event will be predicted incorrectly both in timing and magnitude when not considering the finite size of the rolling elements.

In summary it has been demonstrated that dynamic models of defective bearings that employ point masses to simulate rolling elements cannot accurately predict the characteristics typically observed in the vibration response measured for sharp edged defects. This limitation is overcome in the proposed model by including the finite size of the rolling elements, with results showing that this leads to accurate predictions that compare favourably with experimental findings.

7. Conclusion

This paper has demonstrated the importance of including the finite size of rolling elements into nonlinear multi-body dynamic models of defective rolling element bearings. A method for accounting for the finite rolling element size was presented and included into a nonlinear multi-body dynamic model of a defective bearing, which has the capacity to model a wide range of defect geometries. The vibration response predicted by the proposed model was compared with experimental results for the case of a rectangular shaped, sharp edged defect. This showed that the time-frequency characteristics of the significant events observed in the experimentally measured vibration response were more accurately predicted in comparison to previous models that do not account for the finite rolling element size. These significant events include the low-frequency event that occurs when a rolling element enters the defect entry, the multiple high-frequency events that occur when it exits the defect, and the events that occur when the rolling element strikes the raceway mid-way through the defect. Unlike the previous models, the actual defect geometry does not need to be artificially modified in the simulations to obtain a reasonable agreement with experimental results, and no assumptions on the rolling element path at the defect entry and exit need to be made. The mechanisms that cause inaccuracies in the predicted vibration response when including the rolling elements as point masses instead of finite size objects, as done in previous models, were identified and explained. The proposed model can be used to investigate the dynamic behaviour of defective bearings for a wide range of defect geometries. This will facilitate the development of new diagnostics algorithms, such as defect size estimation techniques for a wider range of defect sizes and geometries.

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