An experimental application of aeroacoustic time-reversal to the aeolian tone

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ABSTRACT

This paper presents an experimental application of the aeroacoustic Time-Reversal (TR) source localization technique for studying flow-induced noise problems and compares the TR results with those obtained using Conventional Beamforming (CB). Experiments were conducted in an Anechoic Wind Tunnel for the benchmark test-case of a full-span circular cylinder located in subsonic cross-flow wherein the far-field acoustic pressure was sampled using two Line Arrays (LAs) of microphones located above and below the cylinder. The source map obtained using the signals recorded at the two LAs without modeling the reflective surfaces of the contraction-outlet and cylinder during TR simulations revealed the lift-dipole nature of aeroacoustic source generated at the Aeolian tone; however, it indicates an error of 3/20th of Aeolian tone wavelength in the predicted location. Modeling the reflective contraction-outlet and cylinder during TR was shown to improve the focal-resolution of the source and reduce side-lobe

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levels in the low-frequency range. The experimental TR results were shown to be comparable to
(a) the simulation results of an idealized dipole at the cylinder location in wind-tunnel flow and
(b) that obtained by Monopole and Dipole CB, thereby demonstrating the suitability of TR
method as a diagnostic tool to analyze flow-induced noise generation mechanism.
I. INTRODUCTION

Microphone array processing techniques can be used to provide important information on the location and strength of the aeroacoustic sources. Such knowledge enables an understanding of the underlying flow-induced noise generation mechanisms essential for reducing noise emissions. A classical array processing technique is the Conventional Beamforming (CB) method\(^1\) that has previously been successfully applied to several aeroacoustic problems such as wind tunnel measurements of airfoil self noise\(^2\), jet-noise\(^3\), aircraft landing gear noise\(^4\), aircraft fly-over tests\(^5\) and field measurements of wind turbine noise\(^6\). The CB method enables the estimation of the direction of sound propagation from sources (typically, assumed to be of a monopole nature) towards a microphone array by means of an appropriate Green’s function\(^1\). In the conventional Delay-and-Sum Beamforming (DSB) method\(^1\), the acoustic pressure recorded at the microphones are delayed and summed coherently to enhance the signal from the source location and minimize it at a location different from the source. Therefore, the source location can be theoretically determined by scanning over a grid of mesh points (or nodes) and determining the location of the maximum. While in principle, the DSB method can be implemented in the time-domain\(^7\), it is most often used in the frequency-domain because its time-domain implementation is computationally expensive. A popular computationally efficient variant of the DSB method is the cross-spectral CB method, implemented in the frequency-domain and differs from the DSB in that it can eliminate the microphone self-noise or the auto-correlation between microphones\(^1,8\).

The frequency-domain cross-spectral CB method\(^1,8\), however, does not allow the visualization of the spatio-temporal evolution of the aeroacoustic fields, thus losing valuable time-domain information. Furthermore, the CB method often makes \textit{a priori} assumptions of the
nature of the source (noted above), thereby immediately restricting the understanding of what the
ture nature of the flow-induced noise source is. Also, it is difficult to directly account for the
effect of boundary conditions such as reflecting boundaries, diffracting surfaces and rigid-
scatterers on the accuracy of the source localization through the use of the CB method.

An alternative array processing technique is acoustic Time-Reversal (TR) which is a
promising source localization method. It finds application in diverse fields such as ultrasonics
for medical imaging, diagnostic and non-destructive testing, long-range communication in
underwater acoustics, in structural dynamics for health monitoring and localizing vibration
sources, localizing sound sources in an outdoor urban area with many buildings, in
propagation of water waves, source identification in electromagnetics and recently, for
detecting gas-leakage in pipelines. The acoustic TR method typically employs the following
two-step approach. In the first-step, the acoustic pressure field radiated by source(s) is recorded
over an array of microphones either completely or partially enclosing it. During the second-step
(constituting TR), the recorded acoustic pressure data is reversed in time and is enforced at the
boundary nodes (corresponding to the microphone locations) as numerical sources in a high-
resolution numerical algorithm that initiates back-propagation of acoustic waves that converge at
the source(s). The superior focusing quality of TR (in comparison to the CB method) is because
the back-propagated waves are simulated by numerically solving in reverse time, the same set of
equations that govern their emission from the source(s) without making a priori assumptions on
the source nature and also enables a visualization of the spatio-temporal evolution of the acoustic
fields (especially near the source). Indeed, the TR simulations are able to focus acoustic waves
(towards the source) by retracing them along the same trajectory created during their emission in
a homogenous stationary medium or a complex heterogeneous random medium with density or
temperature gradients\textsuperscript{18}, in the presence of rigid point-like cloud of scatterers\textsuperscript{19}, thousands of rigid vertical rods\textsuperscript{20} or a finite size rigid scatterer\textsuperscript{21}. Furthermore, the TR method can take into account, the effect of a reflecting surface\textsuperscript{22} and can be implemented (using limited channels or transducers) in a reverberating environment such as a room\textsuperscript{23} or a chaotic mono-crystalline silicon cavity\textsuperscript{24,25}.

The application of TR in the field of Computational Aeroacoustics (CAA) has also received considerable attention in recent works\textsuperscript{22,26-30}. This includes the use of TR for localizing a pulse in a uniform mean flow field\textsuperscript{22,26,27}, characterizing the nature of idealized aeroacoustic sources\textsuperscript{27-29}, developments of techniques for improving its computational implementation\textsuperscript{29} as well as the focal-resolution of sources\textsuperscript{30} and analysing the sound generated by a compressible plane mixing-layer flow\textsuperscript{26}.

A unique blend of numerical CAA simulation and data from experiments, therefore, gives TR a potential advantage over CB to analyze flow-induced noise mechanisms; however, such an experimental application of aeroacoustic TR has received only limited attention\textsuperscript{27}. For instance, Padois \textit{et al.}\textsuperscript{27} presented an application of the TR technique to localize loudspeaker source(s) (modeling a time-harmonic monopole or a dipole source) in a non-uniform shear mean flow using the experimental acoustic pressure time-history measured over one Line Array (LA) of microphones located above a wind tunnel test-section (and outside the flow). It is important to note that Padois \textit{et al.}\textsuperscript{27} essentially apply the TR technique to a \textit{simulated-experimental} aeroacoustics problem because noise is generated by loudspeakers even in the absence of flow. Therefore, the application of aeroacoustic TR to an actual flow-induced noise problem has not yet been demonstrated.
In light of the background provided above, this paper presents, for the first time, an application of the aeroacoustic TR method to an experimental flow-induced noise problem of the benchmark test-case of a full-span circular cylinder placed in a subsonic cross-flow\textsuperscript{31-34}. The tonal sound generated by a cylinder in cross-flow is an important source of flow-induced noise because cylindrical objects are found in a range of engineering applications such as rail pantographs, automobile appendages, aircraft landing gear, periscopes, masts and antennas\textsuperscript{35-38}. Indeed, an analysis and review of the flow-induced noise mechanism due to cylindrical objects is a subject matter of several investigations\textsuperscript{35-38}. Therefore, the test-case of flow-induced cylinder noise is considered here with a view to demonstrate the suitability of the TR method to analyse such class of problems. The objective of this work is to present a detailed analysis of the spatio-temporal evolution of the time-reversed acoustic pressure fields and accuracy of the TR source maps for characterizing the nature of flow-induced noise generated at the Aeolian tone using two LAs of microphones located in the far-field at above and below the circular cylinder, the effect of modeling the geometry of experimental set-up and ignoring mean flow during TR simulation as well as the use of a single LA on source characterization. The experimental TR results are also compared with those obtained using the cross-spectral CB method.

This paper is organized as follows. Section II briefly describes the experimental set-up. Section III analyzes the far-field acoustic spectra generated by the circular cylinder in subsonic cross-flow. Section IV describes the methodology for implementing the aeroacoustic TR simulations, computation of the TR source maps and algorithm for the cross-spectral CB method based on Monopole/Dipole steering vector formulation. Section V presents the results of the forward and TR simulation of an idealized dipole source located in wind tunnel flow, the analysis of which is intended to serve as a reference for interpretation of the experimental TR
results. Section VI presents the experimental TR results obtained using the acoustic pressure data of the Aeolian tone: an analysis of the spatio-temporal evolution of the time reversed acoustic pressure fields and the TR source maps. Section VII presents the CB source maps obtained using the experimental acoustic pressure data and compares them with the corresponding TR source maps (obtained in Section VI without modeling the contraction-outlet facility and the cylinder). The important contributions of this work are then summarized in the concluding Section VIII.

II. EXPERIMENTAL SET-UP AND TEST-MODEL

Experiments were conducted in the Anechoic Wind Tunnel (AWT) at the University of Adelaide. The AWT is nearly cubic having internal dimensions $1.4 \, \text{m} \times 1.4 \, \text{m} \times 1.6 \, \text{m}$ and its walls are acoustically treated with foam wedges providing a near reflection-free environment above 250 Hz. It contains a contraction-outlet that produces a quiet, uniform test-flow and is of a rectangular cross-section of height $h = 75 \, \text{mm}$ and width $w = 275 \, \text{mm}$ with flanges on the top and bottom, each of height $h_f = 40 \, \text{mm}$. The maximum free-stream velocity of the jet and turbulence intensity at the contraction-outlet is $U_\infty \approx 40 \, \text{m} \cdot \text{s}^{-1}$ and 0.33%, respectively $^{38,39}$.

The test-model, a full-span circular cylinder (of diameter $D_o = 4 \, \text{mm}$) is secured between two side-plates attached to the contraction-outlet flange as indicated in Figs. 1(a) and (b), which show a photograph and schematic of the front-view, respectively, of the experimental set-up. It is noted that end-effects are minimized because the cylinder-span $l = 450 \, \text{mm}$ along the $z$ (span-wise) direction is sufficiently beyond the width of contraction-outlet$^{39}$. A schematic of the side-view of the experimental set-up and the co-ordinate system convention is shown in Fig. 1(c) where $x$ is the stream-wise direction and $y$ is the vertical direction. The origin $(x = y = 0)$ is taken on the axis of the contraction-outlet at its opening and the cylinder is mounted on a pivot-hole located downstream at $x = 50 \, \text{mm}$, $y = 0$. 
In this work, a set of experiments were carried out at $U_\infty = \{16, 24, 32\} \text{ m} \cdot \text{s}^{-1}$ with flow issuing out from the contraction-outlet towards the positive $x$ direction as indicated in Fig. 1(c).

III. MEASURING AND ANALYZING THE FAR-FIELD ACOUSTIC SPECTRA

Acoustic measurements were taken with two LAs of microphones aligned parallel to the flow and located 700 mm apart, on opposite sides and equidistant from the cylinder, i.e., at $y = \pm L_y = \pm 350 \text{ mm}$ and are co-planar (located in the $z = 0$ plane which passes through the mid-span of the cylinder) as shown in Figs. 1(a) and (c). Each LA consists of 32 GRAS 40PH 1/4" phase-matched microphones (mounted in a timber-frame) positioned such that the spacing between two consecutive microphones is 30 mm, therefore, the total array length equals 930 mm.

The length of each LA measured upstream from the contraction-outlet is given by $L_{s1} = 96 \text{ mm}$ whilst that measured downstream denoted by $L_{s2} = 834 \text{ mm}$. It is noted that four microphones in both top and bottom LAs are located upstream of the origin (i.e., the contraction-outlet opening) with the microphone nearest to the contraction-outlet being 6 mm upstream as indicated in Fig. 1(c). The 64 microphones in the two LAs were connected to a National Instruments PXI-8106 data acquisition system containing 4 PXI-4496 simultaneous sample/hold Analog-to-Digital Converter (ADC) cards. The data (acoustic pressure time-history) at each of the 64 microphones are recorded at sampling frequency $f_s = 2^{16} \text{ Hz}$ for a sample time of 10 s.

When a cylinder is immersed in uniform flow, vortices of alternate rotation are shed from either side of the cylinder into its wake\cite{34}. This periodic shedding, known as a von Karman vortex street, occurs at a particular frequency, $f_a$, represented in a non-dimensional form by the Strouhal number $St_{D_0} = f_a D_0 / U_\infty$, based on $D_0$. The von Karman vortex street generates
unsteady forces on the surface of the cylinder that support dipole sound sources known as the Aeolian tone\textsuperscript{32,40}.

Figure 2 shows the frequency spectrum (measured in terms of the Power-Spectral Density (PSD)) of the acoustic pressure field due to flow-induced noise of the cylinder at $U_{\infty} = \{32, 24, 16\} \text{ m} \cdot \text{s}^{-1}$ measured at the microphone located in the bottom LA and positioned 24 mm downstream of the contraction-outlet opening. The occurrence of an Aeolian tone at frequency $f_u = \{784, 1208, 1584\} \text{ Hz}$ in the spectrum for $U_{\infty} = \{16, 24, 32\} \text{ m} \cdot \text{s}^{-1}$, respectively, (corresponding to $St_{D_h} = \{0.196, 0.201, 0.198\}$, respectively) suggests a dipole-like nature of the aeroacoustic source expected from the vortex shredding process. This is in agreement with the study of Norberg\textsuperscript{31} which shows that a circular cylinder is expected to produce an Aeolian tone at $St_{D_h} = 0.2$ for Reynolds number $Re_{D_h} = \{4240, 6360, 8480\}$ for $U_{\infty} = \{16, 24, 32\} \text{ m} \cdot \text{s}^{-1}$, respectively, based on $D_0 = 4 \text{ mm}$.

For implementing the TR simulation, the acoustic pressure signals recorded at each microphone were first band-pass filtered (using a high-order Finite Impulse Response (FIR) filter) in 1/3rd octave bands with center frequencies $f_C$ given by (a) 800 Hz, (b) 1250 Hz and (c) 1600 Hz for the spectrum obtained at $U_{\infty} = 16 \text{ m} \cdot \text{s}^{-1}$, 24 m·s\(^{-1}\) and 32 m·s\(^{-1}\), respectively, therefore, $f_u \approx f_C$ implying that the one-third octave band considered, accounts for almost the entire acoustic power contained in the spectrum. Furthermore, the background noise level generated by the free-stream jet is insignificant in comparison to the spectrum generated due to flow-induced noise\textsuperscript{38,39}, especially around the Aeolian tone. Therefore, it is expected to have a negligible contaminating effect on the TR (and CB) results; hence it was not removed from the spectrum prior to band-pass filtering.
IV. SOURCE LOCALIZATION TECHNIQUES: DESCRIPTION OF ALGORITHM

The algorithms for implementing the source localization techniques, namely, aeroacoustic TR simulation and cross-spectral CB for characterizing the flow-induced noise source is described in this section.

A. Aeroacoustic Time-Reverse (TR) simulation

The aeroacoustic TR simulation was implemented by numerically solving\textsuperscript{28-30} the 2-D Linearized Euler Equations\textsuperscript{27} (LEE) using the Pseudo-Characteristic Formulation\textsuperscript{26,41} (PCF) on a rectangular domain \(-L_{x1} \leq x \leq L_{x2}, -L_{y} \leq y \leq L_{y}\) in reverse time \(\tilde{t}\) shown as follows.

\[
\frac{\partial \tilde{p}}{\partial \tilde{t}} = -\frac{\rho_0 c_0}{2} \left\{ \left( \tilde{X}_{\text{linear}}^+ + \tilde{X}_{\text{linear}}^- \right) + \left( \tilde{Y}_{\text{linear}}^+ + \tilde{Y}_{\text{linear}}^- \right) \right\},
\]

\[
\frac{\partial \tilde{u}}{\partial \tilde{t}} = -\frac{1}{2} \left( \tilde{X}_{\text{linear}}^+ - \tilde{X}_{\text{linear}}^- \right) - \tilde{v} \left( -\frac{\partial U_0}{\partial y} \right) - \left( \frac{\rho_0 + \rho_0}{c_0} \right) \left( \frac{\partial U_0}{\partial x} \right),
\]

\[
\frac{\partial \tilde{v}}{\partial \tilde{t}} = -\frac{1}{2} \left( \tilde{Y}_{\text{linear}}^+ - \tilde{Y}_{\text{linear}}^- \right) - \left( -U_0 \right) \frac{\partial \tilde{v}}{\partial x},
\]

where \(\tilde{X}_{\text{linear}}^\pm = \pm \left( c_0 \mp U_0 \right) \left\{ \frac{1}{\rho_0 c_0} \frac{\partial \tilde{p}}{\partial x} \pm \frac{\partial \tilde{v}}{\partial y} \right\}\) and \(\tilde{Y}_{\text{linear}}^\pm = \pm \left( \frac{1}{\rho_0 c_0} \frac{\partial \tilde{p}}{\partial y} \mp \frac{\partial \tilde{v}}{\partial x} \right)\).

The time-reversed acoustic pressure \(\tilde{p}(x,y,\tilde{t})\) signals (recorded during experiments) were enforced at boundary nodes corresponding to the top and bottom LAs which initiates the back-propagation of acoustic wave into the 2-D domain. It is noted that \(L_{x3} = 100\) mm, therefore, the upstream length of the domain for implementing TR simulations is greater than the upstream length of the LAs.

In Eqs. (1-3), \(\tilde{u}\) and \(\tilde{v}\) denote acoustic velocities along the \(x\) and \(y\) directions, respectively, \(\rho_0 = 1.19 \text{ kg} \cdot \text{m}^{-3}\) is the ambient density of the medium (air), the sound speed \(c_0 = 345.75 \text{ m} \cdot \text{s}^{-1}\) (at ambient temperature \(T_0 = 297.47\) K) whilst \(U_0\) represents the spatially-developing shear.
mean flow profile of a 2-D free-jet issuing out from the contraction-outlet measured experimentally using a pitot-tube (connected to a 10 torr baratron with a plastic tube of length 5 m and inner diameter equal to 5 mm). The experimental mean flow profile in the AWT was modeled by

\[ U_0 = U_0(x, y) = \frac{1}{2} U_{max} \times \left(1 + \cosh \left(\frac{\beta \delta}{L_y}\right)\right) \times \text{sech} \left(\frac{\beta \delta - y}{2L_y}\right) \times \text{sech} \left(\frac{\beta \delta + y}{2L_y}\right), \]  

where \( U_{max} = U_{max}(x, U_{\infty}) \), \( \beta = \beta(x) \) and \( \delta = \delta(x) \) are the maximum mean flow, steepness of the shear-layer and half-thickness of the potential-core, respectively; their variation along \( x \) direction (from the contraction-outlet opening) is modeled by an appropriate polynomial-fit using constrained least-squares optimization shown as follows.

Downstream of the contraction-outlet, over the region \( x \geq 0 \),

\[ U_{max}(x, 32 \text{ m} \cdot \text{s}^{-1}) = 31.1746 - 1.1549 x_1 - 2.9524 x_2^2 - 1.8821 x_3^3 + 2.4703 x_4^4 + 1.5011 x_5^5 - 1.4359 x_5^4 - 0.4891 x_5^3 + 0.4070 x_5^2 + 0.0504 x_5^1 - 0.0401 x_5^0, \]  

\[ \beta(x) = 17.8892 - 7.8857 x_1 + 8.7515 x_2^2 - 10.4626 x_3^3 + 39.7521 x_4^4 - 11.9655 x_5^5 - 14.3155 x_5^4 + 4.3221 x_5^3 + 2.2220 x_5^2 - 0.1335 x_5^1 - 0.2600 x_5^0, \]  

where

\[ x_i = \frac{x - 361.0635 \times 10^{-3}}{237.6565 \times 10^{-3}}. \]  

The variation in half-thickness of the potential-core \( \delta(x) \) is given by

\[ \delta = 37.5 \times 10^{-3} \text{ m}, \text{ for } 0 \leq x \leq 374 \text{ mm}, \]  

\[ \delta(x) = (25.3448 - 8.3799 x_2 - 3.1940 x_2^2 - 2.3179 x_2^3 + 1.1574 x_2^4) \times 10^{-3}, \]  

for 374 mm \( \leq x \leq 524 \text{ mm}, \) where

\[ x_3 = \frac{x - 434.4 \times 10^{-3}}{48.8699 \times 10^{-3}}. \]
\[ \delta = 1 \times 10^{-3} \text{ m, for } x \geq 524 \text{ mm.} \quad (15) \]

It is noted that in Eqs. (11) and (14), \( x \) is taken in m whilst the interpolated values of \( U_{\text{max}}(x, 32 \text{ m/s}) \) and \( \delta(x) \) given by Eqs. (9) and (13) are also in m.

For \( x \leq 0 \), i.e., the region upstream of the contraction-outlet opening,

\[ U_{\text{max}} = 32 \text{ m/s}, \quad \beta = 150, \quad \delta = 37.5 \times 10^{-3} \text{ m.} \quad (16) \]

Figure 3 depicts the variation of the spatially-developing shear mean flow of the free-jet at \( U_\infty = 32 \text{ m/s} \) over the 2-D computational domain based on the model given by Eq. (8) and the optimized interpolating polynomial functions obtained above. (The interpolating polynomial for \( U_{\text{max}}(x, 24 \text{ m/s}) \) and \( U_{\text{max}}(x, 16 \text{ m/s}) \) were also similarly obtained using the constrained least-squares optimization, although these are not shown here for brevity.)

It is important to mention that the mean flow direction was reversed towards the negative \( x \) direction in Eqs. (1-5), i.e., \( U_0 \rightarrow -U_0 \) for implementing the TR simulations which was necessary to ensure the TR invariance \(^{26-30}\) of the governing 2-D LEE.

In Eqs. (1-3), the term \( \bar{X}_{\text{linear}}^+ \) denotes the acoustic flux propagating towards the positive \( x \) direction with a diminished speed of \( (c_0 - U_0) \) whilst \( \bar{X}_{\text{linear}}^- \) denotes the acoustic flux propagating towards the negative \( x \) direction with an enhanced speed of \( (c_0 + U_0) \) during TR simulations. Similarly, \( \bar{Y}_{\text{linear}}^\pm \) denotes the fluxes propagating with sound speed \( c_0 \) towards the positive and negative \( y \) directions, respectively. Furthermore, the term \( -U_0 \left( \frac{\partial \tilde{v}}{\partial x} \right) \) in Eq. (3) denotes acoustic disturbances advected by the reversed mean flow towards the negative \( x \) direction, the term \( \tilde{v} \left( -\frac{\partial U_0}{\partial y} \right) \) in Eq. (2) accounts for the refraction of acoustic waves through
the shear-layer (along the $y$ direction) whilst the term $\left( \tilde{u} + \frac{U_0}{c_0} \tilde{p} \right) \left( \frac{\partial U_0}{\partial x} \right)$ in Eq. (2) models the effect of spatially-developing nature (along the $x$ direction) of the free-jet flow on acoustic wave propagation.

1. Computation of spatial derivatives, mesh-resolution and time-integration scheme

The spatial derivative of acoustic pressure and velocities in the opposing fluxes $\left( \tilde{X}^\pm, \tilde{Y}^\pm \right)$ of the PCF were computed using an overall upwind-biased FD scheme$^{29}$ that is formulated using a fourth-order, seven-point optimized upwind-biased FD scheme$^{42}$ at interior nodes and a seven-point optimised backward FD scheme at the boundary nodes$^{43}$. In order to increase the mesh-resolution, two equally spaced nodes were added between each pair of nodes corresponding to the microphone locations. The acoustic pressure time-history at these two extra nodes (required during TR) were obtained by interpolating the experimental data between each pair of microphones using Lagrange polynomial interpolation$^{44}$, resulting in a mesh-size $\Delta x = 10$ mm along the $x$ direction. Equal mesh-size $\Delta y = 10$ mm was also considered along the $y$ direction. The efficiency of implementing the overall upwind-biased FD schemes is increased by recasting them in the following matrix form$^{29}$.

$$\begin{bmatrix} \frac{\partial \phi}{\partial x} \end{bmatrix} \approx \frac{1}{\Delta x} \begin{bmatrix} R_1 \end{bmatrix} \{ \phi \}, \quad \begin{bmatrix} \frac{\partial \phi}{\partial x} \end{bmatrix} \approx \frac{1}{\Delta x} \begin{bmatrix} R_2 \end{bmatrix} \{ \phi \},$$

(Eq. 17a, b)

where $\{ \phi \} = \left\{ \phi_1, \phi_2, \phi_3, ..., \phi_{N_{\text{nodes}}} \right\}^T$ and represents both acoustic pressure and velocities. Equations (17a) and (b) are used for computing the spatial derivatives in acoustic fluxes propagating towards the positive and negative directions, respectively. (The rows and columns of $[R_1]$ and $[R_2]$ matrices are however, not shown for brevity.) It is noted that these overall upwind-biased FD schemes make use of the opposite upwinding directions (or two different
group of stencil points) at a given node $i$ so that the inbuilt dissipation in the upwind-biased FD schemes damps only the unresolved high frequency waves and does not induce spatially growing oscillations with time, necessary to ensure the temporal stability and accuracy of TR simulations\textsuperscript{29}. The spatial derivative in the term $-U_0 \frac{\partial\vec{v}}{\partial x}$ is computed using Eq. (17b) because the direction of mean flow is reversed during TR simulations.

The maximum frequency that may be accurately propagated on this mesh is approximately 8255 Hz as determined from the Dispersion-Relation-Preserving\textsuperscript{42,43} range $\alpha_{DRP} \approx 1.5$ of the fourth-order optimized upwind-biased FD scheme and $c_0 = 345.75$ m/s. The third-order Total-Variation-Diminishing Runge-Kutta scheme\textsuperscript{45} is used for time-integration with a time-step $\Delta t = 1/f_s = 1.5259 \times 10^{-3}$ s implying a $CFL = \{0.55, 0.56, 0.58\}$ for $U_\infty = \{16, 24, 32\}$ m·s$^{-1}$, respectively, for the mesh-size considered.

2. Implementation of Anechoic Boundary Conditions (ABCs)

Enforcing the time-reversed acoustic pressure history at the two LAs (i.e., the Dirichlet boundary conditions\textsuperscript{27}) not only initiates the back-propagation of acoustic wave fronts into the 2-D domain which converge towards the aeroacoustic source location, but also generates outgoing acoustic waves. In order to eliminate the spurious reflections due to these outgoing waves and stabilize the 2-D TR simulations, it was crucial to implement the first-order Clayton–Engquist–Majda (CEM) ABC’s (see Ref. [46]) at all four computational boundaries ($y = \pm L_y$ and $x = \pm L_x$) and the corner ABC’s (see Ref. [47]) at the four corner nodes. The ABC’s at the four boundaries were further reinforced by setting the incoming fluxes to zero at these boundaries\textsuperscript{29}, i.e., $\vec{X}^{\pm}_{\text{linear}} = 0$ at $x = \mp L_x$, respectively, and $\vec{Y}^{\pm}_{\text{linear}} = 0$ at $y = \mp L_y$. The boundary
condition \( \left( \frac{\partial \vec{V}}{\partial x} \right)_{v=L_x} = 0 \) was also implemented to eliminate the incoming spurious numerical waves advected by the fluid necessary for temporal stability of TR\(^{29}\).

3. Use of the Time-Reversal-Sponge-Layer (TRSL) damping or the Superposition technique

Due to the conservation of energy\(^{27}\) and absence of an acoustic-sink\(^{30,48}\) during TR simulation of flow-induced noise sources (of a tonal or broadband nature), the converging wavefronts do not stop at the source but propagate beyond and interfere with flux emanating from the LAs located at the opposite boundary\(^{29}\) unlike the TR simulation of a transient signal such as a pulse\(^{26}\). Therefore, in order to suppress the deteriorating flux-interference effect near the LAs, a Time-Reversal-Sponge-Layer (TRSL) was implemented that damps the fluxes normally incident on the LAs by multiplying them by a Gaussian damping function \( G_{\text{TRSL}}(n) \) that smoothly decays to zero across \( n_{\text{TRSL}} \) nodes adjacent to and including the node on the LA boundary using the following transformations\(^{29}\).

\[
\tilde{Y}_{\text{linear}}(x, \pm L_x \mp n\Delta y) \rightarrow \tilde{Y}_{\text{linear}}(x, \pm L_y \mp n\Delta y) \times G_{\text{TRSL}}(n), \quad (18a, b)
\]

where

\[
G_{\text{TRSL}}(0) = 0
\]

and

\[
G_{\text{TRSL}}(n) = e^{-\frac{1}{2} \frac{1}{\alpha_{\text{TRSL}}^2} \left( \frac{n_{\text{TRSL}} - n}{n_{\text{TRSL}} - 1} \right)^2} \quad \text{for} \quad n = [1, 2, ..., n_{\text{TRSL}} - 1]. \quad (19a, b)
\]

In Eq. (19b), \( \alpha_{\text{TRSL}} \) is the damping coefficient taken equal to 3.5 whilst \( n_{\text{TRSL}} \) (the thickness of the TRSL) is taken equal to 10 in this work. The TRSL damping technique is however, used during TR simulations only for Aeolian tone frequencies \( f_a = \{1208, 1584\} \) Hz. The use of TRSL is not preferred during TR simulations for \( f_s = 784 \) Hz because at such low-frequency (for size of the computational domain considered along the \( y \) direction), the primary side-lobes occur near the LAs and the TRSL generates spurious local maxima region near the LAs which
tend to overwhelm the focal spots, thereby deteriorating the quality of source localization/characterization. Therefore, in order to prevent the problems arising out of flux-interference at the LAs, the superposition technique\textsuperscript{29} is used to implement the TR simulations for $f_a = 784$ Hz. To this end, the time-reversed acoustic pressure fields given by $\tilde{p}_{\text{Top}}(x, y, \tilde{t})$ and $\tilde{p}_{\text{Bottom}}(x, y, \tilde{t})$ obtained by implementing the TR simulations using a single LA located at the top and bottom boundaries, respectively, are superposed, i.e.,

$$\tilde{p}(x, y, \tilde{t}) = \tilde{p}_{\text{Top}}(x, y, \tilde{t}) + \tilde{p}_{\text{Bottom}}(x, y, \tilde{t}),$$

(20)

to obtain the total time-reversed acoustic pressure field $\tilde{p}(x, y, \tilde{t})$ which is equivalent to using two LAs simultaneously. However, in comparison to the use of TRSL, the computational cost of the superposition technique is high because it solves the same set of governing equations twice to obtain the $\tilde{p}(x, y, \tilde{t})$ field\textsuperscript{29}.

4. Scaling the contraction-outlet walls, flanges and the cylinder during TR simulation

The flow-induced noise source generated at the cylinder location during the experiment radiates acoustic waves which are refracted through the shear-layer. The wave fronts propagating upstream of the mean flow are diffracted by the flanges of the contraction-outlet, undergo reflection off its walls and subsequently propagate towards the non-reflective upstream, top and bottom boundaries. (The diffraction phenomenon is pronounced in the low-frequency range\textsuperscript{49}.)

As noted earlier, the TR simulation focuses acoustic waves by numerically solving the same set of differential equations that govern their emission from the source and can also incorporate appropriate boundary conditions. Indeed, the back-propagated acoustic waves during TR simulation are focused at the source by retracing them along the same complex trajectory created during their emission in the presence of multiple scatterers/diffraction objects and rigid
reflecting surfaces. Therefore, with a view to account for the effect of wave diffraction by the
flanges and its reflections off the walls (of the contraction-outlet) on aeroacoustic source
localization, another set of TR simulations were carried out whereby the flanges and walls of the
contraction-outlet were modeled by appropriate rigid wall conditions implemented by setting
\[
\bar{u}(x=0, \ 0.04 \text{ m} \leq y \leq 0.08 \text{ m}, \bar{t}) = 0 \quad \text{and} \quad \bar{u}(x=0, \ -0.04 \text{ m} \leq y \leq -0.08 \text{ m}, \bar{t}) = 0, \quad (21a, b)
\]
\[
\bar{v}(-0.196 \text{ m} \leq x \leq 0, \ y = \pm \ 0.04 \text{ m}, \bar{t}) = 0, \quad (22a, b)
\]
respectively. The rigid cylinder was also modeled by setting the acoustic particle velocities to
zero at the node corresponding to the cylinder location, i.e.,
\[
\bar{u}(x=0.05 \text{ m}, y=0, \bar{t}) = \bar{v}(x=0.05 \text{ m}, y=0, \bar{t}) = 0. \quad (23a, b)
\]
The TR source map (for the flow-induced dipole source at a given Aeolian tone) obtained
without modeling the contraction-outlet walls, flanges and the cylinder are compared with that
obtained with their modeling in an ensuing section.

5. Determining the location and nature of flow-induced noise sources

The TR simulation was implemented over a reverse time-interval \( \bar{t} = [0, 10000\Delta t] \)
whereby the aeroacoustic source location/characteristics were obtained by determining the focal
spots in the Root-Mean-Square (RMS) time-reversed acoustic pressure field\(^{27-30}\) (computed over
the time-interval when a steady-state acoustic field is observed throughout the domain) denoted
by \( \bar{P}_{RMS}^{TR}(x,y) \). The focal spot maximum is termed the focal point. The \( \bar{P}_{RMS}^{TR}(x,y) \) field is
converted to dB scale (with respect to \( p_{ref} = 2 \times 10^{-5} \text{ Pa} \)) and the source map denoted by
\( \bar{P}_{dB}^{TR}(x,y) \) is expressed relative to the focal point(s) whose magnitude is taken as 0 dB, see
Mimani et al.\(^{29,30}\).
B. Cross-spectral Conventional Beamforming (CB)

Cross-spectral CB using both Monopole and Dipole steering vectors was carried out in the one-third octave bands containing the Aeolian tone frequencies (discussed earlier). To this end, the acoustic pressure recorded at each microphone (during experiments) was first high-pass filtered using a 6th order Butterworth filter having a cut-on frequency of 300 Hz. A cross-spectral matrix, $C(f)$, was then computed to estimate the cross-spectral density for each pair of microphones by the Welch’s method\textsuperscript{50} using a Hanning window, Fast Fourier Transform (FFT) size of $2^{13}$ and 50\% window overlap for a frequency resolution of 8 Hz and 150.7 effective blocks (including the reduction due to window overlap).

For each frequency band within the upper and lower frequency limits of the one-third octave band considered, the CB was carried out on a regular grid of 2.5 mm spacing over the domain $-96 \text{ mm} \leq x \leq 504 \text{ mm}$ and $-300 \text{ mm} \leq y \leq 300 \text{ mm}$. The steering vectors used from the grid point $m$ to the microphone $n$ were constructed from the assumed Green’s function, $a_{m,n}(f)$, see Sarradj\textsuperscript{51}.

$$e_{m,n}(f) = \frac{a_{m,n}(f)}{\|a_n(f)\|}, \quad (24)$$

where $a_n(f)$ is the vector of Green’s functions from the grid point to all microphones. It is noted that the monopole Green’s function is given by\textsuperscript{51}

$$a_{m,n}^M(f) = \frac{e^{-j\omega r_{m,n}}}{4\pi \|x_m - x_n\|}, \quad (25)$$
where $\omega = 2\pi f$, $f$ represents the analysis frequency, $x_m$ and $x_n$ are the co-ordinates of the grid point and the microphone $n$, respectively. For a dipole source of orientation denoted by vector $\hat{l}$, the Green’s function is given by

$$a^D_{m,n}(f) = \frac{e^{-i\omega||\mathbf{x}_m - \mathbf{x}_n||}}{4\pi||\mathbf{x}_m - \mathbf{x}_n||} \left( \mathbf{l} \cdot \frac{\mathbf{x}_m - \mathbf{x}_n}{||\mathbf{x}_m - \mathbf{x}_n||} \right).$$  

(26)

The CB output was then calculated using the cross-spectral matrix and the steering vectors. In order to reduce the effect of microphone self-noise, the diagonal removal procedure given by Dougherty was used such that the beamforming output is

$$Y_n(f) = \frac{N}{N-1} \epsilon^H_n(f) C(f) \epsilon_n(f),$$  

(27)

where $N$ is the number of microphones, and the superscript $H$ denotes the conjugate transpose.

The CB source maps were calculated by using a rectangular integration across these frequency bins within the one-third octave frequency band of interest.

Finally, the effect of refraction of waves through the shear-layer is approximated by shifting the grid by $M_\infty h$, where $M_\infty$ is the Mach number and $h$ is the perpendicular distance within the flow (i.e., half-thickness of the potential-core $\delta = 37.5$ mm at the contraction-outlet opening), see Padois et al. This approximation is considered sufficient due to the low Mach number and the small height of the wind tunnel nozzle.

V. SIMULATION OF IDEALIZED DIPOLE IN WIND-TUNNEL FLOW: NUMERICAL VALIDATION OF TR METHOD

This section considers the test-case of a simulation of an idealized (point-like) tonal dipole source located at the cylinder location in a wind-tunnel flow. The objective of this simulation based analysis is twofold: to investigate what effect does the modeling of the contraction-outlet...
walls and flanges have on (a) the directivity of the radiated RMS acoustic pressure field of an idealized dipole source with its axis perpendicular to flow and (b) the corresponding TR source maps. Since the flow-induced noise generated at the Aeolian tone of a cylinder is known to have a lift-dipole type of source nature\(^{31-38}\), the aforementioned analysis will serve as a useful reference for the interpretation of experimental TR results presented later, thus explaining its significance.

A. Forward simulation

The forward simulation of the acoustic field radiated by an idealized dipole source of tonal frequency \(f_0\) in a 2-D free-space was carried out by numerically solving the inhomogeneous 2-D LEE (in the PCF) over a rectangular domain with ABCs (at all boundaries) given by\(^{29,55}\)

\[
\frac{\partial \tilde{p}}{\partial t} = -\frac{\rho_0 c_0}{2} \left\{ \left( X_{\text{linear}}^+ + X_{\text{linear}}^- \right) + \left( Y_{\text{linear}}^+ + Y_{\text{linear}}^- \right) \right\} + c_0^2 S_1, \tag{28}
\]

\[
\frac{\partial \tilde{u}}{\partial t} = -\frac{1}{2} \left( X_{\text{linear}}^+ - X_{\text{linear}}^- \right) - \nu \frac{\partial U_0}{\partial y} \frac{\partial U_0}{\partial x} + \frac{U_0}{\rho_0 c_0} \frac{\partial U_0}{\partial x} + S_2, \tag{29}
\]

\[
\frac{\partial \tilde{v}}{\partial t} = -\frac{1}{2} \left( Y_{\text{linear}}^+ - Y_{\text{linear}}^- \right) - U_0 \frac{\partial \tilde{v}}{\partial x} + \frac{S_3}{\rho_0}, \tag{30}
\]

where

\[
X_{\text{linear}}^+ = \pm \left( c_0 \pm \frac{\tilde{p}}{\rho_0 c_0} \right) \frac{1}{\rho_0 c_0} \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y}, \quad Y_{\text{linear}}^+ = \pm \left( c_0 \pm \frac{\tilde{p}}{\rho_0 c_0} \right) \frac{1}{\rho_0 c_0} \frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x}. \tag{31a-d}
\]

and setting\(^{28-30}\)

\[
S_1(x,y) = 0, \quad S_2(x,y) = F_D \delta(x-x_0) \delta(y-y_0) \sin(2\pi f_0 t), \quad S_3(x,y) = 0. \tag{32}
\]

In Eq. (32), \(F_D = 100 \text{ N} \cdot \text{m}^{-2}\) is the amplitude of the harmonic point-force simulating the idealized dipole source, the known location is given by \(x_0 = 54 \text{ mm}, y_0 = 0\) (taken at the node nearest to the cylinder location \(x = 50 \text{ mm}, y = 0\) whilst \(U_0\) is the spatially-evolving mean shear flow (given by Eq. (8) for \(U_\infty = 32 \text{ m} \cdot \text{s}^{-1}\)). It is noted that the rectangular domain and the mesh

20
size considered during forward simulations are identical to that considered for implementing experimental TR simulations noted in Section IV.A. The forward simulations were implemented for a sufficiently long time-interval given by $t = [0, 10000\Delta t]$ wherein the zero initial conditions were replaced by several periods of time-harmonic response and the acoustic pressure time-history was recorded at the top ($y_0 = 350$ mm) and bottom ($y_0 = -350$ mm) LAs. (Here, the time-step $\Delta t = 5.3059 \times 10^{-6}$ s corresponding to a CFL number equal to 0.2.)

1. **Without modeling the contraction-outlet walls and flanges**

Figures 4(a), (c) and (e) show the RMS acoustic pressure field radiated by an idealized dipole source (with its axis perpendicular to flow) of tonal frequency $f_0 = \{750, 1500, 3000\}$ Hz, respectively, located in a spatially-evolving mean shear flow obtained without modeling the contraction-outlet walls and its flanges over the dynamic range $[0, 80\ dB]$. It is noted that the known source location is represented by a circle $O$ whilst the mean flow direction towards the positive $x$ direction (during forward simulation) is indicated by an arrow. These conventions are followed throughout Section V. Furthermore, the two LAs, contraction-outlet and its flanges are shown by thick white lines and the same symbolic convention is followed henceforth.

Figures 4(a), (c) and (e) exhibit the expected directivity pattern due to an idealized dipole; the RMS acoustic pressure is maximum along the dipole axis perpendicular to the flow, (i.e. along the $y$ direction) whilst a pressure nodal line is observed along the $x$ axis. Furthermore, the RMS acoustic pressure level observed at the two LAs increases with an increase in the tonal frequency of the dipole source.

2. **With the implementation of rigid conditions due to contraction-outlet walls and flanges**

Figures 4(b), (d) and (f) show the RMS acoustic pressure field radiated by an idealized dipole source of tonal frequency $f_0 = 750$ Hz, $1500$ Hz, $3000$ Hz, respectively, located in a
spatially-evolving mean shear flow obtained with the modeling of the contraction-outlet walls and its flanges over the dynamic range $[0, 80 \text{ dB}]$.

It is observed from Figs. 4(b), (d) and (f) that incorporation of the contraction-outlet geometry during forward simulation significantly alters the directivity pattern of the radiated acoustic pressure field which is explained in terms of the wave diffraction phenomenon\textsuperscript{49} at the flanges. For tonal frequency $f_0 = 750 \text{ Hz}$, the corresponding wavelength $\lambda_0 = 461 \text{ mm}$ is significantly greater than the total height (150 mm) of the contraction-outlet facility taken as sum of (a) the contraction-outlet height $h = 75 \text{ mm}$ and (b) that of the flanges $2 \times h_f = 80 \text{ mm}$.

Therefore, during forward simulation, the wave fronts are diffracted by the flanges and the propagation of the diffracted wave fronts result in reflections from the rigid-walls of the contraction-outlet and subsequently, a significant amount of acoustic power being received by the non-reflective upstream boundary as observed from the RMS field shown in Fig. 4(b).

For tonal frequency $f_0 = 1500 \text{ Hz}$, the corresponding wavelength $\lambda_0 = 230.5 \text{ mm}$ is comparable to the total height of the contraction-outlet facility. Therefore, the wave diffraction\textsuperscript{49} phenomenon at the flanges is less pronounced. As a result, the wave reflection at the walls of the contraction-outlet and the acoustic power intercepted by the upstream boundary is substantially less. Rather, due to reflection of incident waves by the flanges, the downstream length of the LAs intercept a larger amount of radiated acoustic flux in comparison to the upstream length as may be observed from Fig. 4(d).

For tonal frequency $f_0 = 3000 \text{ Hz}$, the corresponding wavelength $\lambda_0 = 115.25 \text{ mm}$ is smaller than the total height of the contraction-outlet facility, therefore, the wave diffraction\textsuperscript{49} at the flanges is negligible. Rather, the acoustic wave fronts propagating away from the source and incident on the flanges are completely reflected from it. Indeed, the RMS directivity pattern
shown in Fig. 4(f) resembles the directivity field due to a piston in a rigid baffle and as a result, the downstream length of the LAs intercepts a further greater amount of radiated acoustic flux.

**B. Aeroacoustic TR simulation**

The aeroacoustic TR simulations were implemented by numerically solving Eqs. (1-3) and using the simulated acoustic pressure data recorded at the top and bottom LAs during the forward simulation of the idealized tonal dipole source when the contraction-outlet and its flanges were modeled (corresponding to Figs. 4(b), (d) and (f)). The direction of the spatially-evolving mean shear flow at $U_\infty = 32 \text{ m \cdot s}^{-1}$ was reversed during TR. As discussed in Section IV.A.5, the TR source map (of the idealized dipole source) was obtained by computing the RMS acoustic pressure field when steady-state time-harmonic condition was developed throughout the 2-D domain. The following set of TR simulation was carried out at each tonal frequency: (a) without modeling the contraction-outlet walls and its flanges and (b) with their modeling (by making use of Eqs. (21) and (22)) with a view to investigate the effect of rigid-wall modeling of the contraction-outlet facility on characterization of the idealized dipole source.

1. **Without modeling the contraction-outlet walls and flanges**

   Figures 5(a), (c) and (e) show the TR source maps of the idealized dipole source of tonal frequency $f_0 = 750 \text{ Hz}$, $1500 \text{ Hz}$, $3000 \text{ Hz}$, respectively, obtained using the top and bottom LAs without modeling the contraction-outlet walls and its flanges over a dynamic range given by $[0, \ -10 \text{ dB}]$. It is observed that the TR source maps shown in Figs. 5(a), (c) and (e), exhibit two focal spots (identical in terms of relative magnitude, shape, and size) that are located in proximity, thereby confirming the dipole nature of the idealized source$^{28-30}$. The geometrical center of the two focal points (noted from their respective focal spots) is taken as the predicted location of the dipole source and is denoted by a cross $X$ in the source maps. Furthermore, the
top and bottom LAs located at \( y = \pm 350 \text{ mm} \) boundaries, respectively, are indicated by thick white lines whilst the reversed direction of mean flow (during TR simulations) is indicated by an arrow; the same dynamic range and symbolic conventions are followed henceforth.

The predicted location of the dipole source and the error (taken as the distance between the predicted and known locations expressed as a fraction of a wavelength \( \lambda_0 \) corresponding to the tonal frequency \( f_0 \)) is shown in the 2\(^{nd}\) and 3\(^{rd}\) columns, respectively, of Table I. These values indicate a maximum error of \( \lambda_0/20 \) in the predicted source location, thereby demonstrating the accuracy of the TR simulation (implemented without modeling the contraction-outlet walls and its flanges).

2. With the implementation of rigid-wall conditions due to contraction-outlet walls and flanges

Figures 5(b), (d) and (f) show the TR source maps of the idealized dipole source of tonal frequency \( f_0 = 750 \text{ Hz}, 1500 \text{ Hz}, 3000 \text{ Hz}, \) respectively, obtained using the top and bottom LAs with the incorporation of the contraction-outlet geometry modeled by implementing Eqs. (21) and (22) during TR simulations. Figure 5(b) is comparable to Fig. 5(a); however, the location of the idealized dipole source predicted in Fig. 5(b) is downstream of the known location, thereby resulting in a large error \( 0.15\lambda_0 \) as shown in the 5\(^{th}\) column of Table I. Furthermore, a significant wave reflection at the rigid-walls of the contraction outlet is observed in Fig. 5(b) which is explained by the following discussion. The wavelength \( \lambda_0 = 461 \text{ mm} \) at tonal frequency \( f_0 = 750 \text{ Hz} \) is significantly greater than the total height (150 mm) of the contraction-outlet facility. Therefore, during TR, the back-propagated waves from the upstream LAs are diffracted\(^{49}\) (at the flanges) which directs their propagation downstream of the contraction-outlet. Furthermore, the back-propagated waves from the downstream LAs are also diffracted and
propagate towards the rigid-walls of the contraction-outlet (whereby they are reflected) and the
non-reflective upstream boundary. Due to a continuous interaction of the waves back-
propagating from the upstream and downstream LAs over the region downstream of the known
location, two instantaneous focal-spots are formed throughout TR yielding Fig. 5(b).

Figure 5(d) is however, significantly different from Fig. 5(c); it exhibits two localized
maxima regions formed near the flanges which makes it difficult to characterize the dipole
source nature. (It is for this reason that the predicted location and error are not shown in the 4th
and 5th columns, respectively, of Table I for \( f_0 = 1500 \text{ Hz} \).) This observation is understood by
noting that wavelength \( \lambda_0 = 230.5 \text{ mm} \) at tonal frequency \( f_0 = 1500 \text{ Hz} \) is comparable to the
total height of the contraction-outlet facility. This signifies that the diffraction phenomenon of
waves (at the flanges) is less pronounced; as a result, the downstream lengths of the LAs receive
a larger portion of the radiated acoustic power during forward simulation as suggested by Fig.
4(d). During TR simulation, the waves back-propagating from downstream LAs and incident on
the flanges also experience a limited diffraction, rather, these are reflected from the flanges
towards downstream and continuously interferes with the waves converging at the source from
the downstream LAs resulting in formation of two instantaneous maxima regions near the
flanges throughout TR simulation yielding Fig. 5(d). Forward simulation were also carried out
for the idealized dipole of \( f_0 = 1500 \text{ Hz} \) located further downstream at \( x_0 = 80 \text{ mm}, y_0 = 0 \).

However, in this case, the TR source map obtained with the modeling of contraction-outlet
facility did not exhibit two maxima regions localized near the flanges; rather, the source map
showed two focal spots (located in proximity) similar to those shown in Fig. 5(b). This signifies
that when the dipole source wavelength is comparable to the total height of the contraction-outlet
facility, the effect of incorporating the contraction-outlet geometry during TR need not be
counter-productive if the source is located sufficiently away from the flanges ensuring that span
(along the x direction) of the focal spots does not overlap with the flanges.

Figure 5(f) is similar to Fig. 5(e); it exhibits two focal spots, thereby confirming the dipole
nature. However, unlike Fig. 5(e), a small error of $0.09 \lambda_0$ is observed in the predicted source
location in Fig. 5(f) as indicated in Table I. This observation is explained by noting that the
wavelength $\lambda_0=115.25$ mm (at $f_0=3000$ Hz) is smaller than the total height of the contraction-
outlet facility. Therefore, during TR, the waves incident on the flanges from downstream LAs
undergo a negligible diffraction, rather, these are completely reflected from it. The interaction of
the reflected waves with those converging near the source (from the downstream LAs)
throughout the TR simulation and the relatively smaller size of the focal spots at $f_0=3000$ Hz
(so that it does not overlap with the flanges despite the proximity of the source to the flanges)
result in the formation of two instantaneous focal-spots yielding Fig. 5(f). Indeed, the focal spot
size (along the x direction) and the side-lobe levels in Fig. 5(f) are smaller in comparison to those
observed in Fig. 5(e), indicating an improved focal-resolution.

VI. EXPERIMENTAL TR RESULTS AND DISCUSSION

This section presents the experimental TR results obtained using the acoustic pressure data
of the Aeolian tone generated by the circular cylinder in cross-flow during aeroacoustics
experiments carried out in the AWT (described in Section II) and an analysis of these results.

A. TR simulation without modeling the contraction-outlet walls, flanges and cylinder

1. Spatio-temporal evolution of the time-reversed acoustic pressure field

Figure 6 shows the time-snapshots of the spatio-temporal evolution of the time-reversed
acoustic pressure field $\tilde{p}(x,y,t)$ obtained using the experimental acoustic pressure data
(recorded at the two LAs) of the flow-induced noise generated at the Aeolian tone $f_a=1584$ Hz
without modeling the contraction-outlet geometry and the cylinder during TR simulation. (The colorbar in Fig. 6 represents the acoustic pressure in Pa.) It is noted that the cylinder (located at 
$x_o = 50 \text{ mm, } y_o = 0$) is represented by a circle $O$ and this symbolic convention is followed henceforth in the experimental TR results.

Figures 6(a) and (b) indicate a simultaneous emission of acoustic fluxes from the two LAs during the initial time-instants that propagate into the domain and are about to converge or undergo constructive interference near the cylinder location. Figure 6(c) shows that this is followed by formation of two instantaneous maxima regions (i.e., the instantaneous focal spots) about the cylinder-axis of nearly the same strength but opposite phase, indicating a dipole-source. The simulations reveal that at the source region, the width of the wave-fronts diminish whilst their amplitude significantly increase. However, due to the conservation of energy\cite{27} and absence of an acoustic-sink$^{30,48}$ during TR, the converging wave-fronts do not stop at the source but propagate beyond towards the LA located at the opposite boundary. By virtue of the TRSL damping technique$^{29}$ (implemented using Eqs. (18a) and (b)), the deteriorating flux-interference effect near the LAs is suppressed. Figures 6(d-f) indicate a continuous formation of two instantaneous focal spots throughout the TR simulations, although their instantaneous geometrical center (taken as the instantaneous dipole location) slightly varies over time. To enhance this discussion on the spatio-temporal evolution of the time-reversed acoustic pressure field, the reader is also referred to Multimedia 1 which plays the corresponding TR simulation.

The spatio-temporal evolution pattern of the $\hat{p}(x,y,t)$ field obtained (without modeling the contraction-outlet geometry and the cylinder during TR) using the experimental acoustic pressure data recorded at $f_a = \{1208, 784\}$ Hz were found to be similar to that shown in Fig. 6. However, the size of the instantaneous dipole focal spots during TR simulation at
\( f_a = \{1208, 784\} \) Hz were successively larger (due to low-frequency range) than those observed at \( f_a = 1584 \) Hz in Figs. 6(c-f).

2. Analyzing the TR source maps: Effect of Aeolian tone frequency

Figures 7(a), (c) and (e) show the TR source maps due to the full-span circular cylinder obtained using the experimental acoustic pressure data (recorded at the top and bottom LAs) at the Aeolian tone \( f_a = \{784, 1208, 1584\} \) Hz, respectively, without modeling the contraction-outlet walls, its flanges and the cylinder during TR simulation. It is noted that Figs. 7(a), (c) and (e) present the results of experiments carried out at \( U_\infty = \{16, 24, 32\} \) m s\(^{-1}\), respectively, in the one-third octave bands with \( f_c = \{800, 1250, 1600\} \) Hz, respectively. Figures 7(a), (c) and (e) exhibit a pair of focal spots (of nearly the same magnitude, shape and size) located in proximity. The experimental TR source maps shown in Figs. 7(a) and (e) are observed to be similar to the TR source maps of the idealized dipole source of comparable tonal frequency shown in Figs. 5(a) and (c), respectively. This indicates that the flow-induced noise generated at the Aeolian tone due to a cylinder in a cross-flow has a dipole-type nature and is qualitatively similar to an idealized dipole source (whose axis is perpendicular to the mean flow). Indeed, due to the lift forces responsible for noise generation, the orientation of the focal spots in Figs. 7(a), (c) and (e) is almost above and below the cylinder and thus, the source is termed a lift-dipole\(^3\).

Table II shows the predicted location (indicated by \( X \)) of the flow-induced dipole source obtained in Figs. 7(a), (c) and (e) corresponding to \( f_a = \{784, 1208, 1584\} \) Hz, respectively, and the error which is taken as the distance of the predicted location from the cylinder expressed as a fraction of the corresponding wavelength \( \lambda_a \) of an Aeolian tone. The average error in prediction is \( \approx 0.16\lambda_a \), while the error is largest at the lowest Aeolian tone \( f_a = 784 \) Hz and smallest at the
highest Aeolian tone $f_a=1584$ Hz. This observation is explained as follows. The span-wise coherence length scale of vortex shedding$^{31}$ is larger at lower free-stream velocity ($U_\infty=16 \text{ m} \cdot \text{s}^{-1}$); hence, it is possible that the two co-planar LAs of microphones located in the $x$-$y$ plane passing through the cylinder mid-span and oriented along a direction perpendicular to it will receive phase information from sections along the cylinder span that are similar to a dipole source placed further downstream (of the cylinder) and thus gives a positional error in the TR source map. In other words, a greater length scale of span-wise coherence at low-frequency indicates a 3-D nature of the flow-induced noise problem, therefore, the use of 2-D TR simulation somewhat under-represents the problem of low-frequency Aeolian tone generation which may be more accurately modeled by the computationally intensive 3-D TR simulation.

On the other hand, at higher free-stream velocity ($U_\infty=32 \text{ m} \cdot \text{s}^{-1}$), this positional error in the TR source map will be reduced as the span-wise coherence length scale will be smaller, forming a larger number of compact acoustic dipole sources that radiate individually along the cylinder span which reduces the phase error on the two co-planar LAs of microphones at a given time, thereby indicating that the use of 2-D TR simulation is able to model the high-frequency Aeolian tone generation problem with a relatively greater accuracy. Furthermore, it is noted that accuracy of predicted location may possibly be improved by increasing the upstream and downstream lengths of LAs, (i.e., by using more number of microphones) due to an increased effective angular aperture at the source location$^{29}$.

Table II also indicates that the average Sound Pressure Level (SPL) at the predicted focal points of the dipole source in Figs. 7(a), (c) and (e) increases with an increase in the frequency of Aeolian tone. This is in agreement with the PSD spectrum of the full-span (see Fig. 2) which shows that the highest SPL for $U_\infty=32 \text{ m} \cdot \text{s}^{-1}$.
Figures 7(a), (c) and (e) indicate that with an increase in the Aeolian tone, the longitudinal and transverse size of the dipole focal spots decreases and a more compact orientation of the focal points are obtained. In particular, Fig. 7(a) exhibits focal spots of a relatively larger size (along the flow-wise direction) due to the occurrence of low-frequency Aeolian tone which extend beyond the upstream length of LAs whilst in Figs. 7(c) and (e), the transverse span of the focal spots extends up to a distance close to the upstream end of the LAs. For this reason, a larger upstream domain length was used during TR simulations. The size of the dipole focal spots is quantified in terms of the following two metrics; the transverse spatial resolution (parallel to the LAs) and longitudinal spatial resolution (perpendicular to the LAs) and is taken as Full-Width at Half-Maximum (FWHM) given by sum of the distances corresponding to –6 dB level considered on either side of a focal point\(^9\). The average transverse and longitudinal focal-resolution of the dipole focal spots is quantified in the 2nd and 4th columns, respectively, of Table III for different Aeolian tone frequency in terms of the wavelength \(\lambda_a\) corresponding to \(f_a\). It is observed from Table III that the average transverse and longitudinal size of the focal spots are nearly commensurate with \(\lambda_a\); their average values are given by 1.03\(\lambda_a\) and 0.38\(\lambda_a\), respectively, for the given LA configuration. These average values of the focal-resolution signify the conventional half-wavelength diffraction limit\(^{30,48}\) (at the source vicinity) of the TR method. Furthermore, it is noted that dipole focal spots at \(f_a = \{1208, 1584\} \text{ Hz}\) span a length along the flow-wise direction (i.e., the transverse size) which includes the contraction-outlet flanges because the cylinder is mounted at pivot-hole which is in proximity to the contraction-outlet. At \(f_a = 784 \text{ Hz}\), the transverse size of the focal spots is largest, however, its transverse span does not overlap the flanges because at such low-
frequency, the distance (along $y$ direction) between the focal points is greater than the total height (155 mm) of the contraction-outlet facility.

**B. TR simulation with the modeling of contraction-outlet walls, flanges and cylinder**

1. **Spatio-temporal evolution of the time-reversed acoustic pressure field**

Figure 8 shows the spatio-temporal evolution of the time-reversed acoustic pressure field $\tilde{p}(x, y, \tilde{t})$ obtained using the experimental acoustic pressure data (recorded at the two LAs) of the flow-induced noise generated at the Aeolian tone $f_a = 1584$ Hz with the modeling of the contraction-outlet geometry and the cylinder during TR simulation. (The reader is also referred to Multimedia 2 which plays the corresponding TR simulation.) It is noted that the reverse time-instants shown in Figs. 8(a-f) are the same as those considered in Figs. 6(a-f), respectively.

Figures 8(a) and (b) are similar to Figs. 6(a) and (b), respectively. These time-snapshots indicate a simultaneous back-propagation of acoustic fluxes from the two LAs that are about to converge near the cylinder. Figure 8(c) shows that the waves back-propagating from the LAs downstream of the contraction-outlet impinge and are about to be reflected from the flanges unlike Fig. 6(c). It is noted that because the Aeolian tone wavelength $\lambda_a \approx 218$ mm (at $f_a = 1584$ Hz) is comparable to the total height of the contraction-outlet facility given by 155 mm (indicated earlier), the incident acoustic waves undergo a limited diffraction about the rigid flanges. Due to limited acoustic fluxes diffracted towards the rigid-walls of the contraction-outlet, almost negligible reflection off its walls is observed in Fig. 8(d). Rather, instantaneous local maxima regions are observed near the flanges in Fig. 8(d) due to interaction of the waves reflected from the flanges and that incident towards it from the LAs downstream of the contraction-outlet. Figure 8(e) shows the formation of two instantaneous focal spots (downstream of the cylinder) and is similar to Fig. 6(e). Figure 8(f) is similar to Fig. 8(d) and
indicates the formation of two instantaneous local maxima regions near the flanges at different time-instants during TR simulation.

The spatio-temporal evolution pattern of the $\bar{p}(x, y, t)$ field obtained (with modeling of the contraction-outlet geometry and the cylinder during TR) at $f_a = \{1208, 784\}$ Hz were however, found to be significantly different from Fig. 8. This is explained as follows. At $f_a = 784$ Hz, the acoustic waves radiated from the flow-induced dipole (at the cylinder location) undergo a significant diffraction about the flanges because the corresponding Aeolian tone wavelength $\lambda_a \approx 441$ mm is significantly greater than the total height (155 mm) of the contraction-outlet facility. As a result, the upstream LAs intercept a significant amount of the outgoing acoustic fluxes during experiments (as was explained through numerical simulation in Fig. 4(b)). Therefore, during TR simulation, the back-propagated waves from the upstream LAs also undergo a significant diffraction (about the flanges) which directs their propagation downstream of the contraction-outlet. Furthermore, the low-frequency back-propagated waves from the downstream LAs are also diffracted about the flanges and propagate towards the walls of the contraction-outlet and the non-reflective upstream boundary. Therefore, due to interaction of the waves continuously back-propagating from the upstream and downstream LAs over the region downstream of the cylinder, two instantaneous focal-spots are formed throughout the TR simulation. Indeed, the spatio-temporal evolution of the $\bar{p}(x, y, t)$ field at $f_a = 784$ Hz obtained without and with the modeling of rigid conditions is observed to be somewhat similar.

2. Analyzing the TR source maps: Effect of Aeolian tone frequency and implementation of the rigid-wall conditions

Figures 7(b), (d) and (f) show the TR source maps due to the full-span circular cylinder obtained by enforcing the time-reversed experimental acoustic pressure data (at the two LAs)
corresponding to the Aeolian tone \( f_o = \{784, 1208, 1584\} \) Hz, respectively, with the modeling of
the contraction-outlet geometry and the cylinder during TR simulations by making use of Eqs. (21-23). Figure 7(b) is comparable to Fig. 7(a); however, location of the flow-induced dipole
source predicted in Fig. 7(b) is further downstream of the cylinder, thereby indicating a greater
error. Furthermore, Fig. 7(b) shows a significant wave reflection at the rigid-walls of the
contraction outlet due to the low-frequency waves diffracted about the flanges and incident on it
and is observed to be similar to Fig. 5(b) which shows the TR source map of the idealized dipole
of \( f_o = 750 \) Hz obtained with the modeling of contraction-outlet walls and flanges. Figure 7(d) is
also observed to be comparable to Fig. 7(b); however, at this Aeolian tone wavelength
\( \lambda_o \approx 286 \) mm, the diffraction effect is relatively less pronounced. As a result, due to reflection at
the flanges, localized region of maximum acoustic pressure (whose relative magnitude is slightly
smaller than the focal points) is formed near the flanges whilst the predicted location of the
dipole is relatively closer to the cylinder. Figure 7(f) is however, significantly different from its
counterpart Fig. 7(e). It exhibits two local maxima regions near the flanges (due to reasons noted
in the discussion of Fig. 8) which makes it difficult or nearly impossible to characterize the
dipole source nature and is indeed qualitatively similar to Fig. 5(d).

The 4\(^{th}\) column of Table II indicates that incorporation of the rigid-wall model of the
contraction-outlet geometry and the cylinder (during TR) results in a greater error in the
predicted location of the flow-induced dipole source at the low-frequency Aeolian tone whilst
the error in prediction reduces at a higher frequency. Furthermore, the 5\(^{th}\) column of Table II
indicates that the average SPL at the focal points also marginally increases on incorporation of
the rigid-wall model of the contraction-outlet geometry and the cylinder. The 3\(^{rd}\) column of
Table III indicates a significant improvement in the average transverse focal-resolution of the
dipole focal spots whilst the average longitudinal focal-resolution shown in 5th column of Table III indicates only a marginal change when the rigid-wall modeling is implemented during TR.

In view of the formation of two local maxima regions (near the flanges) in Fig. 7(f) which makes the characterization of the flow-induced dipole source impossible, the predicted source location, average SPL and the average transverse and longitudinal focal-resolution corresponding to \( f_a = 1584 \) Hz are not shown in Tables II and III, respectively. This demonstrates that when the Aeolian tone wavelength \( \lambda_o = 218 \) mm is comparable to the total height (155 mm) of the contraction-outlet geometry and the cylinder in cross-flow (inducing dipole sources) is mounted in proximity to the contraction-outlet opening, the rigid-wall modeling of the contraction-outlet geometry during TR is counter-productive.

C. Effect of neglecting the convective effect of mean flow during TR simulation

The effect of neglecting the convective effect of mean flow during TR simulations on the accuracy of the source localization is investigated in this section. To this end, the TR simulation was implemented by considering a stationary medium, i.e., by setting the mean flow field \( U_0(x, y) = 0 \) in Eqs. (1-3) and enforcing \( \tilde{p}(x, y = \pm L_y, \tilde{t}) \) at nodes of the two LAs corresponding to the Aeolian tone \( f_a = 1584 \) Hz whereby the source map shown in Fig. 9(a) is obtained. It is observed that Fig. 9(a) is nearly identical to the corresponding TR source map shown in Fig. 7(e) obtained by considering the mean flow (modeled by Eq. (8)) during simulation. Indeed, the predicted location of the dipole source, the average SPL at the focal points as well as the focal-resolution obtained in Fig. 9(a) is the same as that obtained in Fig. 7(e) indicated in the 3rd row of Tables II and III. The only (minor) difference apparent in Figs. 7(e) and 9(a) is that primary side-lobe levels are marginally lower in the former due to the incorporation of the mean flow. This signifies that for such low Mach number flows when
$M_0 \leq 0.1$ (approximately), incorporation of the mean flow in the governing LEE may be altogether ignored during TR simulations without affecting the accuracy of source localization.

**D. Comparison of source map obtained using one LA of microphones with that obtained using the two LA configuration**

The effect of using a limited angular aperture of the Time-Reversal Mirror (TRM) during aeroacoustic TR simulations on the characterization of flow-induced noise source generated at the Aeolian tone is examined. To this end, the TR simulations were implemented by enforcing the time-reversed acoustic pressure data (obtained at the Aeolian tone $f_\text{A} = 1584$ Hz) at only the top LA and the corresponding TR source map is presented in Fig. 9(b). (It is noted that the TRSL damping technique was not used at nodes near the bottom LA in this case.) Figure 9(b) exhibits a large elongated focal spot which extends from the top LA boundary to the bottom LA boundary resulting in a poor resolution of the flow-induced dipole source. The predicted location of the source (obtained by determining the focal point) is given by $x_\text{o} = 94$ mm, $y_\text{o} = 60$ mm, thereby indicating a localization error equal to $0.34 \lambda_a$. The source strength (at the focal point) is estimated to be 87.3 dB. Therefore, due to its limited angular aperture, the use of only one LA (for intercepting/recording the radiated acoustic far-field) during aeroacoustic TR simulations is insufficient for characterizing the dipole nature of the flow-induced source and also results in a significant error in the predicted location. Rather, as shown earlier, the use of two LAs located at the top and bottom boundaries on the opposite sides of the cylinder records minimum boundary data (due to its relatively larger angular aperture) required for accurately characterizing the lift-dipole nature of the flow-induced source and localizing it within an average accuracy of $0.16 \lambda_a$ (see Table II). Therefore, the present results (Figs. 7(e) and 9(b)) experimentally corroborates the simulation based results of the
characterization of an idealized dipole source located in a uniform flow field reported in a
previous work\textsuperscript{28}.

VII. CB RESULTS AND COMPARISON WITH TR SOURCE MAPS

The source maps obtained by the CB method are analyzed and compared with the
corresponding TR source maps (obtained without modeling the contraction-outlet facility and the
cylinder) in this section with respect to the source characterization, accuracy of the predicted
location, strength and focal-resolution of the flow-induced dipole source.

A. Monopole steering vector

Figures 10(a), (c) and (e) present the CB source maps obtained using the Monopole
steering vector formulation (henceforth, termed as Monopole CB source maps) for Aeolian tone
frequency $f_a = \{784, 1208, 1584\}$ Hz, respectively. A pair of nearly identical focal spots located
in proximity in each of these source maps indicates the dipole source nature of the flow-induced
noise generated at the Aeolian tone due to a circular cylinder in cross-flow. It is observed that the
dipole focal spots in Figs. 10(a), (c) and (e) are similar to those exhibited by the corresponding
TR source maps shown in Figs. 7(a), (c) and (e), respectively. Indeed, the only noticeable
difference between the Monopole CB and the TR source maps is that side-lobe levels are
significantly lower in the former.

The predicted location of the dipole source, error (expressed as a fraction of the
corresponding Aeolian tone wavelength) and the average SPL observed at the focal points in the
Monopole CB source maps are shown in the 2\textsuperscript{nd} and 3\textsuperscript{rd} columns, respectively, of Table IV. An
average error of $0.15\lambda_a$ is observed in the predicted location of the dipole whilst the average
SPL (at the focal point) increases with an increase in the free-stream velocity $U_\infty$. The 2\textsuperscript{nd} and 3\textsuperscript{rd}
columns of Tables II and IV indicate that the error in the predicted location as well as the
average SPL obtained (at different Aeolian tone frequencies) by the TR and the Monopole CB methods, respectively, are comparable. This similarity is also highlighted in Table V which compares the transverse and longitudinal focal-resolution of the dipole focal spots obtained by the TR and Monopole CB. It is observed from Table V that the average transverse and longitudinal focal-resolution of the focal spots obtained by the TR method is given by $1.03\lambda_a$ and $0.38\lambda_a$, respectively, whilst those obtained by the Monopole CB method are given by $0.93\lambda_a$ and $0.38\lambda_a$, respectively, thereby indicating a similar focal spot size at a given Aeolian tone frequency.

In light of the foregoing quantitative comparisons of the source characteristics, accuracy of predicted location, source strength and focal-resolution, it is therefore, concluded that both TR and Monopole CB method yield similar source maps for a given LA configuration.

**B. Dipole steering vector**

Figures 10(b), (d) and (f) present the CB source maps obtained using the Dipole steering vector formulation (henceforth, termed as Dipole CB source maps) for Aeolian tone frequency $f_a = \{784, 1208, 1584\}$ Hz, respectively. Unlike the TR and Monopole CB source maps, the Dipole CB source maps exhibit a central focal spot (representing the flow-induced dipole source) flanked by side-lobes on either sides. The central focal spot is of an elongated shape oriented along the $x$ direction and its size decreases with an increase in the Aeolian tone frequency. The predicted location of the dipole source obtained from the Dipole CB method (taken as the focal point of the central focal spot) and the error in predicted location is shown in the 4th column of Table IV whilst the average source strength (at the focal point) is tabulated in the 5th column. An average error of $0.13\lambda_a$ is observed in the predicted location which is comparable to that obtained by the Monopole CB and the TR method. Furthermore, the average SPL at source
location at different Aeolian tone frequencies obtained by the Dipole CB are slightly higher than
those predicted by the Monopole CB and about 6 dB higher than those predicted by TR method.

The transverse and longitudinal focal-resolution of the central focal spot exhibited in the
Dipole CB source maps is shown in the 4th and 7th columns of Table V wherein it is observed that
the size of focal spots is nearly commensurate with respect to the Aeolian tone frequencies
\( f_o = \{784, 1208, 1584\} \) Hz. The average transverse and longitudinal focal-resolution of the
central focal spot is given by 0.94\( \lambda_u \) and 0.36\( \lambda_u \), respectively, thereby indicating that the Dipole
CB method yields a focal spot whose size is comparable to the focal spot size obtained by the
Monopole CB and the TR method.

VIII. CONCLUSIONS

This paper has demonstrated, for the first time, an experimental application of aeroacoustic
Time-Reversal (TR) for a benchmark test-case of a full-span circular cylinder located in
subsonic cross-flow. The spatio-temporal evolution of \( \bar{\rho}(x, y, t) \) field and the corresponding TR
source map obtained using the experimental acoustic pressure data recorded at top and bottom
microphone Line Arrays (LAs) (without modeling the contraction-outlet and the cylinder)
indicate the lift-dipole nature of the aeroacoustic source generated at the Aeolian tone frequency. This observation is consistent with the classically known result that the interaction of
a rigid body (typically, smaller than or comparable to a wavelength) with a flowing medium
induces local surface stresses which are of equal magnitude but act in opposite direction to the
surrounding fluid resulting in a dipole source nature. The TR source maps indicate a small
error, approximately 0.15\( \lambda_u \) in the predicted location of the dipole which is attributed to the
partial angular aperture of the two LAs or possibly, the use of the computationally simpler 2-D
Linearized Euler Equation (LEE) solver that under-represents the experimental noise generation
phenomenon, especially at low Aeolian tone frequency. The average transverse and longitudinal
TR focal-resolution of the flow-induced dipole focal spots given by $\approx \lambda_c$ and $\approx 0.4\lambda_c$, respectively, signifies the half-wavelength diffraction limit of the TR method\cite{30,48}.

It is shown that modeling the contraction-outlet and the cylinder during TR simulations marginally improves the transverse focal-resolution of the lift-dipole (given by $0.83\lambda_c$) and also reduces the side-lobe levels in the source map corresponding to the Aeolian tone $f_a = \{784, 1208\}$ Hz occurring in the low-frequency range; however, it yields a larger error in the predicted location. For the Aeolian tone $f_a = 1584$ Hz occurring at a higher frequency (when the wavelength of back-propagated acoustic waves becomes comparable to the total height of the contraction-outlet), incorporation of the contraction-outlet geometry and the cylinder is shown to be counter-productive because it leads to the formation of localized maxima regions near the flanges of the contraction-outlet, thereby making it difficult to characterize the dipole source nature. It is therefore concluded that modeling of the facility geometry and full-span cylinder test-model during TR simulations is not necessary to improve the characterization of dipole source generated at the Aeolian tone in this case. However, the use of 3-D TR simulation is likely to improve the source resolution further.

The experimental TR source maps for the Aeolian tone obtained with/without modeling the contraction-outlet and the cylinder were found to be similar to the TR source map of an idealized dipole source placed at the cylinder location (during forward simulations) in the wind-tunnel flow for the counterpart cases, thereby validating the aeroacoustic TR simulation for localizing/characterizing flow-induced dipole sources.

The source maps obtained by the TR simulation implemented without modeling the contraction-outlet and the cylinder were shown to be comparable to those obtained using
Monopole and Dipole Conventional Beamforming (CB) in terms of accuracy of the predicted location, source strength as well as the transverse and longitudinal focal-resolution of dipole focal spots (see Tables II, IV and V). Furthermore, the shape, size and orientation of the dipole focal spots in the TR source maps were observed to be nearly identical to those of the corresponding Monopole CB source maps for the given LA configuration. These results, therefore, demonstrate the suitability of the TR technique as a diagnostic tool to study/analyze experimental flow-induced noise generation mechanism\textsuperscript{36-39} and as an alternate approach to the more popularly used CB method\textsuperscript{1}.

ACKNOWLEDGMENTS

The authors would like to thank Mr. Ric Porteous for providing experimental data of the mean flow velocity and acknowledge the support of Australian Research Council (ARC) for this work through grant DP 120102134 “Resolving the mechanics of turbulent noise production.”

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TABLE I Comparison of the predicted location of the idealized dipole source of tonal frequencies $f_0 = \{750, 1500, 3000\}$ Hz and error in prediction obtained by the TR method (implemented using simulated data recorded at the top and bottom boundaries during forward simulations) without modeling the contraction-outlet with those obtained with their modeling during TR.

<table>
<thead>
<tr>
<th>Tonal frequency $f_0$</th>
<th>TR without modeling the contraction-outlet</th>
<th>Error</th>
<th>TR with the modeling of contraction-outlet</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted location $(x_0, y_0)$ (in mm)</td>
<td></td>
<td>Predicted location $(x, y)$ (in mm)</td>
<td></td>
</tr>
<tr>
<td>750 Hz</td>
<td>(64, 0)</td>
<td>0.02$\lambda_0$</td>
<td>(124, 0)</td>
<td>0.15$\lambda_0$</td>
</tr>
<tr>
<td>1500 Hz</td>
<td>(44, 0)</td>
<td>0.04$\lambda_0$</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td>3000 Hz</td>
<td>(54, 0)</td>
<td>0</td>
<td>(64, 0)</td>
<td>0.09$\lambda_0$</td>
</tr>
</tbody>
</table>
**TABLE II**

Comparison of the predicted location of the dipole source generated at different Aeolian tone corresponding to \( U_\infty = \{16, 24, 32\} \text{ m}\cdot\text{s}^{-1} \), error in prediction and the average source strength (at the dipole focal points) obtained using the TR method (considering a full-length of the two LAs) without modeling the contraction-outlet and the cylinder with those obtained with their modeling during TR.

<table>
<thead>
<tr>
<th>Aeolian tone frequency ( f_a )</th>
<th>TR without modeling the contraction-outlet and the cylinder</th>
<th>TR with the modeling of contraction-outlet and the cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted location ((x_0, y_0)) (in mm) and Error</td>
<td>Average SPL at focal points (in dB)</td>
<td>Predicted location ((x_0, y_0)) (in mm) and Error</td>
</tr>
<tr>
<td>784 Hz</td>
<td>(134, 10) and 0.19( \lambda_a )</td>
<td>61.2</td>
</tr>
<tr>
<td>1208 Hz</td>
<td>(94, 5) and 0.16( \lambda_a )</td>
<td>80.1</td>
</tr>
<tr>
<td>1584 Hz</td>
<td>(74, 10) and 0.12( \lambda_a )</td>
<td>91.3</td>
</tr>
</tbody>
</table>
TABLE III Comparison of the transverse and longitudinal focal-resolution of the dipole focal spots generated at the Aeolian tone corresponding to $U_\infty = \{16, 24, 32\}$ m·s$^{-1}$ obtained using TR simulations without modeling the contraction-outlet and the cylinder with those obtained their modeling by considering the full length of the two LAs.

<table>
<thead>
<tr>
<th>Aeolian tone frequency $f_a$</th>
<th>Size of the focal spots (relative to $-6$ dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Transverse resolution (Parallel to the LAs)</td>
</tr>
<tr>
<td></td>
<td>Without modeling the contraction-outlet and the cylinder</td>
</tr>
<tr>
<td>784</td>
<td>$1.05\lambda_a$</td>
</tr>
<tr>
<td>1208</td>
<td>$1.04\lambda_a$</td>
</tr>
<tr>
<td>1584</td>
<td>$1.01\lambda_a$</td>
</tr>
</tbody>
</table>
TABLE IV Predicted location of the dipole source generated at different Aeolian tone corresponding to $U_\infty = \{16, 24, 32\}$ m·s$^{-1}$, error in prediction and the average source strength (at the dipole focal points) obtained using Monopole and Dipole CB.

<table>
<thead>
<tr>
<th>Aeolian tone frequency $f_a$</th>
<th>Monopole steering vector CB</th>
<th>Dipole steering vector CB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted location $(x_0, y_0)$ (in mm) and Error</td>
<td>Average SPL at focal points (in dB)</td>
</tr>
<tr>
<td>784 Hz</td>
<td>(109.0, 11.3) and 0.14$\lambda_a$</td>
<td>64.0</td>
</tr>
<tr>
<td>1208 Hz</td>
<td>(104, 3.75) and 0.19$\lambda_a$</td>
<td>83.6</td>
</tr>
<tr>
<td>1584 Hz</td>
<td>(75.3, 7.5) and 0.12$\lambda_a$</td>
<td>96.9</td>
</tr>
</tbody>
</table>
TABLE V Comparison of the transverse and longitudinal focal-resolution of the dipole focal spots generated at the Aeolian tone corresponding to $U_c = \{16, 24, 32\}$ m·s$^{-1}$ obtained using the TR method (without modeling the contraction-outlet and the cylinder) with those obtained using Monopole and Dipole CB.

<table>
<thead>
<tr>
<th>Aeolian tone frequency $f_a$</th>
<th>Size of the focal spots (relative to $-6$ dB)</th>
<th>Transverse resolution (Parallel to the LAs)</th>
<th>Longitudinal resolution (Perpendicular to the LAs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TR Monopole CB Dipole CB</td>
<td>TR Monopole CB Dipole CB</td>
</tr>
<tr>
<td>784</td>
<td></td>
<td>1.05$\lambda_a$ 0.94$\lambda_a$ 0.93$\lambda_a$</td>
<td>0.38$\lambda_a$ 0.36$\lambda_a$ 0.35$\lambda_a$</td>
</tr>
<tr>
<td>1208</td>
<td></td>
<td>1.04$\lambda_a$ 0.89$\lambda_a$ 0.90$\lambda_a$</td>
<td>0.39$\lambda_a$ 0.41$\lambda_a$ 0.37$\lambda_a$</td>
</tr>
<tr>
<td>1584</td>
<td></td>
<td>1.01$\lambda_a$ 0.95$\lambda_a$ 0.98$\lambda_a$</td>
<td>0.37$\lambda_a$ 0.36$\lambda_a$ 0.36$\lambda_a$</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

FIG. 1. (Color Online) (a) Photograph of the experimental set-up: A full-span 4 mm circular cylinder held in a two-sided mounting frame attached to the contraction-outlet in the AWT and two LAs of microphones for recording far-field acoustic pressure. Schematic diagrams (not shown to scale) depicting the full-span cylinder secured in the mounting frame attached to the contraction-outlet depicting the (b) Side-view and (c) Front-view (showing the position of the two LAs of microphones).

FIG. 2. (Color Online) Acoustic spectrum of the cylinder at flow speed (a) \( U_\infty = 32 \, \text{m} \cdot \text{s}^{-1} \), (b) \( U_\infty = 24 \, \text{m} \cdot \text{s}^{-1} \) and (c) \( U_\infty = 16 \, \text{m} \cdot \text{s}^{-1} \) measured at the microphone located in the bottom LA and positioned 24 mm downstream of the contraction-outlet opening (taken as the origin).

FIG. 3. (Color Online) Spatially-developing (evolving) non-uniform shear mean flow profile modeling a subsonic free-jet issuing out of the contraction-outlet in the AWT with the free-stream velocity \( U_\infty = 32 \, \text{m} \cdot \text{s}^{-1} \) at the contraction-outlet opening. The top-hat profile occurring at and near the contraction-outlet opening \((x = 0)\) gradually evolves within the mixing region wherein the potential-core collapses and the jet rapidly spreads into the ambient stationary fluid producing a developing free-shear layer.

FIG. 4. (Color Online) RMS plot depicting the directivity of the radiated acoustic pressure field due to idealized dipole of tonal frequencies: \( f_0 = 750 \, \text{Hz} \) in (a) and (b), \( f_0 = 1500 \, \text{Hz} \) in (c) and (d), \( f_0 = 3000 \, \text{Hz} \) in (e) and (f) simulated through excitation by a point force along the \( y \) direction at \( x_0 = 54 \, \text{mm}, y_0 = 0 \) (almost co-incident with the cylinder location) in the spatially-developing shear mean flow with \( U_\infty = 32 \, \text{m} \cdot \text{s}^{-1} \) (shown in Fig. 3). Parts (a, c and e) are obtained without modeling the contraction-outlet walls and flanges whilst the parts (b, d and f) are obtained with their modeling.

FIG. 5. (Color Online) TR source maps of the idealized dipole source of tonal frequencies: \( f_0 = 750 \, \text{Hz} \) in (a) and (b), \( f_0 = 1500 \, \text{Hz} \) in (c) and (d), \( f_0 = 3000 \, \text{Hz} \) in (e) and (f) obtained using time-reversed acoustic pressure history (recorded during forward simulation) at the top and bottom LAs. Parts (a, c and e) are obtained without modeling the contraction-outlet walls and flanges whilst the parts (b, d and f) are obtained with their modeling. The idealized dipole was simulated through excitation by a point force along the \( y \) direction at \( x_0 = 54 \, \text{mm}, y_0 = 0 \) (almost co-incident with the cylinder location) and implementation of rigid-wall conditions due to contraction-outlet walls and flanges.

FIG. 6. (Color Online) Spatio-temporal evolution of the time-reversed acoustic pressure field \( \tilde{p}(x, y, \tau) \) at the Aeolian tone corresponding to \( U_\infty = 32 \, \text{m} \cdot \text{s}^{-1} \) obtained using the experimental signals recorded at the top and bottom LAs (filtered in the 1/3rd octave band with \( f_c = 1600 \, \text{Hz} \)) without modeling the contraction-outlet walls, flanges and the cylinder at reverse time-instants: (a) \( \tau = 20\Delta t \), (b) \( \tau = 60\Delta t \), (c) \( \tau = 100\Delta t \), (d) \( \tau = 125\Delta t \), (e) \( \tau = 400\Delta t \) and (f) \( \tau = 800\Delta t \).
FIG. 7. (Color Online) Experimental TR source maps of the lift-dipole source generated at the Aeolian tone: $f_a = 784 \text{ Hz}$ corresponding to $U_\infty = 16 \text{ m \cdot s}^{-1}$ in (a) and (b), $f_a = 1208 \text{ Hz}$ corresponding to $U_\infty = 24 \text{ m \cdot s}^{-1}$ in (c) and (d), $f_a = 1584 \text{ Hz}$ corresponding to $U_\infty = 32 \text{ m \cdot s}^{-1}$ in (e) and (f) obtained using the time-reversed signals at the top and bottom LAs. Parts (a, c and e) are obtained without modeling the contraction-outlet walls, flanges and the cylinder whilst the parts (b, d and f) are obtained with their modeling.

FIG. 8. (Color Online) Spatio-temporal evolution of the time-reversed acoustic pressure field $\tilde{p}(x, y, \tilde{t})$ at the Aeolian tone corresponding to $U_\infty = 32 \text{ m \cdot s}^{-1}$ obtained using the experimental signals recorded at the top and bottom LAs (filtered in the 1/3rd octave band with $f_c = 1600 \text{ Hz}$) with the implementation of rigid-wall conditions modeling the contraction-outlet walls, flanges and the cylinder at reverse time-instants: (a) $\tilde{t} = 20\Delta t$, (b) $\tilde{t} = 60\Delta t$, (c) $\tilde{t} = 100\Delta t$, (d) $\tilde{t} = 125\Delta t$, (e) $\tilde{t} = 400\Delta t$ and (f) $\tilde{t} = 800\Delta t$.

FIG. 9. (Color Online) Experimental TR source map of the lift-dipole source generated at the Aeolian tone $f_a = 1584 \text{ Hz}$ obtained using (a) time-reversed signals at the top and bottom LAs by considering a stationary medium (zero mean flow), i.e., ignoring the convective effect of spatially developing shear mean flow during TR simulation and (b) time-reversed signals at only the top LA but considering the spatially developing shear mean flow during TR simulation.

FIG. 10. (Color Online) Experimental source maps of the lift-dipole source generated at the Aeolian tone corresponding to (a) $U_\infty = 16 \text{ m \cdot s}^{-1}$, (c) $U_\infty = 24 \text{ m \cdot s}^{-1}$ and (e) $U_\infty = 32 \text{ m \cdot s}^{-1}$ obtained using monopole steering vector beamforming. Experimental source maps of the lift-dipole source generated at the Aeolian tone corresponding to (b) $U_\infty = 16 \text{ m \cdot s}^{-1}$, (d) $U_\infty = 24 \text{ m \cdot s}^{-1}$ and (f) $U_\infty = 32 \text{ m \cdot s}^{-1}$ obtained using dipole steering vector beamforming.

Submitted Version