Experimental Testing of a Maximum Capture Width Tracking Controller for
Ocean Wave Energy Converters in Irregular Waves

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Abstract

A maximum capture width tracking (MCWT) controller for ocean wave energy converters is investigated via small scale experiments conducted in a wave flume. The MCWT controller is a maximum power point tracking (MPPT) controller modified to account for incoming wave conditions as well as wave energy converter power output. The MCWT controller has been applied to latching control of a small scale oscillating water column, optimising the latching time based on sea state. The performance of the proposed MCWT latching controller is compared to that of an MPPT latching controller in both stationary and transitioning sea states. In stationary seas, it will be shown that both controllers can optimise capture width to within the bounds of certainty that the optimal capture width can be known for a wave energy converter in stochastic waves. In transitioning seas, it will be shown that the MCWT controller is robust to a changing environment, whereas the MPPT controller is not.

Keywords: Oscillating water column, Wave energy converter, Latching control, Maximum power point tracking, Gradient-ascent control, Stochastic control, Irregular waves

1. Introduction

Efficient conversion of ocean wave energy to grid frequency electricity remains an elusive goal despite over 40 years of research and development. The scale of research effort, and the lack of a clear solution to the wave energy conversion problem, are exhibited by the fact that there exist more than 1000 patents for wave energy converters [1] with various operating principles. Feedback controlled oscillating systems, specifically oscillating water columns and heaving buoys, have been a popular solution since the discovery of the point absorber effect [2]. Significant research efforts have since been dedicated to developing efficient feedback controllers which maximise real power extraction in oscillating wave energy converters (WECs), with extensive consideration given to both reactive and passive control methods.

Reactive control methods utilise bi-directional power flow through the power take-off (PTO) to force the WEC to oscillate in phase with the wave exciting force, returning energy from the PTO to the WEC during part of the cycle. Whilst reactive controllers allow a WEC to operate with high efficiency over a range of wave conditions, they are difficult to implement in reality due to the large power loads and forces imposed on the PTO machinery. Reactive controllers require PTO machinery capable of bi-directional power flow and the ability to control force and velocity independently, an example of such a machine being a variable pitch turbine in an oscillating water column that can also act as a compressor [3]. Reactive controllers also suffer from high peak to average power ratios and large PTO forces. Reactive control of a heaving buoy with displacement constraints was studied in [4] where peak power loads were found to be an order of magnitude greater than the mean power and PTO machinery forces exceeded the incident wave force. Similar results

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have also been reported by [5] and [6]. Hence, whilst viable in theory, reactive controllers are difficult and expensive to implement in reality as PTO machinery must be capable of handling large peak power loads, large forces, and bi-directional power flow.

Passive control methods, characterised by unidirectional power flow and low peak to average power ratios, have been of particular interest as they can be implemented with simple, moderately sized PTO machinery, reducing capital expenditure and improving economic viability. A popular passive control method for WECs is latching, which is a non-linear form of phase control. Latching involves locking the WEC in place at the point of zero velocity and releasing it at a more favourable time so that the velocity and exciting force become in phase. Latching was found to be the optimal control method for a heaving WEC when unidirectional power flow or PTO force magnitude constraints were imposed [7]. Optimal latching of a heaving buoy in irregular waves [8], assuming linear dynamics and inviscid flow, increased power absorption by 50-150%, depending on the peak period of the wave spectra. A subsequent study considering viscous losses and friction effects showed that a simple, sub-optimal latching controller could produce 30% more power than the best performing causal reactive controller for the pitching SEAREV device [9]. Hence, latching appears to be an effective, yet simple method of WEC control.

Whilst latching appears promising, studies on optimal latching control typically assume perfect prediction of the future exciting force which is not possible in practice. A recent study on optimal latching of a floating spar-buoy OWC [10] investigated the effect of prediction horizon, being the time period for which the excitation force is known into the future. It was found that latching could significantly increase capture width providing the prediction horizon exceeded the energy period of the sea state. For shorter horizons, however, the optimal latching controller was found to be less effective than a fixed resistive damper. At present future excitation force can only be predicted accurately a few seconds in advance [11], which is inadequate for optimal time-series based control [8, 12]. Hence optimal control on a wave-by-wave basis, whether reactive or passive, is currently considered not possible [13].

Motivated by the difficulty of wave prediction, non-model based adaptive controllers have been developed which optimise WEC PTO parameters based on sea state. Maximum capture width tracking (MCWT), a gradient ascent controller adapted from the maximum power point tracking (MPPT) controllers used in wind and solar energy plants, has been applied to damping control of a submerged WEC in [14] and latching of an OWC in [15]. In [15] it was found that MCWT control was able to optimise WEC capture width to within the bounds of certainty that the optimal capture width could be known in stochastic waves. In non-stationary seas MCWT was found to be robust to a changing environment.

This work validates the results of [15] by experimentally investigating the performance of MCWT latching control of an OWC in irregular waves. For comparison an MPPT latching controller was also tested. Small scale experiments investigate MPPT/MCWT performance in both stationary and transitioning seas, where MCWT superiority will be demonstrated. The small scale experimental rig will be detailed in Section 2, followed by a discussion of experimental methods in Section 3. The MPPT/MCWT algorithms will be introduced in Section 4. Results of Monte-Carlo experiments, performed to investigate the variation in mean OWC power and capture width with time-series representation of the sea state, will be presented in Section 5 and confidence intervals for optimal OWC power and capture width will be defined. Finally, MPPT/MCWT latching control experiments will be detailed in Section 6 considering both stationary and transitioning seas. In stationary seas both the MPPT and MCWT controllers will be shown to optimise OWC capture width to within the confidence bounds that the optimal condition can be known. In transitioning seas it will be shown that MCWT control is robust to a changing environment, whereas MPPT is not.

2. Small Scale OWC and Experiment Setup

The goal of small scale experiments is to replicate the performance of a full scale OWC in a controlled laboratory setting. Small scale experiments were performed in the 32 m wave flume, located in The School of Civil, Environmental and Mining Engineering at The University of Adelaide, South Australia. The flume has width of 1230 mm and maximum depth of 900 mm. A plunger type wave paddle is used for generating small scale water waves.
Table 1: Froudean scale relationships for scaling factor $s$ [16, 18, 19].

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Relationship between scales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length / displacement, m</td>
<td>$s$</td>
</tr>
<tr>
<td>Time, s</td>
<td>$s^{1/2}$</td>
</tr>
<tr>
<td>Mass, kg</td>
<td>$s^3$</td>
</tr>
<tr>
<td>Force, N</td>
<td>$s^3$</td>
</tr>
<tr>
<td>Velocity, m/s$^{-1}$</td>
<td>$s^{1/2}$</td>
</tr>
<tr>
<td>Frequency, Hz</td>
<td>$s^{-1/2}$</td>
</tr>
<tr>
<td>Power, N m/s$^{-1}$</td>
<td>$s^{7/2}$</td>
</tr>
<tr>
<td>Pressure, N m$^{-2}$</td>
<td>$s$</td>
</tr>
</tbody>
</table>

For the dynamics of a full scale OWC to be accurately represented at small scale, the small scale model must maintain geometric, kinematic and dynamic similarity with the full scale system [16, p. 331]. This is achieved by maintaining a fixed ratio between the small and full scale systems for the WEC dimensions, the velocity and acceleration of all body/fluid motions, and the dynamic forces, which is generally not possible. The compromise typically adopted is to maintain the Froude number whilst neglecting changes in the Reynolds number [16, 17, p. 332], a method known as Froudean scaling. The rationale for Froudean scaling is that at small scale the effects of inertia and gravity (associated with the Froude number) dominate those of viscosity and surface tension (associated with the Reynolds number) which are usually negligible for well designed experiments. The Froude number, which describes the ratio of inertia forces to gravitational forces, is given by

$$Fr = \frac{U}{\sqrt{gl}} \propto \frac{\text{Inertia force}}{\text{Gravitational force}} \quad (2.1)$$

where $U$ is fluid velocity, $l$ is the characteristic length of fluid/solid interaction and $g$ is gravitational acceleration. A summary of relationships between scales for Froudean scaling is provided in Table 1, where $s$ is the geometric scale between small and full scale systems.

Experimental scale was chosen based on a Froudean scale relationship between generalised South Australian sea states and the small scale equivalent sea states able to be produced in the wave flume. The desired wave conditions for experiments were small scale replications of the 10th and 50th percentile sea states for South Australian (SA) waters, subsequently denoted $S_{10}$ and $S_{50}$, which are low and medium energy sea states for SA respectively. The operating range of the wave flume is approximately 0.5-2 Hz; frequencies below this range are typically too low in amplitude to be useful, whereas frequencies above this range tend to be quite steep and nonlinear, invalidating the assumptions of linear theory. Further restrictions are placed on scale by considering recommendations for small scale wave experiments with Froudean scaling [16, p. 333], being that the peak period of the irregular wave spectra should be longer than 1.0 s to prevent wave amplitudes being attenuated by surface tension during propagation, and significant wave heights should be less than 100 mm to prevent wave breaking, which is nonlinear and not well represented by Froudean scaling. An experimental scale of 1:80 was chosen as this yielded sea states $S_{10}$ and $S_{50}$ which were producible in the flume, with spectral peak periods greater than 1.0 s and significant wave heights much less than 100 mm.

Characteristics of the 10th and 50th percentile sea states, denoted $S_{10}$ and $S_{50}$ respectively, are listed in Table 2 for both full and 1:80 scale.

The small scale OWC, a schematic of which is shown in Figure 1, was designed specifically for 1:80 scale flume experiments rather than being a scaled down version of a full scale prototype. OWC water chamber geometry was based on the two dimensional (2D) model of [20] for simplicity. The water chamber was constructed from 20 mm transparent acrylic with a width of 1200 mm, being approximately the width of the flume. Two porous baffles were installed inside the OWC chamber to provide damping for cross-modes and sloshing which initially occurred around resonance, known to be problematic in 2D OWC experiments.
Table 2: South Australian sea states [23] and small scale equivalent sea states. Sea state percentile indicated by subscript.

<table>
<thead>
<tr>
<th>Sea state</th>
<th>Full scale</th>
<th>1:80 scale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Significant wave height</td>
<td>Peak period</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>1.47 m</td>
<td>11.6 s</td>
</tr>
<tr>
<td>$S_{50}$</td>
<td>2.45 m</td>
<td>12.7 s</td>
</tr>
</tbody>
</table>

Figure 1: Experiment schematic.

The OWC natural resonance period, which can be varied by adjusting flume depth, is approximately 0.9 s for the configuration considered in Figure 1, which is much shorter than the peak periods of the small scale sea states. The OWC air chamber was designed with a characteristic height of 5.8 m, being the ratio of air chamber volume to water column surface area, to maintain the soft air spring of a full scale OWC and Froudean similarity of the thermodynamics [22]. The small scale OWC installed in the wave flume is displayed in Figure 2a along with a close-up of the small scale PTO in Figure 2b. The small scale PTO consisted of a large orifice with a latching/damping mechanism to control airflow, being a flat plate position controlled by a linear motor. Damping was imposed by positioning the flat plate close to the orifice to throttle the airflow and latching was applied by blocking the air flow through the orifice. A Linmot PS01 – 23 × 160/70 × 210 linear motor was used with position repeatability/accuracy of ±0.05 mm. The linear motor was able to drive the plate from the unlatched position to the latched position and vice versa, a distance of 10 mm, in approximately 10 ms.

Real-time control and data acquisition were performed by a dSpace DS1104 control board. Analogue voltage signals were generated by 2×16 bit DAC channels to control the wave paddle and linear motor position. Data acquisition was performed by 4×16 bit and 2×12 bit ADC channels which sampled analogue voltage signals from five wave probes and one air pressure sensor, the locations of which are indicated in Figure 1. Twin-wire wave probes, used to measure the surface elevation of water waves, the water level at the OWC front wall and the water column level, were constructed in-house and calibrated to an accuracy of ±1 mm. Air chamber pressure was measured by a Honeywell NSC-D-JJN-005N-DUNV differential air.
Table 3: Summary of experiment parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental scale, $s$</td>
<td>1:80</td>
</tr>
<tr>
<td>OWC natural resonance period</td>
<td>0.9 s</td>
</tr>
<tr>
<td>Air pressure sample rate, $F_s = 1/T_s$</td>
<td>1000 Hz</td>
</tr>
<tr>
<td>Wave probe sample rate, $F_w = 1/T_w$</td>
<td>20 Hz</td>
</tr>
<tr>
<td>Number of FFT points, $N_F$</td>
<td>128</td>
</tr>
<tr>
<td>Controller update period</td>
<td>134.4 s</td>
</tr>
<tr>
<td>Latching time step size, $\mu$</td>
<td>0.05 s</td>
</tr>
</tbody>
</table>

pressure sensor with working range ±1250 Pa and calibrated to an accuracy of ±20 Pa. Air chamber pressure was typically within the range ±300 Pa for sea state $S_{10}$ and ±600 Pa for $S_{50}$.

3. Methods

3.1. Sea State Generation

Small scale irregular waves were produced by driving the wave paddle, a position controlled hydraulic ram, with an irregular analogue voltage signal. The irregular voltage signal was created as a time-series realisation of the voltage spectra

$$V_x(\omega) = Y_{ex} Y_{sw}(\omega) A_w(\omega)$$  \hspace{1cm} (3.1)

where $Y_{ex}$ is the transfer function between applied analogue voltage and paddle stroke, $Y_{sw}(\omega)$ is the transfer function between paddle stroke and wave amplitude, and $A_w(\omega)$ is the desired wave amplitude spectra. The transfer function $Y_{ex}$ is approximately 50 mm V$^{-1}$ and independent of frequency provided the paddle velocity is much slower than the dynamics of the hydraulics, which is true for low amplitude linear waves. Wave maker transfer functions are typically derived from linear theory and assume that the volume of water displaced by the wave maker will be equal to the volume of the resulting wave crest. The transfer function for a plunger wave maker is not readily available, however, based on the assumed relation between volume displacement and wave form, $Y_{sw}(\omega)$ will be similar to that of a piston wave maker [24]

$$Y_{sw}(\omega) = \frac{2 \left( \cosh(2kh) - 1 \right)}{\left( \sinh(2kh) + 2kh \right)}.$$  \hspace{1cm} (3.2)
where $\omega$ is the angular wave frequency, with units rad s$^{-1}$, and $kh$ is the dimensionless wave number. The wave amplitude spectra $A_w(\omega)$ is related to the desired spectral power density $S(\omega)$ through

$$A_w(\omega) = \sqrt{2S(\omega) \Delta \omega}$$

(3.3)

where $S(\omega)$ has units m$^2$ Hz$^{-1}$.

South Australian sea states can be characterised as fully developed unimodal seas [23]. Small scale representations of South Australian sea states were generated from ITTC spectra, being fully developed unimodal spectra of Pierson-Moskowitz type [19]. Wave spectra were generated with 154 discrete frequencies $\omega_i$ in the range 0-3 Hz, with resolution $\Delta \omega = 2\pi \times 0.0195$ rad s$^{-1}$. The uniformly distributed frequencies $\omega_i$ were then each offset by a small random value to prevent time-series repetition [25, p. 26], yielding 154 discrete frequencies $\omega_r$ with random spacing: $\omega_r = \omega_i + \theta \frac{2\pi}{\omega_r}$, where $\theta \in [-1,1]$ is a uniformly distributed random variable. The paddle voltage time series is then given by $v(t) = \sum_{r=1}^{154} 2V_r(\omega_r) \Delta \omega \sin(\omega_r - \phi_r)$ where $\phi_r = \theta \pi$ is a random phase, yielding wave elevation time-series $s_w(t) = \sum_{r=1}^{154} 2S(\omega_r) \Delta \omega \sin(\omega_r - \phi_r)$.

The random numbers used for frequency randomising and random phase generation were generated by a Merseenne Twister random number generator initialised with a set seed value. Using a set seed value for the random number generator allowed voltage time-series, and hence wave paddle motion, to be repeatable.

Estimates of incident wave spectra are required to determine incident wave power, and hence OWC efficiency. Incident wave elevation cannot be measured directly in the experiments as the wave field is bi-directional, with incident waves propagating towards the OWC and reflected and radiated waves propagating away from the OWC. Wave decomposition is employed to separate wave measurements into incident and reflected/radiated waves, as in [26], which uses trigonometric functions and dispersion relations from linear wave theory to decompose the bi-directional wave field into separate phasors for the incident and reflected waves. A three probe decomposition method [27] is adopted in this work which uses a least squares analysis to minimise measurement error, which includes error due to noise in the voltage signal, transverse waves/sloshing in the flume, nonlinear waves and phase locked harmonics. Wave elevation samples were obtained by the twin-wire wave probes at a rate $F_w$ of 20 Hz. Finite time estimates of the incident wave spectra were then obtained by taking the fast Fourier transform (FFT) of the wave probe measurements and applying wave decomposition to determine the incident component.

Finite time estimates of the incident wave spectra, $\hat{S}$, are compared with the target discrete ITTC spectra for sea states $S_{10}$ and $S_{90}$ in Figure 3, where it is evident that the estimated incident wave spectra are a poor approximation of ITTC spectra in terms of shape and peak period. Replicating the desired spectra in the wave flume is difficult as locked harmonics in the bi-directional wave field distort the frequency response of the flume, the effect of which is that nearby frequencies have vastly different amplitudes for the same paddle stroke, evident by the spectral amplitude ‘spikes’ in Figure 3. The bi-directional wave field is also amplitude modulated as spectral amplitudes vary in time as individual wave frequencies beat in and out of phase. Unfortunately, these effects are a limitation of the current setup and cannot be avoided. The significant wave height $H_s$ for each spectra was obtained using [16, p. 49]

$$H_s = 4\sqrt{m_0}$$

(3.4)

where $m_0$ is the zeroth spectral moment

$$m_0 = \int_0^\infty S(\omega) d\omega$$

(3.5)

which was evaluated numerically with the upper limit of the integral truncated to 6$\pi$. The estimated incident wave spectra quite well approximate the target ITTC spectra in terms of significant wave height. Considering spectral power, the target ITTC spectra for $S_{10}$ and $S_{90}$ have spectral power levels of 0.21 W and 0.69 W respectively, whereas the finite time estimates have power levels of 0.24 W and 0.92 W respectively. Hence the wave spectra produced in the flume have higher power levels than the target ITTC spectra, with power observed to increase by 12.5% for $S_{10}$ and 25% for $S_{90}$. 

6
Figure 3: Estimated incident wave spectra and target ITTC spectra. Wave decomposition performed with a 1024 point FFT with rectangular window. Irregular wave time-series length 807 s.
3.2. Estimates of wave power, OWC power and capture width

The maximum capture width tracking (MCWT) algorithm investigated in this work attempts to maximise mean capture width for a sea state, CW. Determining mean capture width requires knowledge of both incident wave power and mean OWC power. As real ocean waves are random neither the mean wave power, OWC power or capture width for a sea state can ever be known exactly. Rather, these quantities can only ever be estimated from a finite time-series with some level of certainty. The mean power incident upon the OWC from a unidirectional, irregular wave spectra is

\[ J = W \int_0^\infty \frac{2 \rho g^2 D(kh)}{4\omega} S(\omega) \, d\omega \] (3.6)

where \( W \) is the width of the flume, \( S(\omega) \) the incident wave spectra, \( \rho \) the density of water and \( D(kh) = \left[ 1 + \frac{2kh}{\sinh(2kh)} \right] \tanh(kh) \) the depth function [28] with \( kh \) the dimensionless wavenumber, being the solution to the dispersion relation \( \omega^2 / g = k \tanh(kh) \), which is solved using [29]. The discrete form of Equation (3.6) can be obtained by replacing the continuous integral with the sum of discrete FFT frequencies, and replacing the integration variable \( d\omega \) with the FFT frequency resolution \( \Delta \omega = 2\pi F_w / N_F \), yielding the incident wave power estimate

\[ \hat{J} = \rho g^2 \frac{\pi F_w}{N_F} \sum_{i=1}^{N_F/2} \frac{D(k_i h)}{\omega_i} \hat{S}(\omega_i) \] (3.7)

where \( N_F \) is the number of FFT points, of which only the first \( N_F / 2 \) frequencies, i.e., below the Nyquist frequency, are considered, and \( F_w \) is the sample rate of the wave probes. The number of FFT points, \( N_F = 128 \), was minimised to increase the degrees of freedom of the spectral estimates \( \hat{S}(\omega) \) through increasing the number of averages [16, 30], reducing the variance of the wave power estimates \( \hat{J} \). Rectangular windowing is used for all irregular wave power estimates to reduce distortion of the power spectra. Multiple spectral measurements \( \hat{S}(\omega) \) were combined using Welch’s method of overlapped averages [31]. Specifically, wave power estimates \( \hat{J} \) were each constructed from 21 overlapped averages of \( \hat{S}(\omega) \), generating a new estimate every 134.4 s. This corresponds to approximately 20 min of wave data at full scale which is the standard duration for time-series recordings of sea states to ensure stationarity [16, p. 435], although sea states can generally be considered stationary for up to 3 h [32].

OWC power was determined as the hydrodynamic power in the water column available to be transferred to the PTO. Ideally, OWC power should be evaluated as the aerodynamic power dissipated by the PTO, however, multiple irregular wave experiments conducted using similar wave time-series found aerodynamic power to be an unreliable and unrepeatable quantity, whereas hydrodynamic power was quite repeatable between runs. The discrete instantaneous hydrodynamic power, which describes the work done on the air chamber by the motion of the water column, is given by

\[ P_k = p_k \dot{z}_k A_{owc} \] (3.8)

where \( p_k \) and \( \dot{z}_k \) are the discrete OWC chamber pressure, measured by the pressure sensor, and water column velocity respectively at sample \( k \), and \( A_{owc} \) is the surface area of the water column, which is 0.12 m². Water column velocity was determined as the numerical derivative of the sampled water column elevation \( z_k \)

\[ \hat{z}_k = F_w (z_k - z_{k-1}) \] (3.9)

with \( z_k \) sampled by the OWC chamber wave probe with a zero-order hold at the rate \( F_w \). In determining water column velocity, the sample rate for water column elevation should be maintained as slow as possible to minimise noise from numerical differentiation whilst being fast enough to capture the water column dynamics; 20 Hz was found to be a good compromise, being at least an order of magnitude greater than incident wave frequencies. Air pressure samples \( p_i \) were taken with a zero-order hold at rate \( F_a = 1000 \text{ Hz} \) to capture the bursting airflow that occurred during unlatching. As air pressure was sampled at a much higher
rate than water column elevation, the air pressure samples were buffered and averaged over \( T_w = 1/F_w \) to obtain an averaged pressure \( p_k \)

\[
p_k = \frac{T_s}{T_w} \sum_{i=1}^{50} p_i = \frac{1}{50} \sum_{i=1}^{50} p_i
\]  

(3.10)

from which the discrete instantaneous hydrodynamic power was determined, as in Equation 3.8. Instantaneous hydrodynamic power samples were stored in a zero-overlap buffer of length \( n = 21 \frac{T_s}{T_w} \), corresponding to 134.4 s of OWC power measurements. The mean OWC hydrodynamic power for the sea state is then estimated as

\[
\hat{P} = \frac{1}{n} \sum_{k=1}^{n} P_k.
\]

Finally, the mean capture width for the sea state is estimated as the ratio of the mean OWC hydrodynamic power to the mean incident wave power

\[
\hat{C}W = \frac{\hat{P}}{\hat{J}}.
\]  

(3.11)

4. Maximum Power Point/Capture Width Tracking Algorithms

Maximum capture width tracking (MCWT) is a gradient-ascent type algorithm which maximises capture with respect to a chosen variable, in this case latching time. MCWT works by continually perturbing the latching time and observing the associated change in capture width, the goal being to perturb such that capture width increases. The MCWT latching control algorithm is

\[
T_{L,k+1} = \left| T_{L,k} + \mu \sgn \left( \frac{\hat{C}W_k - \hat{C}W_{k-1}}{T_L,k - T_{L,k-1}} \right) \right|,
\]  

(4.1)

which is a discrete time algorithm where \( T_{L,k} \) is the latching time at sample \( k \), \( \hat{C}W_k \) is the estimate of the mean OWC capture width at sample \( k \), \( \mu \) is the latching time step size with dimensions of seconds, and ‘\( \sgn \)’ is the signum function. The OWC is latched when the water column becomes instantaneously stationary, as in [33], then the MCWT controller determines the time interval that the OWC will remain latched. MCWT was developed as a robust alternative to the standard maximum power point tracking (MPPT) algorithm, which was found to behave erroneously in changing environmental conditions [15, 34, 35, 36]. The MPPT latching control algorithm tested for comparison, is

\[
T_{L,k+1} = \left| T_{L,k} + \mu \sgn \left( \frac{\hat{P}_k - \hat{P}_{k-1}}{T_{L,k} - T_{L,k-1}} \right) \right|,
\]  

(4.2)

where \( \hat{P}_k \) is the estimate of the mean OWC power output at sample \( k \).

Both the discrete MPPT and MCWT algorithms have two independent degrees of freedom; the update rate and the step size [37, 38]. The update rate controls the rate at which the controller responds to changes in the mean power/capture width estimate, and hence changes in incident wave conditions. A fast update rate is desirable for the controller to respond quickly to changes in incident wave conditions, however, the update rate is also inversely proportional to the amount of new information used to influence the next controller action, and hence the variance in the power/capture width estimate. The step size \( \mu \) controls the resolution of the MPPT/MCWT algorithm and determines how aggressively the controller responds to changes in the mean power/capture width output. In this work, the step size \( \mu \) is set to 0.05 s and the update period to 134.4 s.
5. Latching Control Surface and Confidence Intervals for Optimal Capture Width

Gradient ascent controllers are effective when the variable being optimised is convex with respect to the variable being perturbed. For MCWT, with a similar argument applying to MPPT, this means that the capture width vs. latching time curve should be convex with a well defined global maxima at $LT_{opt}$, as then MCWT will continually perturb $LT$ such that $LT \to LT_{opt}$. The latching time vs. mean power/capture width characteristics of the small scale OWC were determined experimentally for $S_{10}$ and $S_{50}$ by performing multiple experimental runs for each sea state, changing only the fixed latching time between runs. A single wave time-series realisation of each sea state was used to minimise the variance between power/capture width results. The duration of the wave time-series for each sea state was $807\text{s}$, the equivalent of $2\text{h}$ at full scale. Data logging was commencing after an initial transient start-up period of $64\text{s}$.

The latching time vs. mean power/capture width characteristics for $S_{10}$ and $S_{50}$ are displayed in Figure 4 along with the mean incident wave power recorded for each latching time, with averages being taken over the entire time-series. Incident wave power fluctuates between latching times as the power in the standing wave field is dependent on the waves reflected and radiated by the OWC, with waves produced by the nonlinear water column motion distorting the wave field. The latching time vs. power characteristics for both $S_{10}$ and $S_{50}$ appear convex about an optimal latching time of $0.275\text{s}$, however, these characteristics become non-convex as latching time is increased. The non-convex behaviour is likely due to distortions in the wave field due to waves radiated by the the nonlinear water column motion, as these waves will reflect off the wave paddle and travel back to the OWC as incident waves. Unfortunately, this cannot be avoided in the present experiments as the wave paddle does not have a wave cancelling ability. A local power maxima is also found to occur for $S_{50}$ at latching time $0.6\text{s}$ coinciding with an increase in the incident wave power.

Converse to OWC power, the latching time vs. capture width characteristics for $S_{10}$ and $S_{50}$ appear convex with peak capture width occurring at a latching time of approximately $0.275\text{s}$ in both cases. Both sea states have the same optimal latching time due to the non-linear PTO. If the PTO were linear, the optimal latching time for each sea state would be solely dependent on the phase difference between the water column velocity and chamber pressure. The optimal fixed latching time for each sea state can be estimated for the linear system as

$$LT_{opt} = \frac{T_e - T_{res}}{2}$$  \hspace{1cm} (5.1)

where $T_e$ is the energy period of the sea state with $T_e = 1.166T_p$ for ITTC spectra [39], and $T_{res}$ is the resonant period of the OWC. Optimal fixed latching times for the OWC with linear PTO are $0.202\text{s}$ for $S_{10}$ and $0.272\text{s}$ for $S_{50}$. For the OWC with non-linear PTO, however, an increase in chamber pressure due to latching causes a corresponding increase in PTO damping. This is beneficial for the small scale OWC, which is a lightly damped system with chamber pressures typically within the range $\pm25\text{Pa}$ for $S_{10}$ and $\pm50\text{Pa}$ for $S_{50}$ in the absence of latching, as increasing PTO damping increases the work done on the air volume by the water column, i.e., the hydrodynamic power. Consequently, for the low amplitude sea state $S_{10}$, the optimal latching time for the non-linear OWC is longer than that of the linear system as the PTO is too underdamped, and increasing the latching time serves to also increase the damping. For moderate amplitude sea state $S_{50}$ chamber pressure, and hence PTO damping, is increased due to increased wave amplitudes, and the optimal latching time for the linear OWC is approximately equal to the $0.275\text{s}$ value obtained experimentally for the non-linear OWC. Similar behaviour was observed in [15] for a simulated OWC with Wells turbine; the variation in optimal latching times between the $10^{th}$ and $90^{th}$ percentile sea states for SA waters was only about $0.75\text{s}$ whereas the difference in peak periods was $2.3\text{s}$. Hence, for the non-linear OWC, the optimal latching time is affected not only by wave period but also wave amplitude, with the optimal latching time decreasing for increasing wave amplitudes.

In [15], considering a latching controlled OWC in simulated sea states, capture width was shown to be a Gaussian random variable and confidence intervals for optimal capture width were defined. Confidence intervals for the (infinite time) mean capture width $CW$ were determined from measurements of the wave time-series. These confidence intervals
indicated the degree to which optimal capture width could be known for the WEC in stationary seas based
on the amount of time-series information available. This procedure will be repeated here for experimentally
determined quantities where it will be shown that, in addition to capture width, confidence intervals can
also be defined for both mean incident wave power and OWC power.

A set of 75 estimates were obtained for each of the mean incident wave power $\hat{J}$, OWC power $\hat{P}$, and
capture width $\hat{CW}$ for each sea state, displayed in Figure 4, for the optimal fixed latching time of 0.275 s.
Estimates were taken over 134.4 s each, being the small scale equivalent of 20 min of sea state data, and
were tested for Gaussian behaviour using Chi-square goodness of fit tests. The hypothesis that each set of
estimates were Gaussian failed to be rejected in all cases, i.e. for both sea states, at the significance level $\alpha$
of 5%. Consequently, confidence intervals can be defined for the mean incident wave power, OWC power and
capture width at the optimal fixed latching time of 0.275 s. For brevity this procedure will be demonstrated
only for capture width, however, the same method has been used to determine confidence intervals for mean
incident wave power and mean OWC power.

As the set of estimates $\hat{CW}$ can be regarded as Gaussian, $\hat{CW}$ can be expressed as

$$\hat{CW} = CW + z\sigma_{CW}$$

where $CW$ and $\sigma_{CW}^2$ are the true mean and variance respectively of $CW$ for a stationary sea state. It is
impossible to ever know $CW$ or $\sigma_{CW}^2$ as determining the true mean and variance of the set $CW$ would require knowing every possible value that $CW$ could take, which would require an infinite number of estimates, and thus an infinite time-series. Instead, $CW$ and $\sigma_{CW}^2$ must be estimated from the finite sample set. The mean capture width $CW$ was taken as the mean of the 75 capture width estimates $\hat{CW}$. Whilst not technically the true mean of $\hat{CW}$, the standard deviation, or random error, in $CW$ obtained by this method is approximately 1%, which is acceptable. Similarly, the true variance $\sigma_{CW}^2$ of $CW$ can never be determined. Instead, the variance of the finite sample set, $\sigma_s^2$, is determined, from which confidence bounds for $\sigma_{CW}^2$ can be found. As $CW$ is Gaussian, the sample variance $\sigma_s^2$ determined from $N$ independent estimates is a Chi-Square variable $\chi^2$.

\[
\text{Prob}\left( \frac{n\sigma_s^2}{\chi^2_{n,\alpha/2}} \leq \sigma_{CW}^2 < \frac{n\sigma_s^2}{\chi^2_{n,1-\alpha/2}} \right) = 1 - \alpha
\]  

(5.3)

where $\chi^2_{n,\alpha/2}$ is the Chi-square variable with $n = N - 1$ degrees of freedom and significance level $\alpha$, and $\text{Prob}$ indicates probability. Considering significance level $\alpha = 0.05$ and $N = 75$,

\[0.7424\sigma_s^2 \leq \sigma_{CW}^2 < 1.4203\sigma_s^2.
\]  

(5.4)

In this work, the maximum variance $\sigma_{CW}^2 = 1.4203\sigma_s^2$ has been used to produce conservative estimates for the confidence intervals for $CW$.

Confidence intervals of significance level $\alpha = 0.05$ can now be determined for $CW$ at the optimal fixed latching time of 0.275 s:

\[
\text{Prob}\left( \frac{z_{\alpha/2}}{\sqrt{\sigma_{CW}} \leq \frac{CW - CW}{\sigma_{CW}} \leq z_{1-\alpha/2}} \right) = 1 - \alpha
\]

(5.5)

where the standard definition $z_{\alpha} \equiv \text{Prob}(z < z_{\alpha}) = 1 - \alpha$ has been adopted. Confidence intervals for mean incident wave power, mean OWC power and mean capture width for the optimal fixed latching time of 0.275 s, i.e., optimal power and capture width, are displayed in Figure 4 for both sea states. It is apparent that, due to the slow decay in capture width with latching time, there exist a large range of latching times which yield capture width within the confidence bounds for optimal. In a real-life OWC, considering power output by the turbine as opposed to hydrodynamic power in the water column, this latching time range is expected to be much narrower, as in [15].

6. MPPT/MCWT Latching Control Experiments

The MPPT/MCWT latching controllers were tested in stationary and transitioning seas considering both slow and rapid sea state transitions. Sea state transitions are characterised in this work as either slow or rapid depending on the temporal change in wave power. The wave power estimate can be considered as a signal corrupted by a disturbance [15]

\[
\hat{J} = J + \varepsilon
\]  

(6.1)

where the signal, $J$, is the mean sea state power and the disturbance, $\varepsilon$, is a stochastic variable dependent on the number of degrees of freedom in the estimate. The change in estimated wave power between samples can be expressed similarly
\[ \Delta \dot{J} = \Delta J + \Delta \varepsilon \]  

where the signal in this case is \( \Delta J \), being the change in mean sea state power, with \( \Delta \) the difference operator;

\( \Delta x = x_k - x_{k-1} \). Slow transitioning seas are characterised in this work as \( |\Delta \varepsilon| > |\Delta J| \), where the power fluctuations between consecutive samples exceed the change in mean sea state power. In slow transitions mean wave power will increase/decrease gradually over time, however, consecutive estimates will show fluctuating power levels, not exhibiting a consistent increase/decrease in power over time. Rapidly transitioning seas are then characterised as \( |\Delta J| > |\Delta \varepsilon| \), where the change in mean sea state power between samples exceeds the power fluctuation. In rapid transitions wave power samples will show a consistent increase/decrease in power over time.

6.1. MPPT/MCWT latching control in slow transitioning seas

Experimental runs with slow transitioning seas considered a progression of three sea states. Runs began with stationary sea state \( S_{10} \) before transitioning from \( S_{10} \) to \( S_{50} \) (denoted \( S_{10/50} \)) and concluding with stationary \( S_{50} \). The irregular wave time-series was produced as the weighted sum of waves generated from \( S_{10} \) and \( S_{50} \), with the instantaneous sea state given by \( S(t) = (1 - \alpha(t))S_{10} + \alpha(t)S_{50} \), where \( \alpha(t) \in [0, 1] \) was ramped linearly from 0 to 1 to produce the transition, as in [15, 40]. Sea state durations/transitions were 1244 s each, corresponding to 3 h at full scale.

Experimental results for the MPPT/MCWT controllers in the three sea state progression are shown in Figure 5 where, for stationary seas, the optimal fixed latching time has been indicated as well as confidence intervals for the incident wave power, optimal OWC power and optimal capture width. During the transition \( S_{10/50} \) mean wave power is shown to gradually increase over time but fluctuates between consecutive samples. This indicates that \( |\Delta \varepsilon| > |\Delta J| \), meaning the sea state transition can be characterised as slow. For the initial sea state \( S_{10} \) both the MPPT and MCWT controllers rapidly optimise the latching time, producing capture width within the confidence bounds for optimal for the duration of the stationary sea state. The sea state transition \( S_{10/50} \) does not significantly affect either controller, with latching times remaining relatively constant for the duration of the transition. A close observation of MPPT results, however, reveals that OWC power, and hence also latching time, follows the change in wave power during the transition almost exactly, the exception being the first step in the transition where wave power decreases by an almost negligible amount. MCWT behaves similarly with regard to OWC power, i.e. following the wave power, however, the latching time does not exactly follow the change in wave power, deviating from that of MPPT towards the end of the transition. This indicates that for MCWT the latching time is not coupled to the wave power, as it is for MPPT. In non-stationary transitioning seas it is difficult to provide a benchmark for controller performance as the optimal latching time, OWC power and capture width are unknown. It is evident though that both controllers maintain relatively constant capture width during the transition, with neither exhibiting any capture width decrease due to changing sea state. Whilst this is not a definitive proof of controller efficacy, the fact that controller performance does not degrade in the non-stationary environment is promising. After the transition, when the sea state becomes stationary \( S_{50} \), both controllers track latching time close to optimal, yielding capture width within the confidence bounds for optimal for the duration of the sea state.

6.2. MPPT/MCWT latching control in fast transitioning seas

Experimental runs with rapid sea state transitions were performed for both increasing and decreasing seas. A series of two sea states were considered in each case; runs began with a sea state transition, being either \( S_{10/50} \) or \( S_{50/10} \) depending on whether seas were increasing or decreasing respectively, then remained stationary at the target sea state for the remainder of the run. Similar to the slow transitions, irregular wave time-series were produced as the weighted sum of waves generated from \( S_{10} \) and \( S_{50} \) with the instantaneous sea state given by \( S(t) = \sqrt{1 - \alpha(t)}S_{10} + \sqrt{\alpha(t)}S_{50} \), where \( \alpha(t) \in [0, 1] \) was ramped linearly from 0 to 1 or vice versa for increasing or decreasing seas respectively. A square root transition characteristic was employed...
Figure 5: Experimental results for MPPT/MCWT latching controllers in slowly increasing seas. Optimal fixed latching times for sea states indicated by dashed horizontal line. Confidence bounds/intervals for mean values at the optimal fixed latching time of 0.275 s indicated by solid horizontal lines/shaded areas.
in an attempt to produce a steady change in wave power over the transition, as $\Delta J \propto \Delta A^2$, where $A^2$ is the square of incident wave amplitude. Sea state durations/transitions were reduced from 1244\,s to 806.4\,s, being the small scale equivalent of 2\,h, to increase the change in wave power between consecutive samples.

Experimental results for MPPT/MCWT control in rapidly increasing seas are shown in Figure 6, where confidence intervals for incident wave power, optimal OWC power and optimal capture width have been indicated for stationary seas as well as the optimal fixed latching time. Wave power results for transition $S_{10/50}$ show a continual increase in power, indicating that $|\Delta J| > |\Delta \epsilon|$. In increasing seas it is evident that MCWT adjusts the latching time to increase capture width. MPPT, in contrast, adjusts the latching time unidirectionally as OWC power continues to increase with incident wave power and independent of controller action, causing the sign in Equation (4.2) to remain positive despite decreasing capture width. When the sea state becomes stationary at $S_{50}$ MCWT tracks latching time close to optimal, maintaining capture width within the confidence bounds for optimal. MPPT, however, fails to track optimal latching time as the controller becomes “stuck”, oscillating the latching time about 0.7\,s which yields capture width below the confidence bounds for optimal. This occurs due to a combination of large wave power oscillations, resulting in large OWC power oscillations, and low sensitivity of OWC power to changes in latching time, evident by the shallow slope of the OWC power vs. latching time characteristic in Figure 4. This causes the controller to become “stuck” as the change in OWC power due to a change in latching time, $\Delta P(\Delta LT)$, is exceeded by the change in OWC power due to the oscillating wave power, $\Delta P(\Delta J)$, causing the latching time to fluctuate with the incident wave power. This behaviour has been observed for MPPT control simulations in [15].

Experimental results for MPPT/MCWT control in rapidly decreasing seas are shown in Figure 7. Wave power results for transition $S_{50/10}$ show a continual decrease in power for most of the transition, indicating that $|\Delta J| > |\Delta \epsilon|$. In decreasing seas MCWT adjusts the latching time to increase capture width. MPPT, in contrast, exhibits dither, where the latching time oscillates back and forth. Dither occurs as OWC power continues to decrease independent of MPPT action, causing the sign in Equation (4.2) to remain negative, causing a significant reduction in capture width during the transition compared to MCWT. Once the sea state becomes stationary at $S_{10}$ MCWT tracks optimal latching time, yielding capture width within the confidence bounds for optimal. Similarly, MPPT rapidly adjusts the latching time to track optimal, producing capture width equivalent to that of MCWT.

7. Conclusion

This work continued on from [15] by experimentally testing the performance of a maximum capture width tracking (MCWT) latching controller for an OWC in irregular waves. For comparison, a maximum power point tracking (MPPT) latching controller was also tested. A small scale oscillating water column (OWC) with latching capability was used as the test apparatus and experiments were conducted in a wave flume. Experiments were performed in stationary seas to determine the characteristics of mean OWC power and capture width vs. latching time, which were revealed to be convex functions about an optimal fixed latching time, and thus able to be optimised by gradient ascent type controllers. Monte-Carlo experiments in stationary seas were performed to investigate the distribution of incident wave power, mean OWC power and mean capture width estimates; all of which were found to be Gaussian random variables and confidence intervals for optimal values were defined.

The MCWT and MPPT latching controllers were tested in stationary and non-stationary seas. In stationary seas both the MPPT and MCWT controllers optimised capture width to within the confidence bounds that optimal capture width could be known. In slow transitioning seas neither controller was significantly affected by the changing environment, however, it was evident that MPPT adjusted the latching time largely in response to fluctuations of incoming wave power, whereas MCWT did not. In rapidly transitioning seas MCWT was shown to be robust to the changing environment, consistently adjusting the latching time to increase capture width. The MPPT controller was not robust to the changing environment, exhibiting either
Figure 6: Experimental results for MPPT/MCWT latching controllers in rapidly increasing seas. Optimal fixed latching times for sea states indicated by dashed horizontal line. Confidence bounds/intervals for mean values at the optimal fixed latching time of 0.275 s indicated by solid horizontal lines/shaded areas.
Figure 7: Experimental results for MPPT/MCWT latching controllers in rapidly decreasing seas. Optimal fixed latching times for sea states indicated by dashed horizontal line. Confidence bounds/intervals for mean values at the optimal fixed latching time of 0.275s indicated by solid horizontal lines/shaded areas.
uni-directional latching time changes in increasing seas or dither in decreasing seas, both of which caused significant reductions in capture width. These results are in agreement with MPPT/MCWT controller behaviours predicted in [15] for a full scale OWC with Wells turbine. Finally, it is worth noting that the efficacy and robustness of MCWT was verified in an a reactive wave environment with amplitude modulated waves, making control optimisation more difficult, and poor estimates of the incident wave power. Thus, the authors are confident that MCWT can be an effective control method for real wave energy converters.

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