Analytical validation of an explicit finite element model of a rolling element bearing with a localised line spall.

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Abstract

In this paper, numerically modelled vibration response of a rolling element bearing with a localised outer raceway line spall is presented. The results were obtained from a finite element (FE) model of the defective bearing solved using an explicit dynamics FE software package, LS-DYNA. Time domain vibration signals of the bearing obtained directly from the FE modelling were processed further to estimate time–frequency and frequency domain results, such as spectrogram and power spectrum, using standard signal processing techniques pertinent to the vibration-based monitoring of rolling element bearings. A logical approach to analyses of the numerically modelled results was developed with an aim to presenting the analytical validation of the modelled results. While the time and frequency domain analyses of the results show that the FE model generates accurate bearing kinematics and defect frequencies, the time–frequency analysis highlights the simulation of distinct low- and high-frequency characteristic vibration signals associated with the unloading and reloading of the rolling elements as they move in and out of the defect, respectively. Favourable agreement of the numerical and analytical results demonstrates the validation of the results from the explicit FE modelling of the bearing.

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1. Introduction

Studying the vibration response characteristics of rolling element bearings, not only defective \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28\] but also non-defective (ideal) \[29, 30, 31, 32\], has been a subject of significant interest to many researchers due to their wide usage in rotating machinery across various industries. While non-defective bearings can produce cyclic vibrations \[29, 30, 31, 32\], also referred to as variable compliance vibrations \[33\], which are caused by the cyclic variation of the stiffness of a bearing assembly as a varying number of rolling elements support the applied load, the presence of various defects within rolling element bearings produces undesirable vibrations that can eventually lead to machinery breakdown if not diagnosed in time. Since the development of the first analytical model in 1984 \[1\], often referred to as a classical model in the literature \[34\], several authors have developed analytical \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\] and FE models \[21, 22, 23, 24, 25, 26, 27, 28\] to simulate the vibration of bearings with localised defects on the raceways (both inner and outer) and rolling elements. The analytical models can be categorised \[34\] into periodic impulse-train \[1, 2, 3, 4\], quasi periodic impulse-train \[5, 6, 7, 8\], and nonlinear multi-body dynamic models \[9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\], whereas FE models \[21, 22, 23, 24, 25, 26, 27, 28\] are dynamic and have been solved using commercial software packages \[35\].

The periodic impulse-train models \[1, 2, 3, 4\] simulate force impulses that are considered to be periodically generated by a localised point spall. For stationary outer race defects \[3, 4\], these models produce impulses of equal amplitude with similar characteristics (shape, height, and width), but for rotating inner race and rolling element defects \[1, 2, 3, 4\], the impulses were modulated according to the static load distribution in bearings \[36, 28\]. Random fluctuations in the period...
of impulses due to defects \[5\] that are caused due to the slip of rolling elements were added to the periodic impulses-train models to develop quasi-periodic (or aperiodic) impulse-train models \[5, 6, 7, 8\].

Using valuable insights from the impulse-train models \[1, 2, 3, 4, 5, 6, 7, 8\], nonlinear, multi-body dynamic models \[9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\] were later developed to relatively better simulate the vibration response of bearings with localised defects. These models incorporated parts of a bearing and associated housing that is contrary to the impulse-train models. For simplification, the multi-body models \[9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\] considered various assumptions, such as: 1) inner and outer rings could not bend or flexurally deform due to their rigid coupling with shaft \[9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20\] and bearing housing \[9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20\], respectively, except the model in Ref \[18\], which considered the outer ring as deformable, 2) rolling elements within a bearing were either excluded or considered to have no mass \[9, 10, 11, 13, 14, 15, 16\] except in Refs \[12, 17, 18, 20\], 3) slip of the rolling elements was ignored \[9, 10, 11, 12, 14, 15, 16, 17\] leading to simulating periodic impulses, whereas some \[13, 19, 18, 20\] consider the slippage, 4) except the models in Refs \[17, 20\], centrifugal forces acting on the rolling elements as they rotate during the operation of a bearing were ignored \[9, 10, 11, 12, 13, 14, 15, 16, 17\]. The main emphasis of the multi-body models was to generate vibration time-traces, and subsequently perform an envelope analysis \[37\] to primarily predict the defect-related frequency components for model validation purposes. Although validated, the aforementioned assumptions led to the accuracy of the multi-body models being compromised in the sense that they could only partially predict the vibration characteristics of defective bearings. For instance, the models accurately estimated the values of the bearing defect frequencies; but not the amplitudes of the defect-related impulses \[4, 11, 12, 15, 16\]. While the difference between the predicted and measured vibration amplitudes has been reported as high as 60,000 percent \[16\], for some models \[4, 11, 12\], the predicted instantaneous amplitudes were corrected to match experimental measurements with no explanation or justification pro-
vided. In summary, from the review [34] of the multi-body models, no single publication addresses all the aforementioned limitations. In the work presented in this paper, no assumptions such as those mentioned above were considered during the FE modelling of a rolling element bearing. Furthermore, a majority of the multi-body models could not predict the low-frequency vibration characteristic signatures pertinent to the unloading of the rolling elements on their entrance in a defect [25, 28]; this unloading event can also be referred to as de-stressing. Based on previous experimental [38, 25, 28] and numerical [25, 28] findings, some authors [19, 20] have adapted their models to generate the low-frequency event. However, in the FE model presented here, the low-frequency unloading event was generated without the application of any specific conditions during the simulation, and the presence of low-frequency characteristic signatures pertinent to this event is demonstrated through the time–frequency analysis, which is lacking in the existing multi-body modelling.

A few researchers [21, 22, 23, 24] have also developed FE models of ball bearings with localised raceway defects. In FE modelling and methods using commercially available software packages [35], a number of assumptions can be minimised leading to better accuracy in results compared to analytical models. However, certain values and choices for parameters in FE methods still need judicious assumptions and problem-related inputs like material model and properties, damping, boundary conditions and loads in addition to adequate meshing of a model. A recent review [34] of these models has highlighted that the accuracy of the FE models [21, 22, 23, 24] was compromised because either the outer ring or its outer surface was modelled as rigid similar to the aforementioned multi-body models [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. This artificially over-stiffened the bearing structure that led to unrealistic (high) instantaneous acceleration levels of the order of 4,000 g [23] and 15,000 g [24] compared to the corresponding experimental measurements of 100 g and 10 g in references [23] and [24], respectively. The significant mismatch between the predicted and measured amplitude levels reported for the multi-body models [2, 3, 11, 12, 14, 15, 16] remains a problem with the existing FE models. Furthermore, several errors
and ambiguities associated with the FE models \cite{21, 22, 23, 24} and results have been discussed in reference \cite{34}. An extensive review of existing impulse train \cite{1, 2, 3, 4, 5, 6, 7, 8}, multi-body \cite{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}, and FE \cite{21, 22, 23, 24} models that simulate the vibration response of bearings with local defects has recently been published by the authors of this paper \cite{34}.

In this paper, results from an explicit FE modelling of a rolling element bearing that has a localised outer raceway line spall are presented. The development of the FE model was presented by the authors of this paper in an earlier publication \cite{25}. Therefore, the model is not fully described here; however, its brief description is provided for completeness and convenience. Unlike previous models in the literature, the bearing components in the FE model in question \cite{25}, from which the results are presented here, were modelled as flexible parts, which provides relatively better representation of the stiffness of the bearing, hence its structural vibration response. Had the parts of the bearing been modelled as rigid instead of flexible, it would have caused artificial over-stiffening of the bearing structure that would have consequently compromised the accuracy of the modelling results. Contrary to previous models \cite{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 21, 22, 23, 24}, the explicit FE model in question \cite{25} accurately predicts the low-frequency characteristic vibration signatures pertinent to the unloading of the rolling elements when they move into the defect, in addition to the predominantly high-frequency reloading of the rolling elements when they move out of the defect \cite{25, 28}; this reloading event can also be referred to as re-stressing. The vibration time-traces obtained directly from the FE modelling of the defective bearing were processed further to estimate time–frequency and frequency domain results, such as spectrogram and power spectrum. Through the time domain analysis, the accurate bearing kinematics, defect frequencies, predicted by the FE model, are discussed. Time–frequency analysis of the results is presented to demonstrate the generation of the distinct low-frequency unloading and predominantly high-frequency reloading events. For the frequency domain analysis, the most commonly used technique, envelope analysis \cite{37}, was used to show that demodulated power
spectrum shows fundamental defect frequency and harmonics. The presentation of the results from various analysis along with relevant discussion leads to the analytical validation of the explicit FE model of the bearing.

A description of the FE model of a rolling element bearing with a local outer raceway line spall is briefly mentioned in Section 2. Analyses of the numerical FE modelled vibration results are presented in Section 3 along with their analytical validation. Time domain analysis of the modelled vibration signals is presented in Sections 3.1 and 3.2. In Section 3.3, time–frequency analysis of the modelled vibration signals is presented that highlights the frequency characteristics of the distinct unloading and reloading vibration signatures. Frequency domain analysis that shows the power spectrum of the vibration signals is presented in Section 3.4. Finally, conclusions of the work are described in Section 4.

2. Explicit Dynamics Finite Element Model of a Defective Rolling Element Bearing

A description of the FE model of a bearing with a local outer raceway line spall has been provided by the authors of this paper in an earlier publication [25]. Therefore, as mentioned earlier, the FE model is not fully described here; however, key features of the model along with the boundary conditions are briefly mentioned for completeness and convenience. The description of the model in reference [25] includes all the necessary steps to build the FE model of the bearing that include material selection and properties, meshing, setting suitable contact parameters between the rolling elements and bearing raceways (both outer and inner raceways), application of boundary conditions and loads to the model so as to effectively simulate its operation in railway applications, and several analysis and control settings. A commercial explicit dynamics FE software package, LS-DYNA [35], was used to model and solve the kinematics and dynamics of the rolling element bearing.
2.1. Brief description of the FE model of the bearing

The two-dimensional FE model of a bearing comprises inner and outer rings, a cage, twenty-four rolling elements and an adapter. Material steel having modulus of elasticity $E = 200$ GPa, density $\rho = 7850$ kg/m$^3$ and Poisson's ratio $\nu = 0.3$ was used to model the bearing. A localised rectangular defect was modelled, which was a line spall, of circumferential length $L_d = 10$ mm and height (or depth) $H_d = 0.2$ mm located at the top of the outer raceway. The location of the defect was chosen such that it is under maximum load during the simulation so as to achieve a full contact between the rolling elements and raceways as the rolling elements traverse through the defect. After the adequate discretisation of the model into finite elements using uniform mesh element size of 0.5 mm, the segment-based surface-to-surface contact [35] with low coefficient of friction $\mu = 0.005$ between the bearing raceways and rolling elements, and the cage and rolling elements was implemented. The chosen frictional coefficient is comparable to that generally recommended for rolling element bearings in practice [36, Chapter 12]. A global damping of 2 percent was applied to the FE model. Higher values of damping, 3–5 percent, were also tested, but these higher values caused the rotational speed of the rolling elements to slow down relative to the modelled rotational speed of the inner raceway resulting in incorrect bearing kinematics. Therefore, the results corresponding to high values of damping were not considered. Figure 1 shows the meshed FE model of the rolling element bearing. The rectangular defect at the top of the outer raceway that is not clearly visible in Figure 1a is shown in Figure 1b.

For the complete description of the FE model of the bearing along with all the boundary conditions used, please refer to the earlier publication in reference [25] by the authors of this paper.
Figure 1: Images of the FE model of the defective rolling element bearing.

(a) The meshed FE model of the bearing and adapter.

(b) A partially zoomed image of Figure 1a showing the rectangular defect on the outer raceway.
3. Analyses and analytical validation of the FE modelled vibration response of the defective rolling element bearing

This section presents analyses of the vibration response obtained from the FE modelling of the defective bearing [25]. The analyses were conducted using standard signal processing techniques commonly used for the vibration-based monitoring of bearings. The analyses are divided into time domain, time–frequency domain, and frequency domain. Each analysis serves a specific purpose as follows:

- time domain analysis of the FE modelled vibration results highlights the presence of the defect-related impulses separated by a period that correlates to the bearing outer race defect frequency $f_{bpo}$ — this indicates correct bearing kinematics,

- time–frequency analysis of the modelled vibration results demonstrates distinct frequency characteristics of the unloading and reloading events when the rolling elements move in and out of the defect, respectively — low-frequency entry and predominantly high-frequency exit impulses, although low-frequency pulses are also generated at the exit of the rolling elements as their rattling between the raceways [25, 26, 27, 28] excites a broad range of the resonance frequencies of the bearing — this indicates correct energy concentration or power of the vibration signals in distinct frequency bands, and

- frequency domain analysis of the modelled results highlights the fundamental defect frequency $f_{bpo}$ and harmonics through the envelope (or demodulated) power spectrum results — this indicates the presence of the defect, and hence correct bearing kinematics.

The FE modelling results presented here correspond to a radial load $W$ of 50 kN applied to the bearing and a rotational speed $n_s$ of 500 RPM at which the bearing was run. While the applied load corresponds to half the load capacity of the bearing, the run speed equates to a train speed of approximately 95 km/hr for
a 1 m wheel diameter. The results of various analysis of the modelled vibration response are structured in a way that clearly demonstrates that the FE model of the bearing \[24\] predicts the complex dynamics of the system with reasonable accuracy.

3.1. Numerical acceleration time-trace

Acceleration of rolling element bearings is commonly measured during their vibration-based condition monitoring followed by the implementation of the envelope analysis technique \[37\] to detect the defect-related bearing frequencies. Similarly, the acceleration results from the FE modelling of the bearing are presented in this section. In addition to the acceleration, velocity and displacement results were also obtained and are presented in Section 3.2 to help identify important vibration characteristics that were not apparently visible in the raw or unfiltered acceleration time-trace.

Figure 2 shows the unfiltered time-trace of the numerically obtained acceleration \(a_y\) (in the global Cartesian \(y\)-direction) at a node on the outer ring. As indicated in the plot, the three impulses are separated by 0.011 seconds. This period correlates to 90.91 Hz that is the bearing outer race defect frequency, (ball pass frequency outer race (BPFO)), \(f_{\text{bpo}}\). Analytically, it can be estimated as \[36, \text{Chapter 25, page 994}\] \[
 f_{\text{bpo}} = \frac{f_s \times N_r}{2} \left( 1 - \frac{D_r}{D_p} \cos \alpha \right) \tag{1}
\] where, \(f_s\) is the bearing run speed, \(N_r\) the number of rolling elements, \(D_r\) the rolling element diameter, \(D_p\) the bearing pitch diameter, and \(\alpha\) is the contact angle. Pertinent to the dimensions of the bearing and run conditions modelled for this study, the analytical estimation of \(f_{\text{bpo}}\), calculated using Equation (1), is 90.07 Hz. The difference of 0.9 percent between the numerical and analytical \(f_{\text{bpo}}\) is because that the FE modelling of the bearing presented here accounts for the rolling elements’ slipping, whereas the analytical formula, Equation (1), does not.
In contrast to previous FE models, where unrealistically high instantaneous acceleration levels of $10^7 \text{ g}$, 4,000 g, and 15,000 g were shown, the model presented here produces realistic acceleration levels of approximately 180 g. One of the reasons for this improvement compared to the previous FE models is that the bearing components were modelled as flexible parts to correctly represent their stiffness. Other reasons, which could not be compared with previous models due to lack of details provided in the literature, include optimal meshing of the model to ensure continuous rolling element-to-raceway contact, correct implementation of the segment-based surface-to-surface contact, and the use of a value for the coefficient of friction that is comparable to the recommended practical value in rolling element bearings. These boundary conditions were discussed during the development of the FE model of the bearing in an earlier reference by the authors of this paper.

From the favourable agreement of the numerical FE and analytical results of
the $f_{bpo}$, it can be seen that the FE model of the bearing has accurately modelled and acquired the basic bearing kinematics. But the modelled acceleration time-trace contains numerical noise.

### 3.1.1. Numerical rolling contact noise

From Figure 2, the instantaneous peak levels of the impulses, separated by $f_{bpo}$, are approximately of the order of $\pm 180$ g, whereas the non-impulsive acceleration signals (between the impulses) peak at about $\pm 50$ g. To seek the frequencies associated with the numerical noise, power spectrum of the acceleration time-trace in Figure 2 was calculated using Welch’s method. Although the signal was sampled at 100 kHz, the simulation was run for 30 ms, resulting in only 3010 samples after the interpolation based on the minimum output time interval. Therefore, the acceleration signal was zero-padded with $2^{15}$ FFT points to interpolate the power spectral estimate so as to achieve a frequency resolution of 3 Hz. Figure 3 shows the narrow band power spectrum of the FE modelled acceleration $a_y$ signal. The tone at 4671 Hz represents the numerical noise.

The authors of this paper in an earlier publication [25] have presented a hypothesis and explained that the numerical noise is primarily caused by the contact interaction of the polygonised rolling elements and bearing races. The word polygonised refers to the transformation of the circular rolling elements to multi-point polygons as a result of their meshing. The rotation of the polygonised rolling elements causes minor impacts as their edge-points contact both inner and outer races resulting in the noise. This numerical noise is referred to as rolling contact noise and is an artefact of the FE model. It is shown in our earlier publication [25] that the tone at 4671 Hz is generated as the corners of the polygonised rolling elements contact the outer race; this is referred to as rolling element-to-outer raceway rolling contact noise frequency $f_{\text{noise}}^o$.

Based on the power spectrum of the acceleration results in Figure 3 a second-order notch filter with a quality factor of 15 was designed to eliminate the tonal noise at 4671 Hz. The narrow-band power spectrum of the unfiltered and notch
Figure 3: Power spectral density of the nodal acceleration $a_y$ time-trace shown in Figure 2, highlighting one of the dominant numerical noise frequencies, $f_{\text{noise}}^o = 4671$ Hz observed in the FE simulation results.
filtered acceleration $a_y$ time-traces is compared in Figure 4. To clearly see the difference between the two spectra, the results in Figure 4a are zoomed from 4–6 kHz, and the corresponding plots are shown in Figure 4b. It can be seen that the tone at the numerical rolling element-to-outer raceway rolling contact noise frequency $f_{noi}^{s}$ has been attenuated by approximately 25 dB without affecting the majority of the response. However, the power spectrum at the frequencies within the filter bandwidth is affected slightly.

Using the filter, the numerical acceleration $a_y$ signal was notch filtered and is shown in Figure 5 along with the unfiltered acceleration signal from Figure 2 using a gray-coloured, dashed line for comparison. The effect of filtering the noise is evident in Figure 5; the instantaneous levels of the non-impulsive acceleration signals plummet from $\pm 50$ g to approximately $\pm 20$ g. However, there is still some residual noise that is a result of the rolling elements slipping as they contact the cage slots in addition to the variations in the adaptive time-stepping, which is an inherent feature of the FE software used here [35], as discussed in earlier references [25, 27, 28]. This behaviour is random and therefore, the (noise) frequencies cannot be estimated and subsequently filtered without affecting the overall vibration signals.

3.2. Time domain analysis

Although the form and content of a typical vibration signal are similar whether it is the acceleration, velocity or displacement, in the present study, it was found useful to investigate the numerically modelled velocity and displacement time-traces in addition to the acceleration results. It is shown in this section that the investigation of the velocity and displacement time-traces helped identify some important vibration characteristics that were not apparently visible in the acceleration time-trace. Due to the usefulness of the velocity and displacement time-traces, they were also included in the subsequent time–frequency and frequency domain analyses in addition to the acceleration results. This also helped in demonstrating the analytical validation of the simulation results of the FE model of the bearing [25].
(a) Power spectral densities of the unfiltered and notch filtered acceleration $a_y$ time-traces, highlighting the tonal noise at $f_{noise}^{num} = 4671$ Hz for the unfiltered time-trace.

(b) Comparison of the power spectral densities shown in Figure 4a on a zoomed frequency scale of 4–6 kHz, highlighting the attenuation of the tonal noise by 25 dB after filtering.

Figure 4: Power spectrum of the FE modelled, unfiltered and notch filtered, acceleration $a_y$ time-traces shown in Figure 5 for a radial load $W$ of 50 kN and rotational speed $n_s$ of 500 RPM.
Figure 5: Effect of filtering out the rolling element-to-outer raceway rolling contact noise at $f_{\text{noise}}^{\alpha} = 4671$ Hz on the FE modelled acceleration $a_y$ time-trace shown in Figure 2 for a radial load $W$ of 50 kN and rotational speed $n_s$ of 500 RPM.
The numerically modelled velocity $v_y$ and displacement $u_y$ time-traces at the same node where the acceleration $a_y$ signal was obtained, are shown in Figures 6a and 6b, respectively. Both figures compare the unfiltered and notch filtered nodal velocity and displacement time-traces. The effect of filtering out the numerical rolling contact noise frequency $f_{\text{noise}}$ is clearly evident in Figures 6a and 6b similar to that in Figure 5. The defect-related impulses are separated by 0.011 s that correlates to $f_{\text{bpo}}$ of 90.91 Hz, and they are clearly evident in the velocity $v_y$ time-trace in a similar way to the acceleration $a_y$ signal shown in Figures 2. In contrast, for the displacement $u_y$ signal in Figure 6b, the impulses are not as clear as for the cases of acceleration $a_y$ and velocity $v_y$ signals; however, the change (fall and rise) in the displacement signal indicates the passage of the rolling elements through the defect, which is discussed in the following paragraphs.

It has been established in the literature [38, 39, 25, 28, 34, 40] that the entrance of a rolling element into a defect is a low-frequency de-stressing/unloading event, whereas its exit from a defect is a predominantly high-frequency re-stressing/reloading impulsive event that excites a broad range of the structural resonant modes of a bearing [37] including both low- and high-frequency modes. In other words, although the exit of the rolling elements is generally characterised by (impulsive) energy in high-frequency regions, low-frequency pulses are also generated [25, 28, 34] during the exit. It is crucial for the numerically modelled vibration response from the developed explicit FE model of the bearing [25] to show such characteristics for model validation, in addition to correctly acquiring the bearing kinematics, which has already been demonstrated in Figures 2 and 6. Although the unloading event has earlier been experimentally validated [38, 39, 25, 28], previous models [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 21, 22, 23, 24] could not predict the vibration signals associated with this event without hypothesising the shape of the defect and path of the rolling elements [19, 20, 39].

The observation of the numerical acceleration $a_y$ time-trace of the defective bearing in Figure 5 does not reveal a distinction between the low- and pre-
Figure 6: FE modelled, unfiltered and notch filtered, velocity $v_y$ and displacement $u_y$ time-traces of the bearing for a radial load $W$ of 50 kN and rotational speed $n_s$ of 500 RPM.
dominant high-frequency unloading and reloading events, respectively. In fact, the entrance of the rolling elements into the outer race defect is not visible compared to their exit indicated by the impulses whose period correlates to $f_{bpo} = 90.91$ Hz. In contrast, careful observation of the notch filtered numerical velocity $v_y$ in Figure 6a provides some indication of change in the signal characteristics before the defect-related impulsive signals.

Without the unfiltered results, the notch filtered nodal velocity time-trace is reproduced in Figure 7a for clarity, along with some markers and associated annotations. Throughout this paper, the elliptical markers indicate the de-stressing/unloading of the rolling elements as they move in the defect, whereas the rectangular markers indicate their re-stressing/reloading as they move out of the defect. Despite being noisy, the distinction between the vibration signatures pertinent to the unloading and reloading events is evident in the FE modelled velocity time-trace.

From the understanding gained from Figure 7a, the unloading and reloading events can also be distinguished from the nodal displacement $u_y$ results in Figure 6b. For clarity, the notch filtered displacement time-trace is reproduced in Figure 7b along with relevant indications to highlighting the unloading and reloading of the rolling elements.

The unloading of the rolling elements is clearly evident in the notch filtered velocity $v_y$ and displacement $u_y$ time-traces in Figure 7 as indicated by the elliptical markers, which agrees with previous experimental findings [38]; however, no such signatures are visible in the notch filtered acceleration $a_y$ results in Figure 5. It implies that the low-frequency vibrations signatures pertinent to the de-stressing event in the acceleration time-trace are masked by the (stochastic) numerical noise.

Time–frequency analysis, which is commonly used to investigate the energy or power distribution in a typical vibration signal, can be used to check the frequency content of the two events for the modelled vibration signals. It will also help in identifying if the numerical acceleration signal contains the low-frequency de-stressing signals, which is crucial to the validation of the modelling.
Figure 7: FE modelled, notch filtered, velocity $v_y$ and displacement $u_y$ time-traces shown in Figure 6, indicating the de-stressing (unloading) and re-stressing (reloading) events.
results from the FE model of the bearing \[25\].

3.3. *Time–frequency analysis*

One of the common forms for preliminary investigation of the energy content of a time-varying signal is to plot its spectrogram, also referred to as the short-time Fourier transform (STFT). For a time-varying signal \(x(t)\), moved over a time window \(w(t)\), the STFT is given by [11 Chapter 3, page 130]

\[
\text{STFT}(\tau,f) = \mathcal{S}(\tau,f) = X(t,f) = \int_{-\infty}^{\infty} x(t) w(t-\tau) \exp(-i2\pi ft) \tag{2}
\]

The aims of presenting the time–frequency analysis of the FE modelled vibration time-traces are to: understand the frequency characteristics of the unloading and reloading of the rolling elements, identify and distinguish between the energy content of the unloading and reloading events, identify if the numerically modelled acceleration \(a_y\) signal shown in Figure 5 contains the low-frequency unloading signatures, and to aid in estimating the spall size.

Figures 8a, 8b, and 8c show the spectrogram plots of the numerically modelled nodal acceleration \(a_y\), velocity \(v_y\), and displacement \(u_y\) results, respectively, discussed in the preceding sections. The numerical noise was removed by de-trending the vibration time-traces along the time \((x-)\) axis. In other words, the mean of the power spectral density for each frequency was removed. The low- and high-frequency regions corresponding to the unloading and reloading of the rolling elements are highlighted using appropriate markers. The distinction between the two events is clearly evident in all spectrogram plots. While the energy pertinent to the unloading event is below 3 kHz, the impulses associated with the reloading event have energy in the high-frequency region between 10 kHz and 25 kHz consistently across all the spectrogram plots of the vibration signals. In addition, there is also a slight indication of the low-frequency content at the timings corresponding to the re-stressing events causing an overlap of the low- and high-frequency signatures. This is because when the rolling elements re-stress between the inner and outer raceways, multiple defect-related impulses
Figure 8: Spectrogram plots of the FE modelled, unfiltered, acceleration $a_y$, velocity $v_y$, and displacement $u_y$ time-traces indicating the de-stressing (unloading) and re-stressing (reloading) events.

(a) A spectrogram of the FE modelled, unfiltered, acceleration $a_y$ time-trace shown in Figure 2.

(b) A spectrogram of the FE modelled, unfiltered, velocity $v_y$ time-trace shown in Figure 6a.

(c) A spectrogram of the FE modelled, unfiltered, displacement $u_y$ time-trace shown in Figure 6b.
are generated that excite a broad range of the resonance modes of the bearing including both low- and high-frequency modes [25, 28].

Compared to the notch filtered time-trace of the numerical acceleration \( a_y \) in Figure 5, where the de-stressing/ unloading of the rolling elements could not be seen, the time–frequency analysis of the acceleration signal in Figure 8a facilitated the appearance of this event.

As the de-stressing signals are mainly characterised by signals in low-frequency regions, it was found useful to low-pass filter the numerical acceleration \( a_y \) time-trace. This was implemented using a Butterworth filter of third-order and a cut-off frequency of 2.5 kHz, and the low-pass acceleration time-trace is shown in Figure 9. It is evident from this plot that the de-stressing signals that were not visible in the unfiltered acceleration \( a_y \) time-trace in Figure 5 can now be clearly identified. It is worthwhile noting that despite the partial removal
of the high-frequency impulses corresponding to the reloading event due to the low-pass filtering, the impulses are still visible. This is because the reloading/re-stressing event excites a broad range of frequencies that include both low- and high-frequency resonance modes of a bearing as mentioned earlier. Therefore, the impulses related to the re-stressing event that remained after the low-pass filter effectively have low-frequency characteristics. The instantaneous level of the re-stressing impulses in Figure 9 is significantly reduced from approximately 180 g to 20 g as a result of the low-pass filtering. It should be noted that the low-pass filter has only been implemented to enhance the unloading of the rolling elements and the low-pass filtered signals are not used for further data analysis.

The capability of the explicit FE model [25] described here to predict the unloading or de-stressing of the rolling frequency is novel and unique, compared to previous multi-body analytical [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18] and FE models [21, 22, 23, 24], which could not predict this event. The results from the time–frequency analysis have identified the distinct entry- and exit-related vibration signatures that agree with existing knowledge in the literature [38, 39, 25, 28, 34, 40]. Frequency domain analysis of the modelled vibration results is presented in the next section with an aim to predict the fundamental defect frequency $f_{bpo}$ and harmonics.

3.3.1. Spall size estimation

As discussed in an earlier publication by the authors of this paper [25], time separation between the distinct de-stressing and re-stressing events for the simple case of a localised line spall can be used to determine the average size (length $L_d$) of the defect as

$$L_d = \frac{2\pi f_c D_o \Delta t_{\text{event}}}{2}$$

where, $L_d$ is the length of the defect, $2\pi f_c = \omega_c$ is the angular velocity of the rolling elements, $D_o$ is the outer race diameter, and $\Delta t_{\text{event}}$ is the difference between the consecutive de-stressing or re-stressing events. From the time and time–frequency domain analysis in Figures [7] [8] and [9] the time separation be-
tween the consecutive events $\Delta t_{\text{event}} = 4$ ms that corresponds to the estimated length $L_d = 9.5$ mm. It compares favourably to the actual modelled spall length of 10 mm.

3.4. Frequency domain analysis

Frequency domain analysis, commonly referred to as spectral analysis, enables the transformation of a time-trace into its equivalent frequency domain representation by taking the Discrete Fourier Transform (DFT) of the time-trace. In digital signal processing, spectral analysis is typically performed using the Fast Fourier Transform (FFT) algorithm. The Fourier transform $\mathcal{F}$ of a time-varying signal $x(t)$ is defined as

$$
\mathcal{F} \{ x(t) \} = X(f) = \int_{-\infty}^{\infty} x(t) \exp(-i2\pi ft) \, dt
$$

One of the main purposes of performing spectral analysis on bearing vibration data is to identify major frequency components, such as defect-related fundamental, harmonics and side bands, which can indicate the presence of a defect. The envelope analysis technique [37], which is widely used for bearing diagnosis, has been implemented in this study.

A prerequisite associated with the envelope analysis technique is that the most suitable frequency band for demodulation is chosen. While an estimate may be determined from the time–frequency analysis, spectral kurtosis [42] and kurtograms [43] are commonly used to find the frequency band with the highest content of impulsive energy. It is important to show the implementation of these techniques and identify the frequency band for demodulation for the modelled vibration signals.

3.4.1. Spectral kurtosis

Spectral kurtosis extends the concept of (global) kurtosis to that of a function of frequency, which indicates the impulsiveness of the signal in a frequency band
Spectral kurtosis $\mathcal{K}$ of a time-varying signal $x(t)$ can be defined as

$$\mathcal{K}\{x(t)\} = \text{SK}(f) = \frac{\langle |X(t,f)|^4 \rangle}{\langle |X(t,f)|^2 \rangle^2} - 2 \quad (5)$$

where, the squared magnitude, $|X(t,f)|^2$ — the spectrogram — returns the power spectrum at time $t$ and its average over time, $\langle |X(t,f)|^2 \rangle$ — the power spectral density.

For various window lengths $N_w$ (power of 2), Figures 10a, 10b, and 10c show the spectral kurtosis of the notch filtered numerical acceleration $a_y$, velocity $v_y$, and displacement $u_y$ time-traces shown in Figures 5, 7a, and 7b, respectively. A common characteristic of these spectral kurtoses plots is that the defect-related impulsivity within the vibration signals is approximately between 12 kHz and 22 kHz. These findings are similar to those shown in the spectrogram plots in Figures 8a, 8b, and 8c, where the energy of the defect-related (re-stressing) impulses can be seen to have consistently concentrated between 10 kHz and 25 kHz. These results provide an indication of the frequency bands for the envelope analysis; however, as the SK varies for various window lengths, the frequency bands need to be judiciously chosen. A kurtogram helps to determine the optimal frequency band.

3.4.2. Kurtogram

Kurtogram [43] refers to the representation of SK as a function of both frequency and window length. A kurtogram is basically a cascade of spectral kurtoses obtained for different values of the STFT window length $N_w$, but for a much finer grid.

Figures 11a, 11b, and 11c show the full kurtogram plots of the numerically modelled acceleration, velocity, and displacement results, shown in Figures 5, 7a, and 7b, respectively. The results consistently show that the defect-related impulsivity is concentrated around 20 kHz. Although the information in spectral kurtosis and kurtogram plots is similar, one can achieve precise information from the latter due to the mapping of the former over a wide range of STFT window.
Figure 10: Spectral kurtosis plots of the FE modelled, notch filtered, acceleration $a_y$ time-trace shown in Figure 5.

(b) A spectral kurtosis plot of the FE modelled, notch filtered, velocity $v_y$ time-trace shown in Figure 7a.

(c) A spectral kurtosis plot of the FE modelled, notch filtered, displacement $u_y$ time-trace shown in Figure 7b.

Figure 10: Spectral kurtosis plots of the FE modelled, notch filtered, acceleration $a_y$, velocity $v_y$, and displacement $u_y$ time-traces corresponding to a radial load $W$ of 50 kN and rotational speed $n_s$ of 500 RPM for various window lengths $N_w$. 

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Figure 11: Kurtograms of the FE modelled, notch filtered, acceleration $a_y$, velocity $v_y$, and displacement $u_y$ time-traces for a radial load $W$ of 50 kN and rotational speed $n_s$ of 500 RPM.
lengths.

3.4.3. Envelope analysis

Envelope analysis, considered a benchmark for bearing diagnostics [37], comprises bandpass filtering a signal in a high frequency region where the defect-related impulses are amplified by structural resonances followed by amplitude demodulation to generate the envelope signal. Power spectrum of the envelope signal contains desired diagnostic information in the form of fundamental defect frequency, side bands and harmonics.

In this paper, envelope analysis [37] has been carried out using the Hilbert transform [44]. For a real-valued time signal $x(t)$, the Hilbert transform $\mathcal{H}$ is the imaginary component of its complex analytic signal $\hat{x}(t)$. These variables are related as

$$\mathcal{H}\{x(t)\} = \hat{x}(t)$$  \hspace{5mm} (6)

$$\hat{x}(t) = x(t) + i\tilde{x}(t)$$  \hspace{5mm} (7)

Unlike the Fourier transform $\mathcal{F}$, which transforms a signal from the time to frequency domain or vice-versa, the Hilbert transform $\mathcal{H}$ keeps the signal in the same domain; however, it causes a phase shift of $90^\circ$ in the frequency domain or a quarter of a wavelength in the time domain [44].

From the kurtogram plots in Figures 11a, 11b, and 11c, the impulsive frequency band of 18–23 kHz was chosen for demodulating the numerical bearing vibration signals, and subsequently the envelope analysis technique was implemented. The power spectrum of the envelope signals was estimated using Welch’s method with 50% overlap. As the simulated signals have only 3010 data points, they were zero-padded with $2^{16}$ FFT points in order to smoothly interpolate the results to achieve a frequency resolution of 1.5 Hz.

Figures 12a, 12b, and 12c show the envelope power spectrum of the numerical acceleration $a_y$, velocity $v_y$, and displacement $u_y$ time-traces. The tonal peaks at the fundamental and harmonics, as indicated in the figures, correspond to $f_{bpo}$. The envelope power spectra of the vibration time-traces follow the standard
Figure 12: Envelope of the FE modelled, band-pass filtered, acceleration $a_y$, velocity $v_y$, and displacement $u_y$ time-traces for $W = 50$ kN and $n_s = 500$ RPM; the vertical lines indicate the fundamental $f_{bpo}$ and its harmonics.
pattern of having the highest amplitude of the fundamental and decreasing amplitudes for subsequently harmonics. These envelope power spectra shown in the figures clearly indicate the presence of a defect within the rolling element bearing, which shows the explicit FE of the bearing has accurately simulated the complex dynamics of the system.

3.5. Summary of the numerical results

In the preceding sections, standard signal processing techniques, applicable to the vibration-based monitoring of rolling element bearings, were used to analyse the numerically modelled vibration signals obtained using the FE model of the bearing. The main emphasis of the analyses was to discuss the verification of the modelled results based on the relevant analytical and experimental knowledge from the literature, such as bearing kinematics \( f_{bpo} \) through the time and frequency domain analyses, and distinct frequency characteristics of the signals of the unloading and reloading events through the time-frequency analysis. A favourable comparison between the numerical and analytical \( f_{bpo} \), indicating accurate modelling of the bearing kinematics, has been shown in Figures 2 and 6. The envelope power spectrum plots shown in Figure 12 also show the fundamental and subsequent harmonics at \( f_{bpo} \). The low- and predominantly high-frequency vibration signatures pertinent to the unloading and reloading of the rolling elements have been demonstrated not only in the notch filtered vibration time-traces in Figures 7 and 9 but also in the spectrogram plots in Figure 8.

4. Conclusions

Results from an explicit FE model of a rolling element bearing with a localised outer raceway line spall are presented. Time domain vibration signals obtained directly from the modelling were presented so as to discuss the validation of the results. A detailed analysis of the time domain results, that include acceleration, velocity and displacement time-traces, showed that the modelled
signals contain characteristics corresponding to unloading and reloading of the
rolling elements as they move in and out the defect, respectively. The vibration
time-traces were processed further to estimate time–frequency and frequency
domain results using standard signal processing techniques applicable to the
vibration-based monitoring of rolling element bearings. Spectrogram plots of
the modelled vibration signals helped highlighting the low- and high-frequency
regions pertinent to the unloading and reloading of the rolling elements, respecti-
vely. Demonstrated by the spectrogram plots, it was found useful to low-pass
filter the acceleration time-trace to enhance the low-frequency vibration char-
acteristics pertinent to the unloading of the rolling elements as they move in
a defect. The commonly used envelope analysis method was implemented to
show the fundamental outer raceway defect frequency and harmonics in the fre-
quency spectrum. A logical approach to presenting the numerically modelled
results was followed with an aim to presenting the analytical validation of the
modelled results. The time and frequency domain analyses showed that the FE
model of the defective bearing generates accurate bearing kinematics, defect fre-
quency components, whereas the time–frequency analysis clearly highlighted the
distinct low- and predominantly high-frequency vibration signatures pertinent
to the unloading and reloading of the rolling elements, respectively. Through
these analysis techniques, analytical validation of the FE modelled results was
demonstrated.

Acknowledgments

This work was conducted as a part of ARC Linkage funded project LP110100529.

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