Hybrid Digital Control of Piezoelectric Actuators

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Abstract

Nanopositioning, as a core aspect of nanotechnology, concerns the control of motion at nanometre scale and is a key tool that allows the manipulation of materials at the atomic and molecular scale. As such it underpins advances in diverse industries including biotechnology, semiconductors and communications.

The most commonly used nanopositioner is the piezoelectric actuator. Aside from being compact in size, piezoelectric actuators are capable of nanometre resolution in displacement, have high stiffness, provide excellent operating bandwidth and high force output. Consequently they have been widely used in many applications ranging from scanning tunnelling microscopes (STM) to vibration cancellation in disk drives. However, piezoelectric actuators are nonlinear in nature and suffer from hysteresis, creep and rate-dependencies that reduce the positioning accuracy.

A variety of approaches have been used to tackle the hysteresis of piezoelectric actuators including sensor-based feedback control, feedforward control using an inverse-model and charge drives. All have performance limitations arising from factors such as parameter uncertainty, bandwidth and sensor-induced noise.
This thesis investigates the effectiveness of a synergistic approach to the creation of hybrid digital algorithms that tackle challenges arising in the control of non-linear devices such as piezoelectric actuators. Firstly, a novel digital charge amplifier (DCA) is presented. The DCA overcomes inherent limitations found in analog charge amplifiers developed in previous research.

In order to extend the DCA operational bandwidth, a complementary filter was combined with the DCA along with a non-linear black-box model derived using system identification techniques. To maximize the model accuracy a novel method is utilized that reduces error accumulation in the model. This method is generally applicable to many dynamic models. A non-linear model is also used with a data fusion algorithm to ensure the DCA does not exhibit drift, an issue common to most of charge amplifiers.

The proposed hybrid digital system is evaluated and it is shown that hysteresis is significantly decreased, while operational bandwidth is extended with no displacement drift. Experimental results are presented throughout to fully validate the proposed system.
Declarations

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Notation

+\(x\) Positive electrode in the x direction
-\(x\) Negative electrode in the x direction
+\(y\) Positive electrode in the y direction
-\(y\) Negative electrode in the y direction
\(b\) Bias in semi-linear ANN
\(b\) Damping of a piezoelectric actuator
\(C\) Centre of a cluster
\(C_L\) Capacitance of a load capacitance
\(C_p\) Capacitance of a piezoelectric actuator
\(C_s\) Sensing capacitor
\(C_{series}\) Capacitor in series with a piezoelectric actuator
\(D\) Density function
\(D\) Electrical displacement (charge per unit area)
\(d\) Piezoelectric material constants (Chapter 2)
\(\hat{d}\) Displacement of a piezoelectric actuator (other chapters)
\(\hat{d}_{ANN}\) Output displacement of the ANN model
\(d_a\) Actual displacement
\(d_d\) Desired displacement
\(\hat{d}_{GDCDE}\) Output displacement of the GDCDE unit
$d_t$  Reference displacement
$E$  Electric field
$e$  Error
$e_{ls}$  Error of an electric current
$e_m$  Error at the output of the NARX model
$e_{q_{opt}}$  Error in the optimal output charge
$F$  Artificial neural network
$f$  Output of fuzzy rules and FISs
$f$  Function
$f_{critical}$  Break (critical) frequency of a system
$F_{ext}$  Force imposed from an external mechanical source
$F_t$  Transduced force from an electrical domain
$H(s)$  Continuous transfer function between $V_o(t)$ and $V_s(t)$
$H(z)$  Discrete transfer function between $V_o(t)$ and $V_s(t)$
$I_L$  Electric current of a load
$I_n$  Electric Current source
$I_p$  Electric current of a piezoelectric actuator
$I_s$  Electric current of a sensing resistor/capacitor
$k$  Proportional gain
$K$  Gain (Charge to displacement)
$K_c$  Closed loop gain
$k_p$  Stiffness of a piezoelectric actuator
$m$  Mass of a piezoelectric actuator (Chapter 2)
The number of rules of a FIS (Others)
$MSE_{e_{opt}}$  Mean square error (MSE) of $e_{q_{opt}}$
\( N \)  
Number of previous displacements (Chapter 4)  
Activation function (Chapter 6)

\( n \)  
Number of inputs to a FIS (Chapter 4)  
Number of data elements in each column of raw data (Chapter 6)

\( n_t \)  
Number of terms of discrete sum function

\( p \)  
Consequent parameter of fuzzy inference system

\( q \)  
Consequent parameter of fuzzy inference system

\( q_{\text{actual}} \)  
Actual charge across piezoelectric actuator

\( q_{\text{DCA}} \)  
Charge calculated by DCA unit

\( q_{\text{desired}} \)  
Desired charge

\( q_{\text{in}} \)  
Input charge

\( q_L \)  
Charge across a load

\( q_{\text{measured}} \)  
Measured charge across a piezoelectric actuator

\( q_{\text{model}} \)  
Charge calculated by the NARX model

\( q_{\text{optimal}} \)  
Optimal output charge

\( q_p \)  
Charge across a piezoelectric actuator

\( r \)  
Consequent parameter of fuzzy inference system

\( r_a \)  
Range of influence

\( r_b \)  
Squash factor

\( r_d \)  
Displacement (Output) order

\( r_{\text{de}} \)  
Discrete delay time

\( R_{\text{inputADC}} \)  
ADC input resistance

\( R_L \)  
DC impedance of a load

\( R_p \)  
Protection circuit resistor

\( R_s \)  
Sensing resistor
\( R_t \)  Total resistance
\( r_u \)  Input order
\( r_v \)  Piezoelectric voltage (input) order
\( r_y \)  Output order
\( S \)  Integration area
\( S \)  Strain of a piezoelectric actuator
\( s \)  Sampling (index)
\( s \)  Compliance matrix
\( T \)  weight of connections of the ANN
\( T \)  Stress
\( t \)  Time
\( t_d \)  Delay time (Dead time)
\( t_f \)  Final time of operation
\( T_p \)  Electromechanical transformer ratio
\( T_s \)  Sampling time
\( u \)  Input to the model
\( V \)  Applied voltage
\( v \)  piezoelectric actuator velocity
\( V_{bias} \)  Bias voltage at the input of an ADC
\( V_{i} / V_{in} / V_{in} \) Input voltage
\( v_{me} \)  Voltage across the Maxwell elasto-slip element
\( V_o \)  Output of the voltage amplifier
\( V_p \)  Voltage across a piezoelectric actuator
\( V_{pext} \)  Last extremum value of applied voltage
\( V_{ps} \)  Vector of the present and a number of previous piezoelectric-
voltage values

$V_{\text{ref}}$  Reference voltage

$V_s$  Sensing voltage across the sensing resistor/Capacitor

$V_{\text{si}}$  Strain-induced voltage

$v_t$  Back-emf from the mechanical side of a piezoelectric actuator

$\mathbf{W}$  Weight of connections of the ANN

$w$  Weight of a rule in a FIS

$w_s$  DCA weighting coefficients

$W_{\text{ANN}}$  ANN weighting coefficients

$w_b$  NARX model weighting coefficients

$W_{\text{GDCDE}}$  GDCDE weighting coefficients

$x$  A datum in clustering

$x$  Input to the activation function $N$

$y$  Output of the model

**Symbols**

$\alpha$  Switching value of Preisach model

$\beta$  Switching value of Preisach model

$\gamma_{\alpha\beta}$  Elementary hysteresis operator

$\Delta$  Nonlinear impedance

$\epsilon$  Permittivity

$\mu (.)$  Membership grade in FISs

$\mu (..)$  Weighting function of the Preisach model

$\mu_n$  Mean

$\sigma_n$  Standard deviation

$\omega_c$  Cut-off frequency
1 Introduction

Nanotechnology is the multidisciplinary science of manufacturing and manipulation of objects down to nanometre size. Nanopositioning is a core aspect of nanotechnology and concerns the control of motion at nanometre scale and is a key tool that allows the manipulation of materials at the atomic and molecular scales. As such, it underpins advances in diverse industries including biotechnology, semiconductors and communications. Nanopositioning systems require precise actuation on the nanometre scale and they are generally driven using solid state actuators manufactured using smart materials.

The most commonly used nanopositioner is the piezoelectric actuator. Aside from being compact in size, they are capable of nanometre resolution in displacement, have high stiffness, provide excellent operating bandwidth and high force output (Devasia et al., 2007). Consequently they have been widely used in many applications ranging from scanning tunnelling microscopes (STM) (Wiesendanger, 1994) to vibration cancellation in disk drives (Ma and Ang, 2000).

However, piezoelectric actuators suffer from hysteresis and creep which reduce the positioning accuracy and are the most important challenges in the use of piezoelectric actuators (Leang and Devasia, 2002). A number of
previous control methods have attempted to tackle these issues; however the problems are still not completely solved.

### 1.1 Aims and objectives

The aim of this research is to investigate the role of hybrid digital algorithms in tackling challenges arising in the control of non-linear systems. In this research, hysteresis, which is the main source of piezoelectric actuator non-linearity, was chosen in order to evaluate the performance of the proposed hybrid digital approach. The proposed control systems are tested and evaluated on a piezoelectric stack actuator AE0505D44H40 from NEC and piezoelectric tube actuator PI-PT130.24 from PI.

Thus the objectives of this research are the:

- Extensive investigation of digital approaches to implement charge amplifiers.
- Creation of innovative, holistic models, appropriate to the linearization, compensation and control of piezoelectric actuators.
- Investigation of hybrid methods to improve the accuracy of models.
- Investigation of multiple conceptual technologies, such as Artificial Neural Networks (ANNs), fuzzy logic and complementary filters to improve the tracking performance and extend the operational bandwidth of actuators.
1.2 Publications arising from this thesis

The research outcomes of this thesis have led to the generation of refereed publications, including journal papers and refereed conference papers which are listed in the following:

**Refereed Journal Papers:**


Under-review Journal Papers:


Refereed Conference Papers:


1.3 Preview of the thesis

This thesis comprises, in part, manuscripts that have been published or submitted for publication in international peer-reviewed journals in accordance with the ‘Academic Program Rules 2013’ approved by the Research, Education and Development Committee of the University of Adelaide. In addition three conference papers, which are also relevant to the present work, are included in the appendix. This section provides brief descriptions of each publication and the links between publications in order to show how the objectives of this thesis are achieved.

Chapter 2 covers previous literature regarding the displacement control of piezoelectric actuators including feedback voltage drive, feedforward voltage drive and charge drive. This chapter shows that while there has been much research into the control of piezoelectric actuators, there is an opportunity to explore digitally implemented charge amplifiers within an integrated hybrid system. To establish the field of knowledge, this chapter also includes some general background on piezoelectric actuators and their behaviour.

Chapter 3 presents a novel digital charge amplifier (DCA). It is discussed how the DCA can reduce hysteresis and improve the linearity of piezoelectric actuators. This easily implemented, digital charge drive approach opens up the possibility of integration with other control methods such as model-based methods to improve the performance of the displacement controller which are investigated in the subsequent chapters.

Chapter 4 consists of two journal papers. The first paper is focused on black-box modelling of a piezoelectric stack actuator. Appropriate selection
of the inputs to the model is an important task in black-box modelling. In this paper, current and previous values of voltage, previous values of the displacement and for the first time extremum values of input voltage and/or displacement in each cycle of operation are used as an input to the model. In addition, in order to understand the role of each input in the model and to choose the most appropriate inputs, fuzzy subtractive clustering and neuro-fuzzy networks are used.

The second paper proposes a novel hybrid method to increase the accuracy of the model. This is achieved through the use of a method that employs the velocity signal, which is related to the current passing the piezoelectric actuator, to reduce the effect of error accumulation on the output displacement of the model. This method is then theoretically and experimentally verified.

In Chapter 5 a model-based drift correction technique is proposed to remove the drift and improve the tracking performance of the DCA. It uses data fusion to integrate the reliability and short term accuracy of the DCA, with the long term accuracy of the non-linear model, to realise the benefits of both techniques. Experimental results clearly show the elimination of drift and improvements in tracking performance.

In order to extend the DCA operational bandwidth, a novel hybrid digital method is proposed in Chapter 6. In this method a non-linear model was designed and trained to estimate displacement based on the piezoelectric voltage at low frequencies. The model and charge-based displacement estimators were used together through a complementary filter to increase the bandwidth of displacement estimation and control. In addition, the system is
designed to be capable of driving grounded-loads such as piezoelectric tube actuators.

Finally, conclusions and future work is presented in Chapter 7.

References


2 Background and Literature

Review

The purpose of this chapter is to provide a general background on piezoelectric actuators and to review all relevant literature relating to controlling piezoelectric actuators, mostly focussing on charge drives.

This chapter is structured around four sections. Section 2.1 discusses how piezoelectric actuators work. In Section 2.2, the non-linearities associated with piezoelectric actuators are explained. Control of piezoelectric actuator displacement, including displacement feedback voltage drive, feedforward voltage drive and charge drive, is described in Section 2.3. Section 2.4 presents the conclusions.

2.1 Piezoelectric actuators

The Curie brothers discovered the piezoelectric effect in 1880. They realised that compressing a piezoelectric material results in an electric charge. After further investigation, they also found the inverse piezoelectric effect, where an applied electric charge results in deformation of the material. They eventually built a new type of actuator called ‘the piezoelectric actuator’ (Ballato, 1996).
Figure 2.1 shows a schematic of a single layer piezoelectric actuator. The lead zirconate titanate (PZT) ceramic, which is a popular piezoelectric material, is in the centre, with electrodes above and below. The PZT will be deformed when an electric field is applied. Single layer piezoelectric actuators are sometimes called “moving capacitors” because, from the electric point of view, they are similar to a capacitor with the exception that they move.

In order to increase actuating forces and displacements, a number of single layer piezoelectric materials are often assembled in series to build piezoelectric stack actuators.

Another type of piezoelectric actuator is the piezoelectric tube actuator which is most widely used in Scanning Probe Microscopes (SPMs) to manipulate matter at the nanometre scale (Moheimani, 2008). In traditional SPMs, tripod positioners formed from three piezoelectric stack actuators were employed for movement in the x, y and z directions. Binnig and Smith (1986) proposed the use of piezoelectric tube actuators instead. Compared to tripod positioners, piezoelectric tube actuators offer better accuracy, higher bandwidth and an easier manufacturing process (Devasia et al., 2007), while
their smaller size simplifies vibration isolation (Chen, 1992). Piezoelectric tubes are the foremost actuators in atomic force microscopy (AFM) (Abramovitch et al., 2007, Kuiper and Schitter, 2010) and are likely to remain the most widely used positioning actuators in other micro-scale and nano-scale positioning tasks for several years (Moheimani, 2008), such as displacement of fibre optics (Leung et al., 2008) and ultrasonic applications (Hui et al., 2010).

Figure 2.2 shows a schematic of a piezoelectric tube actuator. Typically the tube has one grounded inner electrode and four equally distributed outer electrodes. The bottom of the tube is fixed and the top moves in the $x$, $y$ and $z$ directions.

An applied voltage between an external and the internal electrode will bend the tube and the sign of the applied voltage determines the direction in which it bends. This means that by applying a positive and negative voltage with equal magnitudes on the $-x$ and $+x$ electrodes, the tube will bend in the $x$ direction. Similarly, it can move in the $y$ direction. To actuate the tube in the $z$ direction, equal voltages should be applied to all four external electrodes.
2.2 Non-linearities in piezoelectric actuators

The relationship between piezoelectric actuator displacement and voltage suffers from hysteresis and creep, which are nonlinear in nature and reduce positioning accuracy (Leang and Devasia, 2002). These non-linearities are explained by the misalignment of molecular dipoles, which constitute piezoelectric materials, and internal power dissipation within the piezoelectric actuators (Vautier and Moheimani, 2005).

Piezoelectric actuators exhibit hysteresis when driven by voltage amplifiers. Hysteresis depends on a combination of both the currently applied voltage as well as the previously applied voltage (Kuhnen and Janocha, 1998). In practice, it means that, for similar values of applied piezoelectric voltage, the piezoelectric actuator has a variety of displacement values, and this cannot be described with linear models. Hysteresis should not be confused with phase lag which is a linear effect, while hysteresis is a nonlinear effect.
The hysteresis loop has sharp reversal peaks at the extrema, while the phase lag has the shape of an ellipse with more rounded shape at the extrema (Moheimani and Fleming, 2006).

Figure 2.3 illustrates a hysteresis loop when the piezoelectric stack actuator AE0505D44H40 from NEC is driven by a voltage amplifier. For the displacement range of 11.54μm, the maximum hysteresis is 1598 nm for a driving frequency of 10 Hz. Therefore the maximum hysteresis is about 14% of the displacement range.

![Hysteresis loop between applied voltage and displacement](image)

**Figure 2.3: Hysteresis loop between applied voltage and displacement**

The overall voltage to displacement relationship of piezoelectric actuators is rate-dependent. In the literature, this effect is explained in two different ways. In some articles (Yanding et al., 2013, Smith et al., 2000, Vautier and Moheimani, 2005, Devasia et al., 2007) the hysteresis effect is assumed to be rate-dependent, while other authors (Croft and Devasia, 1998, Wu and
Zou, 2007, Adriaens, 2000) have assumed that hysteresis is rate-independent and that the rate-dependent behaviour is because of the linear dynamics of piezoelectric actuators. The second explanation is more convenient for modeling purposes, as it allows the linearity and non-linearity of piezoelectric actuators to be treated separately.

Another form of non-linearity in piezoelectric actuators is called creep. It is the result of the remnant polarization which continues to change after the applied signal reaches its final value and typically is an issue at low frequencies (Meeker, 1996).

Figure 2.4 shows the AE0505D44H40 piezoelectric actuator displacement response to a 40 V input step signal at time 0s. It can be seen that it takes 4.5 seconds to reach the final value (11.5 µm). The displacement continues to change after the input voltage reaches its final value (in this case 40V).

![Figure 2.4: Creep in a piezoelectric actuator](image)

The total error in the displacement of piezoelectric actuators is the result of both hysteresis and creep effects. Hysteresis and creep together can cause as
much as 40% error in the position of piezoelectric actuators (Barrett and Quate, 1991).

2.3 Control of displacement

Because hysteresis and creep affect the positioning accuracy of the piezoelectric actuator, to achieve high accuracy the piezoelectric nonlinearities have to be compensated for. This section reviews different approaches to controlling the displacement of piezoelectric actuators with this compensation.

2.3.1 Displacement feedback voltage drive (sensor-based control)

In sensor-based control, the displacement of a piezoelectric actuator is measured (by a variety of sensors such as strain gauges, capacitive sensors, optical sensors and eddy current sensors) and a feedback controller is used to equalize the desired displacement with the actual displacement measured by the sensor (Figure 2.5).

![Figure 2.5: Displacement feedback control](image)

Proportional-integral-derivative (PID) controllers are very common for this purpose. Although they provide accurate positioning, especially at low frequency, stability is an important issue in feedback controllers and at high
frequencies, because of the resonance frequency of the actuator, the system may become unstable if the feedback gains are too high. Adaptive control (Shieh et al., 2004), (Tan and Baras, 2005) and robust control (Pare and How, 1998, Jonsson, 1998) are two common methods which have been utilized to tackle this issue. Other issues with feedback controllers are the physical size of sensors, sensor-induced noise, high cost, tracking lag and limited bandwidth (Fleming and Leang, 2008).

2.3.2 Feedforward voltage drive (model-based control methods)

Feed-forward control can be used to compensate for the non-linearities of a piezoelectric actuator using an inverse mathematical model of the system dynamics (Wang et al., 2011, Zhao and Jayasuriya, 1995, Krejci and Kuhnen, 2001). Two approaches are common to find the inverse model. In the first approach, the non-linearities of a piezoelectric actuator are modelled, e.g., the Preisach model (Ge and Jouaneh, 1995) or the Maxwell resistive model (Goldfarb and Celanovic, 1997b), and then the model is inverted. In the second approach, the inverse model (as an example, using Preisach techniques) is directly calculated and utilized to drive the piezoelectric actuator (Croft et al., 2001). Generally the second approach requires less effort because there is no need to find the original model. In such feedforward methods, the inverse model is cascaded with the piezoelectric actuator.

Compared to the feedback displacement control method, this method has reduced hardware complexity since displacement sensors are not required. However, the parameter sensitivity and modelling complexity are the main
challenges in designing feedforward controllers. The next section provides an overview of piezoelectric modelling.

### 2.3.2.1 Modelling piezoelectric actuators

A well-known model of piezoelectric actuators is that described by the standards committee of the IEEE (Meeker, 1996), which presents linear equations to describe piezoelectric behaviour. These equations are represented in matrix notation as follows:

\[
S_p = s_{pq} T_q + d_{kp} E_k \tag{2.1}
\]

\[
D_k = d_{kp} T_p + \epsilon_{ki} E_i \tag{2.2}
\]

where \(S\) is the strain, \(s\) is the compliance matrix, \(T\) is the stress, \(d\) are the piezoelectric material constants, \(E\) is the electric field, \(D\) is the electrical displacement (charge per unit area) and \(\epsilon\) is the permittivity. The subscripts \(p, q = 1,2,3,4,5,6\) and \(k\) and \(i = 1,2,3\) represent different directions within the material coordinate system as shown in Figure 2.6.

![Figure 2.6: Directions within the material coordinate system](image-url)
Equation (2.1) describes the inverse piezoelectric effect and Equation (2.2) describes the direct piezoelectric effect as described in Section 2.1. These equations represent both the strain and the electric displacement as being linearly dependent on the stress and the electric field. These linear equations, however, have proved inadequate in many instances due to the significant nonlinearities such as hysteresis and creep in piezoelectric actuators. If these non-linearities are not considered in modelling, a large displacement error is generated. The Maxwell resistive capacitor (MRC) model (Goldfarb and Celanovic, 1997a) and the Preisach model (Ge and Jouaneh, 1995) are two important nonlinear mathematical modelling methods.

An electromechanical model, which is shown in Figure 2.7, is proposed by Goldfarb and Celanovic (1997a and 1997b). It describes both the mechanical and electrical aspects of piezoelectric actuators, and the relationship between them.

![Figure 2.7: The electromechanical model of a piezoelectric actuator](image-url)
According to Figure 2.7, the behaviour of the piezoelectric actuator is described by:

\[ q_p = T_p d + C_p v_t \]  \hspace{1cm} (2.3)

\[ v_{in} = v_{mrc} + v_t \]  \hspace{1cm} (2.4)

\[ v_{mrc} = \text{MRC}(q_p) \]  \hspace{1cm} (2.5)

\[ F_t = T_p v_t \]  \hspace{1cm} (2.6)

\[ m\ddot{d} + b\dot{d} + k_p d = F_t + F_{ext} \]  \hspace{1cm} (2.7)

where \( q_p \) is the overall charge across the piezoelectric actuator, \( T_p \) is the electromechanical transformer ratio, \( d \) is the displacement of the actuator, \( C_p \) is the linear capacitor, \( v_t \) is the back-emf from the mechanical side, \( v_{in} \) is the applied voltage, Maxwell Resistive Capacitor (MRC) models the non-linearity behaviour of the piezoelectric actuator, \( v_{mrc} \) is the voltage across the MRC unit, \( F_t \) is the transduced force from the electrical domain, \( m \), \( b \), and \( k_p \) are the mass, damping, and stiffness of the actuator, and \( F_{ext} \) is the force imposed from the external mechanical source.

The behaviour of molecular dipoles, from which the piezoelectric materials are formed, is the main reason for the non-linear behaviour of piezoelectric actuators. When a sufficient electric field is applied to a piezoelectric actuator, the direction of each dipole will be changed to align with the electric field. The movement of each dipole depends on the initial orientation and the strength of the electric field. The non-linear behaviour is due to the change in the directions of all the dipoles in the piezoelectric actuator.
The MRC method is a linear parametric method of modelling the non-linear hysteresis behaviour of a piezoelectric actuator. In this method, each dipole is modeled by an elasto-slide element. Each elasto-slide element, which consists of an ideal spring coupled to a massless block, has a hysteretic behaviour and the summation of all the elasto-slide elements models the hysteresis behaviour of piezoelectric actuators. Therefore this model is a piece-wise approximation of the hysteresis and is based on physical principles (Goldfarb and Celanovic, 1997b).

In the MRC model, because the hysteresis is modeled by a combination of elements, the number of parameters is relatively large. Therefore, such a model is not very suitable for controller design purposes.

In 1995, the Preisach phenomenological model, originally devised by the German physicist in 1935 to describe hysteresis phenomena in magnetic systems (Preisach, 1935), was used to model piezoelectric actuators (Ge and Jouaneh, 1995) and is now widely accepted (Boukari et al., 2011, Liaw and Shirinzadeh, 2011, Zhang et al., 2009).

The relation between the displacement of the piezoelectric actuator, $d(t)$, and the voltage across the piezoelectric actuator, $V_p(t)$, is expressed by:

$$d(t) = \iint_{\alpha \leq \beta} \mu(\alpha, \beta) \gamma_{\alpha\beta}[V_p(t)] d\alpha d\beta$$  \hspace{1cm} (2.8)

where $\mu(\alpha, \beta)$ is a weighting function of the Preisach model and $\gamma_{\alpha\beta}$ are the elementary hysteresis operators with switching values $\alpha$ and $\beta$ shown in Figure 2.8.
Because the hysteresis operators $\gamma_{\alpha\beta}$ are two position relays, the double integration in Equation (2.8) can be represented as a summation of weighted relays connected in parallel as shown in Figure 2.9.

The Preisach model fails to model asymmetric hysteresis loops and this has been addressed in a variation, the Prandtl-Ishlinskii model (Zareinejad et al., 2010). Other extended and modified versions of Preisach model have been

Recently a variety of black box models has been used in the modelling of piezoelectric actuators. They include Multi-Layer Perceptrons (MLPs) (Dong et al., 2008, Yang et al., 2008, Zhang et al., 2010), Radial Basis Function Networks (RBFNs) (Dang and Tan, 2007), Recurrent Fuzzy Neural Networks (RFNNs) (Lin et al., 2006b), wavelet neural networks (Lin et al., 2006a), Nonlinear Auto-Regressive Moving Average models with eXogenous inputs (NARMAX models) (Deng and Tan, 2009) and non-standard Artificial Neural Networks (ANNs) (Hwang et al., 2001).

Contemporary, universal approximators are available with proven ability in system modeling; hence, the modeling technique is not a critical issue. In this thesis, Nonlinear Auto-Regressive models with eXogenous inputs (NARX models), commonly used for classical system identification (Nelles, 2001), are utilized.

According to the NARX structure, for a single input-single output system such as a one-dimensional piezoelectric actuator (Nelles, 2001),

\[
y(t) = f(u(t-t_d), u(t-t_d-T_s), ..., u(t-t_d-r_u T_s), y(t-T_s),
\]
\[
y(t-2T_s), ..., y(t-r_y T_s)),
\]

where \( u \) is input to the model (applied voltage), \( y \) is output of the model (displacement), \( T_s \) is sampling time, \( t_d \) is delay time, and \( r_u \) and \( r_y \) are input and output orders respectively. \( f \) can be approximated by any nonlinear function. More explanation is provided in Chapter 4.

The disadvantage of NARX models is that the current output error depends on previous output errors and hence the estimation error may accumulate.
Error accumulation is a serious problem for a NARX model and in an extreme situation the model can become unstable (Nelles, 2000). More detail about this issue is discussed in Chapter 3.

2.3.3 Charge drive

Comstock (1981) showed that if a piezoelectric actuator is driven by charge instead of voltage, the hysteresis loop is reduced significantly. Figure 2.10 shows a simplified electromechanical model (Goldfarb and Celanovic, 1997a and Moheimani and Vautier, 2005) of a piezoelectric actuator which is used to explain why the hysteresis loop is reduced by charge driven methods. The model consists of a voltage source, $V_{si}$, which is the strain-induced voltage, a piezoelectric capacitance $C_p$, a nonlinear impedance $\Delta$ which models the hysteresis and a parallel resistor $R_L$ which models the charge leakage. As described in Goldfarb and Celanovic (1997a) the displacement of the piezoelectric actuator is proportional to the charge across the capacitor $q_p$.

![Figure 2.10: Lumped parameter model of a piezoelectric actuator](image-url)
Using the model in Figure 2.10 as a starting point, initially the effect of $R_L$ is ignored to simplify the discussion. Because of the nonlinear impedance $\Delta$, when the piezoelectric actuator is driven by a voltage source, the applied voltage is not linearly related to the voltage or the charge across the capacitor $q_p$. Thus the effect of hysteresis can clearly be noted between the applied voltage and the output displacement.

In contrast, if the voltage amplifier is replaced by a charge amplifier, the charge $q_p$ across the capacitor $C_p$ is equal to the applied charge $q_{in}$ and, because the nonlinear impedance $\Delta$ does not have any effect on the charge $q_p$, the hysteresis is removed and the output displacement is linear with the applied charge $q_{in}$. However, the DC impedance of the load, $R_L$, causes parasitic charge leakage which limits the application of the charge amplifier at low frequencies. To determine this, the model may be simplified by neglecting the effects of the nonlinear impedance and $V_{si}$ (Clayton et al., 2008). Hence the simplified model will only have the capacitor $C_p$ in parallel with $R_L$. Therefore the transfer function between $q_{in}$ and $q_p$ is given by:

$$\frac{q_p(s)}{q_{in}(s)} = \frac{R_L C_p s}{R_L C_p s + 1}$$

Equation (2.10) is a high-pass filter with a cut-off frequency $f_c = \frac{1}{2\pi R_L C_p}$. It means that at low frequencies $q_p(s)$ is not equal to $q_{in}(s)$. Therefore the charge amplifier has poor low frequency performance because of the parasitic charge leakage of the piezoelectric actuator (Spiller et al., 2011).
2.3.3.1 The implementation of charge amplifiers

Despite the low frequency limitations, many approaches have been used to implement charge amplifiers since they were first used in 1981 by Comstock. The aim of this section is to review the literature on the implementation of charge amplifiers. Charge amplifiers are broadly classified as either capacitor insertion, time controlled current amplifiers and sense capacitor.

2.3.3.2 Capacitor insertion

The capacitor insertion method proposed by (Kaizuka and Siu, 1988) is one of the simplest ways to reduce hysteresis. In this method, a capacitor is inserted in series with a piezoelectric actuator and a voltage source drives the series combination. It has been shown that the charge across the piezoelectric actuator is proportional to the voltage source and, because the charge is also proportional to the displacement (Newcomb and Flinn, 1982, Fleming and Moheimani, 2005), the displacement source is proportional to the voltage.

To explain this method more completely, the piezoelectric actuator is considered a non-linear capacitor (Goldfarb and Celanovic, 1997a), meaning that the capacitance of the piezoelectric actuator is changing over time. A change in the capacitance will cause a change in the charge across the piezoelectric actuator and this change is the main reason for the hysteresis loop. The sensitivity factor $\frac{dq_p}{dC_p}$ quantifies this effect. Where $q_p$ is the charge across a piezoelectric actuator and $C_p$ is the capacitance of the actuator. A reduction in the sensitivity factor means that the charge on the piezoelectric actuator is less sensitive to the change of the capacitance of the piezoelectric actuator with a consequent reduction in the hysteresis loop.
As shown in Minase et al. (2010), when the piezoelectric actuator is driven by a voltage amplifier the sensitivity factor is

\[
\frac{dq_p}{dC_p} = V_p,
\]  

(2.11)

where \( V_p \) is the voltage across the actuator. While, when it is driven by the capacitor insertion method, it is

\[
\frac{dq_p}{dC_p} = \left( \frac{C_{\text{series}}}{C_{\text{series}} + C_p} \right) V_p,
\]

(2.12)

where \( C_{\text{series}} \) is the inserted capacitor. It can clearly be seen that when the capacitor insertion method is used the sensitivity factor is reduced by a factor

\[
a = \frac{C_{\text{series}}}{C_{\text{series}} + C_p}.
\]

(2.13)

Therefore this reduction in the sensitivity factor causes a reduction in the hysteresis loop. In other words, the series capacitor operates as a charge regulator across the piezoelectric actuator.

However, in the capacitor insertion method the applied voltage is divided between the piezoelectric actuator and the inserted capacitor. As a result, less voltage will be applied across the piezoelectric actuator, reducing the maximum displacement range for a given voltage. In other words, to get the same displacement range a higher voltage source is required.
2.3.3.3 Time controlled current amplifier

A current amplifier can be used for charge control because the charge on the piezoelectric actuator is equal to the integral of the applied electric current. This method has been described by a number of authors (Newcomb and Flinn, 1982, Dorlemann et al., 2002, Fleming and Moheimani, 2003, Ru et al., 2008, Spiller et al., 2011). Figure 2.11 provides a simplified diagram of a basic current amplifier. The closed-loop with high gain, \( k \), equalizes the reference signal \( V_{\text{ref}} \) with the sensing voltage \( V_s \). Therefore, the load current, \( I_L \), is given by

\[
I_L = \frac{V_{\text{ref}}}{R_s},
\]

(2.14)

where \( R_s \) is a sensing resistor. In other words, this is a current amplifier with gain \( \frac{1}{R_s} \).

![Figure 2.11: Basic current amplifier](image)
Newcomb and Flinn (1982) implemented a charge regulator based on a current amplifier. They used two constant current sources to regulate the current on the piezoelectric actuator as shown in Figure 2.12. The current source $I_1$ provides a positive current on the piezoelectric actuator, which causes extension of the piezoelectric actuator, while $I_2$ applies a negative current which causes contraction. The voltage $V_{in}$ controls the switch for applying either a positive or negative current. The amount of charge on the piezoelectric actuator can be regulated by controlling the time interval of the positive and negative $V_{in}$.

![Figure 2.12: Simplified diagram of current source piezo regulation (Newcomb and Flinn, 1982)](image)

Furutani and Iida (2006) improved the Newcomb and Flinn (1982) method by using current pulse modulation to control the charge on the piezoelectric actuator as shown in Figure 2.13. It consists of current sources, current sinks and controlling switches. The switches control the current going to the piezoelectric actuator. At any one time only one switch is on. Current sources are used to increase the piezoelectric actuator charge whereas sinks decrease the charge. To generate rapid displacements, large sources are used.
and for small and accurate displacements, small current sources or sinks are used (Furutani and Iida, 2006). Compared to Newcomb and Flinn (1982), this method is more costly because more current sources are needed.

Figure 2.13: Schematic of current pulse driving method (Furutani and Iida, 2006)

Ru (2008) also modified the basic current amplifier, utilizing a switch with two sensing resistors. For quick positioning and fast response, a smaller sensing resistor is selected by the switch, allowing more charge to the piezoelectric actuator, while for precise positioning a larger resistor is selected.

None of these methods address the problem of voltage drift caused by dielectric leakage of the piezoelectric actuator and current leakage of the current source. They can therefore only work for a limited time.

2.3.3.4 Sense capacitor

Figure 2.14 shows a diagram of an ideal charge amplifier with a sensing capacitor with capacitance $C_s$. A high gain feedback loop is used to equalize the voltage across the sensing capacitor $V_s$ with the reference voltage $V_{ref}$. 
In the Laplace domain, the load current $I_L(s)$ is equal to $V_{\text{ref}}(s)C_s$. Because $I_L(s)$ is also equal to $q_L(s)s$, then

$$q_L(s) = C_s V_{\text{ref}}(s),$$

(2.15)

where $q_L$ is the charge across the piezoelectric actuator. In other words, this is a charge amplifier with gain $C_s$.

In practice, because the sensing capacitor is not ideal, and due to dielectric leakage of the piezoelectric actuator, the current $I_L$ will contain a DC component. As the sensing capacitor $C_s$ integrates the current $I_L$, the voltage and the charge across the sensing capacitor will drift. The output voltage, $V_o$, will therefore drift and finally saturate at the maximum output voltage after a period of time.

The main complexity in designing this type of charge amplifier is to solve this drift problem. The two common methods to practically implement charge amplifiers while removing drift will be described in the following.
The first method proposed by Comstock (1981) is shown in Figure 2.15. It consists of an ideal charge amplifier and an initialization circuit which is periodically activated by a timer. The initialization circuit uses three switches to discharge the actuator and return the DC voltage across the actuator to zero in order to avoid saturation.

As can be seen from Figure 2.15 when switch 1 is closed, the sensing capacitor will be short circuited and the voltage across it will be set to zero. This discharges the sensing capacitor and avoids saturation.

**Figure 2.15: Charge controlled piezoelectric actuator (Comstock, 1981)**

In the circuit shown in Figure 2.15, the timer drives switch 2 which connects the op-amp output to the inverting input through feedback resistor $R_2$. Because $R_1$ is connected to ground, the amplifier gain will be set to $Gain = 1 + \frac{R_2}{R_1}$ during the initialization. Switch 3 is used to divert the op-amp input
from the normal input to the damped oscillator input. This switch is closed
during the initialization. The damped oscillator produces a down-ramped
oscillation at low frequency, consisting of a low frequency sine wave
modulated with a down ramp signal. The applied voltage oscillates around
the mid-range displacement of the actuator. In this manner, eventually the
system will settle at mid-range, point A in Figure 2.16, and the system will
switch to normal mode. This point is equivalent to zero displacement and
zero charge, which is the starting point at which time it can be switched to
normal mode (Comstock, 1981).

Figure 2.16: Hysteresis of charge controlled piezo (Comstock, 1981)

Main et al. (1995) also used the Comstock (1981) method with an additional
current buffer at the output of the amplifier to improve the amplifier
bandwidth. In both methods, the sudden discharge of the sensing capacitor
through switch 1 causes output signal distortion at low frequencies and also
undesirable high frequency disturbance (Fleming and Moheimani, 2003).
The second method to avoid drift is to employ a DC feedback path, as shown in Figure 2.17. As with the ideal charge amplifier, the feedback loop is utilized to equalize the voltage across the sensing capacitor with the reference voltage $V_{\text{ref}}$. The DC feedback path consists of resistors $R_1$ and $R_2$ which model the current leakage of the input terminal of the op-amp and current leakage of the sensing capacitor and piezoelectric actuator. In practice, the values of these resistors are not known. Therefore these resistors are replaced with additional resistance in order to manage the sensing voltage drift (Fleming and Moheimani, 2005).

![Figure 2.17: Circuit diagram of a charge amplifier with DC feedback path](image)

Because of the DC feedback path, the charge amplifier has a different transfer function from the ideal charge amplifier. The transfer function between the load charge and the reference voltage is
where $q_L(s)$ is the load charge, $C_s$ is the sensing capacitor and $C_p$ is the linear capacitance of the piezoelectric actuator. Equation (2.16) is a high pass filter with cut off frequency

$$
\omega_c = \frac{1}{R_tC_p}
$$

At frequencies well below $\omega_c$, the impedance of resistors $R_1$ and $R_2$ are much smaller than the impedances $C_p$ and $C_s$. Therefore, $C_p$ and $C_s$ are negligible. Hence

$$
V_p(s) = \frac{R_1}{R_2} V_{ref}(s)
$$

which is a voltage amplifier with gain $\frac{R_1}{R_2}$. Therefore at low frequencies because of the DC feedback path, the system is a voltage amplifier which cannot compensate the hysteresis loop of the piezoelectric actuator and hence there is a nonlinear relation between the reference signal and the displacement.

Many improvements have been made to the low frequency performance of the charge amplifier with DC feedback path. Fleming and Moheimani (2004) developed a hybrid DC-accurate charge amplifier for linear piezoelectric positioning. They added a voltage feedback loop to improve the low frequency response. However, compared to the charge amplifier
with DC feedback path the new voltage feedback limits the operational bandwidth. Their system works as a voltage amplifier at low frequencies and therefore it is still a non-linear system at low frequencies.

Yi and Veillette (2005) introduced a charge control system, utilizing an inverting amplifier in a feedback loop to drive a piezoelectric actuator. A lead compensator is also used to ensure the stability of the feedback loop. Moreover, a DC feedback path is utilized to eliminate the drift. However, the minimum frequency which the system can deal with as a charge controller is limited by the DC feedback path. This system is also unable to drive a grounded-load.

Fleming and Moheimani (2005) modified the charge amplifier with DC feedback path to be suitable for driving a grounded-load such as a piezoelectric tube actuator. As with Yi and Veillette (2005), they alleviated the drift problem by using a DC feedback path which limits the minimum frequency of the charge control operation. Given that a piezoelectric actuator is driven by an ideal charge amplifier, because of the voltage drop across the sensing capacitor, the maximum displacement achievable will be reduced.

2.4 Conclusion

The literature suggests that the main issue with designing a charge amplifier is drift, which can cause saturation of the load. While a number of authors have neglected the drift problem others have utilized additional analog circuitry to avoid it. In spite of this, additional analog circuitry often prevents the wide acceptance of charge drive methods (Kaizuka and Siu, 1988) and increases their implementation complexity and cost. Furthermore, this additional circuitry added to charge drive methods tackle the limitations
of operation, results in limitation of the lowest frequency of operation. As a consequence, charge amplifiers have not been widely applied.

As proposed charge methods adopted to date were implemented using analog circuitry. An opportunity exists to investigate the potential of a digital charge amplifier. A significant advantage of digital-based systems is their low-cost. Furthermore, digital systems should eliminate problems of drift due to ageing and temperature effects, as well as problems of reproducibility due to component variations arising from manufacturing tolerances that plague analog processing techniques. Most importantly, digital systems will make it possible to incorporate charge control with other displacement control methods, such as model-based control. Thus there is an opportunity to explore a digitally implemented charge amplifier within a hybrid system that can integrate charge control with other displacement controls such as model-based controls to achieve new levels of performance.

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3 Digital Charge Amplifier

3.1 Introduction

As mentioned in the previous chapter, analog charge amplifiers are expensive to implement due to the complexity of the analog circuitry. This chapter presents a novel digital charge amplifier (DCA) which is the foundation development of this thesis. The design and analysis of the DCA is presented and it is shown that the DCA can reduce hysteresis and improve the linearity of piezoelectric actuators. In order to evaluate the performance of the DCA, the behaviour of the DCA in respect of linearity, frequency response and voltage drop are presented in the experimental results.

This easily-implemented, digital charge drive approach opens up the possibility of integration with other displacement control techniques such as model-based methods to improve the performance of the displacement controller.
3.2 Implementation and analysis of an innovative digital charge amplifier for hysteresis reduction in piezoelectric stack actuators

# Statement of Authorship

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## Author Contributions

By signing the Statement of Authorship, each author certifies that their stated contribution to the publication is accurate and that permission is granted for the publication to be included in the candidate's thesis.

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Implementation and analysis of an innovative digital charge amplifier for hysteresis reduction in piezoelectric stack actuators

Mohsen Bazghaleh, Steven Grainger, Ben Cazzolato and Tien-Fu Lu

Abstract. Smart actuators are the key components in a variety of nanopositioning applications, such as scanning probe microscopes and atomic force microscopes. Piezoelectric actuators are the most common smart actuators due to their high resolution, low power consumption and wide operating frequency but they suffer hysteresis which affects linearity. In this paper, an innovative digital charge amplifier is presented to reduce hysteresis in piezoelectric stack actuators. Compared to traditional analog charge drives, experimental results show that the piezoelectric stack actuator driven by the digital charge amplifier has less hysteresis. It is also shown that the voltage drop of the digital charge amplifier is significantly less than the voltage drop of conventional analog charge amplifiers.

Keywords: Piezoelectric actuators; digital implementation; hysteresis; charge control.

1 Introduction

Piezoelectric stack actuators have been widely used in many applications ranging from fuel injection systems (MacLachlan et al., 2004) to vibration cancellation in disk drives (Ma and Ang, 2000). Aside from being compact in size, they are capable of nanometre resolution in displacement, have high stiffness, provide excellent operating bandwidth and high force output.
(Devasia et al., 2007). A piezoelectric stack actuator is assembled using multiple layers of piezoelectric materials which are placed in series and wired in parallel as shown in Figure 1. The sign of the applied voltage, $V$, determines whether the actuator should expand or contract.

![Piezoelectric stack actuator construction](image)

**Figure 1: Piezoelectric stack actuator construction**

Typically piezoelectric stack actuators are driven by voltage, however voltage driven methods suffer from hysteresis and creep which are nonlinear in nature and reduce the positioning accuracy (Leang and Devasia, 2002).

Creep is the result of the remnant polarization which continues to change after the applied signal reaches its final value and is normally an issue at low frequencies (Meeker, 1996). Hysteresis is due to the polarization of microscopic particles (Damjanovic, 2005) and depends on both the currently applied voltage as well as that previously applied (Kuhnen and Janocha, 1998). Physically it means that, for a similar voltage input, the piezoelectric actuator may have different displacement values. The relationship between applied voltage and displacement also changes subject to the amplitude and the frequency of the applied voltage, thus it is both non-linear and dynamic in nature (Adriaens, 2000).
A variety of approaches have been used to tackle the hysteresis of piezoelectric actuators. Feed-forward control uses an inverse-model, such as the Preisach model (Ge and Jouaneh, 1995) or Maxwell resistive model (Goldfarb and Celanovic, 1997b), to compensate for hysteresis. But due to uncertainties in the piezo parameters, the feedforward inverse-model technique can suffer from a lack of robustness. Another approach is displacement feedback control (Shieh et al., 2004) but this is limited by sensor-induced noise, high cost, additional complexity and limited bandwidth (Fleming and Leang, 2008).

Charge regulator and capacitor insertion techniques have also been used (Minase et al., 2010). The capacitor insertion method (Kaizuka and Siu, 1988) involves using a capacitor in series with the piezoelectric actuator and, while it is effective in dealing with hysteresis, significantly reduces the operating range because of the voltage drop across the capacitor. Charge regulation was first introduced by Comstock (1981) who showed that by regulating the charge across a piezoelectric actuator the hysteresis is reduced significantly. However a charge amplifier approach has historically been complicated and expensive to implement due to the complexity of the analog circuitry required to meet stringent performance targets (Fleming and Moheimani, 2005). Additionally, in the ideal charge amplifier, the load capacitor is charged up because of the uncontrolled nature of the output voltage and once the output voltage reaches the power supply rails, the charge amplifier saturates. To avoid this undesirable effect additional methods have been used which increases the implementation complexity (Fleming and Moheimani, 2004). Comstok (1981) used an initialization circuit to short circuit the sensing capacitor periodically; however, it causes undesirable disturbances at high frequencies. Fleming and Moheimani (2004) used an additional voltage feedback loop to improve the low...
frequency response. But it works as a voltage amplifier at low frequencies. Therefore the piezo still suffers from hysteresis at low frequencies.

In this paper, a new digital charge amplifier is presented. It significantly reduces the hysteresis effect of piezoelectric actuators. It is more cost effective than analog charge amplifiers and this new design significantly addresses the reduced voltage drop. The design and analysis of the digital charge amplifier are presented in Section 2. Experimental investigations are in Section 3 followed by a conclusion in Section 4.

2 Design and analysis

In a typical analog charge amplifier, sensing capacitors are used for integrating electric currents. However, since these capacitors are not ideal they suffer from dielectric charge leakage which is one of the reasons for drift and saturation of loads (Clayton et al., 2006). The main operating principle in the digital charge amplifier is to calculate the charge signal by digitally integrating the electric current passing the piezoelectric actuator. The sensing capacitor is replaced by a sensing resistor whose voltage drop can be measured to provide a signal from which the electric current can be deduced and thereby avoids the dielectric charge leakage of capacitors.

2.1 Digital charge amplifier configuration

Figure 2 shows the digital charge amplifier that forms the basis of this work. It consists of a voltage amplifier, digital to analog converter (DAC), analog to digital converter (ADC) and a digital signal processor (DSP). A sensing resistor is placed in series with the piezoelectric stack actuator.
The DSP unit calculates the charge across the piezoelectric actuator, \( q_p(t) \), by integrating the piezoelectric actuator current.

\[
q_p(t) = \int I_s(t) \, dt. 
\]  

(1)

Because of the ADC input resistance \( R_{\text{inputADC}} \), the piezoelectric actuator current is given by

\[
I_s(t) = \frac{V_s(t)}{R_s || R_{\text{inputADC}}}. 
\]  

(2)

where \( V_s(t) \) is the sensing voltage across the sensing resistor.

Substituting (2) into (1) gives

\[
q_p(t) = \int \frac{V_s(t)}{R_s || R_{\text{inputADC}}} \, dt. 
\]  

(3)

The input resistance is in parallel with sensing resistor \( R_s \) so the total resistance is
Substituting (4) in (3) gives

\[ q_p(t) = \frac{1}{R_T} \int V_s(t) dt. \]  

Therefore the charge across the piezoelectric actuator is equal to the integral of the voltage across the sensing resistor divided by the total resistance. A high gain feedback loop is used to equate the actual charge with the desired charge.

### 2.2 Transfer function of the digital charge amplifier

The transfer function can now be derived. By considering a capacitor, \( C_L \), as a model of the piezoelectric actuator (Fleming and Moheimani, 2005, Huang et al., 2010), the transfer function between the output of the voltage amplifier \( V_o(t) \) and sensing voltage \( V_s(t) \) is:

\[ H(s) = \frac{V_s(s)}{V_o(s)} = \frac{R_T C_L s}{R_T C_L s + 1} \]  

By taking the Z transform, the discrete transfer function will be:

\[ H(z) = \frac{V_s(z)}{V_o(z)} = \frac{z^{-1}}{z - e^{-T_s C_L}} \]  

where \( T_s \) is the sampling time. Moreover, the transfer function of the digital charge drive algorithm is
\[ V_s(z) = k(q_{\text{desired}}(z) - \frac{T_s}{(z-1)R_T}V_s(z)) \]  \hspace{1cm} (8)

where \( k \) is the closed loop gain and \( q_{\text{desired}}(z) \) is the desired charge. Substituting (8) in (7) results in

\[ z - e^{\frac{T_s}{R_T C_L}}V_s(z) = k(q_{\text{desired}}(z) - \frac{T_s}{(z-1)R_T}V_s(z)). \]  \hspace{1cm} (9)

By considering \( V_s(z) = \frac{q_{\text{actual}}(z)R_T(z-1)}{T_s} \), where \( q_{\text{actual}}(z) \) is the actual charge across the piezoelectric actuator, the discrete transfer function from the input (desired charge) to the output (actual charge) is given by

\[ q_{\text{actual}}(z) = \frac{T k}{R_T} \frac{z - e^{\frac{T_s}{R_T C_L}} + \frac{T k}{R_T}}{z - e^{\frac{T_s}{R_T C_L}} + \frac{T k}{R_T}}. \]  \hspace{1cm} (10)

For a typical piezoelectric stack actuator with a capacitance of 3.4 µF using the digital charge amplifier with a 100 Ω sensing resistor and sampling frequency 40 kHz, the frequency response has zero dB gain with approximately zero phase-shift up to 10 kHz. This is the theoretical bandwidth. In Section 3 the practical bandwidth will be shown.

3 Experimental investigations

3.1 Experimental setup

The proposed technique was validated experimentally by using a piezoelectric stack actuator AE0505D44H40 from NEC. The displacement
of the piezoelectric stack actuator was measured using a strain gauge from Vishay Electronics (EA-06-125TG-350), which is solely used to evaluate the performance of the DCA. All digital algorithms, estimation and control functions were written in Matlab/Simulink then compiled for a dSPACE DS1104 development platform. Matlab/Simulink was also used for actuator characterization (Figure 3).

Figure 3: The experimental setup

3.2 DCA linearity results

Figure 4 illustrates the improvement in linearity offered by the new digital charge amplifier compared to a standard voltage amplifier. Figure 4a shows the hysteresis loop when the piezo is driven by a voltage amplifier and Figure 4b shows the results when the piezo is driven by the DCA. It can be seen that the hysteresis loop is significantly reduced in the DCA. For a displacement range of 11.54μm, the digital charge amplifier has maximum hysteresis of 144 nm while it is 1598 nm for the voltage amplifier for a
driving frequency of 10 Hz. Therefore, the digital charge amplifier has reduced the hysteresis by 91%.

Figure 5 illustrates the displacement trajectories and tracking errors under voltage drive and DCA. A 5 Hz triangle wave with three different displacement ranges (4 µm, 8 µm and 20 µm) were chosen which correspond to 10%, 20% and 50% of the maximum displacement range.

Table 1 shows a comparison of the maximum absolute errors for voltage drive and DCA. For displacement range of 4 µm, 8 µm and 20 µm, the voltage-driven displacement errors were reduced by 77.15%, 89.54% and 91.14% using the DCA.

Table 1: Comparison of errors between the voltage drive and the DCA

<table>
<thead>
<tr>
<th>Displacement range</th>
<th>Voltage drive</th>
<th>DCA</th>
<th>Reduction</th>
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<tr>
<td>4 µm</td>
<td>3.35%</td>
<td>0.76%</td>
<td>77.15%</td>
</tr>
<tr>
<td>8 µm</td>
<td>5.02%</td>
<td>0.52%</td>
<td>89.54%</td>
</tr>
<tr>
<td>20 µm</td>
<td>7.18%</td>
<td>0.64%</td>
<td>91.14%</td>
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Figure 4: Experimental hysteresis loop of a piezoelectric stack actuator AE0505D44H40 subjected to a 10 Hz sine wave driven by a) a voltage amplifier and b) DCA.
Figure 5: The percentage of displacements and errors for 5 Hz triangle waves with 4, 8 and 20 µm displacement ranges.
3.3 Frequency response of the output displacement to input charge

Figure 6 illustrates experimentally measured displacement versus charge at different frequencies. This shows that at higher frequencies the area within the loop between charge and displacement increases. This loop is not hysteresis but rather a linear phase shift. Figure 7 illustrates the frequency response of displacement output to charge input for the stack piezo. It can be seen that for frequencies higher than 100Hz the phase lag starts to increase, which makes it undesirable for higher frequency tracking applications. As described by Goldfarb and Celanovic (1997a) the relation between the charge and the displacement of the piezo introduces a phase lag as shown in Figure 7 from the nature of the piezoelectric actuator itself.

3.4 Low frequency limitation of the DCA

Because of the RC circuit formed by the sensing resistor and the piezo, at low frequencies the voltage drop across the sensing resistor decreases significantly. This causes a reduction in the signal-to-noise ratio (SNR) which is shown in Figure 8. To make the data comparable, the displacement range is set to be ±5 μm. Therefore the accuracy of measured charge decreases with decreasing frequency, which causes an increase in the error of the output displacement. As shown in Figure 8, increasing the sensing resistance can increase the voltage across the sensing resistor and therefore improve the SNR. However, increasing the sensing resistor can cause the sensing voltage to reach the maximum ADC input voltage at lower frequencies and therefore limits the bandwidth of the DCA.
Figure 6: Measured displacement versus charge for a sinusoidal input of 1 Hz, 20 Hz, 100 Hz and 500 Hz.
Figure 7: Measured frequency response of the output displacement to input charge
Figure 8: Signal to noise ratio (SNR) of the sensing voltage for $R_s$ equal to 50 $\Omega$ (Red), 500 $\Omega$ (Green) and 5 K$\Omega$ (Black)

3.5 Voltage drop

The voltage drop across any sensing element limits the piezoelectric actuator voltage range. Table 2 compares the maximum voltage drop for two hysteresis linearization techniques from the literature that have been evaluated experimentally (a grounded charge amplifier (Fleming and Moheimani 2005, Minase et al., 2010), capacitor insertion (Kaizuka and Siu, 1988, Minase et al., 2010)) and the DCA at different frequencies. To make the data comparable, the displacement range is set to be $\pm$5$\mu$m. It can be seen that at lower frequencies the DCA voltage drop is significantly smaller than the voltage drop of the grounded charge amplifier or capacitor insertion method, thus maximizing the displacement that can be achieved for a given supply voltage.
### Table 2: Comparison of voltage drop and percentage of voltage drop using three alternative methods at four different frequencies

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Grounded charge amplifier ($C_s = 30 , \mu F$)</th>
<th>Capacitor insertion ($C_s = 1 , \mu F$)</th>
<th>DCA ($R_s = 50 , \Omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 Hz</td>
<td>2.72(12%)</td>
<td>51.42(72%)</td>
<td>0.01(0.05%)</td>
</tr>
<tr>
<td>1 Hz</td>
<td>2.41(11%)</td>
<td>52.38(72%)</td>
<td>0.02(0.1%)</td>
</tr>
<tr>
<td>10 Hz</td>
<td>2.48(11%)</td>
<td>53.28(72%)</td>
<td>0.25(1.2%)</td>
</tr>
<tr>
<td>75 Hz</td>
<td>2.32(11%)</td>
<td>53.53(72%)</td>
<td>1.93(8.8%)</td>
</tr>
</tbody>
</table>

#### 3.6 Choosing an appropriate sensing resistor

Choosing the appropriate sensing resistor is an important task when using the DCA. The sensing resistor should not be so small that the noise on the measured charge increases significantly and also it should not be so large that $V_s$ becomes more than the input ADC limitation at the required bandwidth. The value of the sensing resistor should be set for each application. To select the best value for $R_s$, the maximum displacement at the maximum frequency should be considered. Then $R_s$ should be calculated such that the sensing voltage will not be greater than the maximum ADC voltage. Using the highest possible sensing resistor increases the SNR at low frequencies. As an indicator, if the maximum displacement and the maximum frequency for a particular application are 25 $\mu m$ and 100 Hz respectively, then $R_s$ should be set to 50 $\Omega$ to avoid saturation on the ADC and if the acceptable SNR is taken as 30 dB then from Figure 8, the minimum operational frequency will be 3 Hz.

### 4 Conclusion

An innovative digital charge amplifier has been introduced to reduce the non-linear behaviour of a piezoelectric actuator. For the experiments
conducted, it was found that the charge amplifier reduced the hysteresis by 91% at 10 Hz. Compared to a grounded charge amplifier, the DCA has less voltage drop, does not need gain tuning and is far more cost effective. This easily implemented, digital charge drive approach opens up the possibility of integration with other displacement control methods such as model-based methods to improve the performance of the displacement controller which will be investigated in future work.

References


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4 Non-linear Modelling of Piezoelectric Actuators

4.1 Introduction

The DCA, which has been presented in the previous chapter, shows some improvement in performance compared to traditional analog charge drives. However, the principal difficulty in using this technique is drift of the displacement output. In the digital charge amplifier, the analog to digital converter (ADC) is not ideal and suffers from offset voltage and drift. This issue, together with dielectric leakage of the piezoelectric actuator, causes a bias voltage across the sensing resistor and due to the integration of the voltage bias the measured charge will drift, resulting in miscalculation of the actual charge across the piezoelectric actuator. The output voltage will also drift until it reaches the power supply rails.

In order to find a drift-free method, which later will be integrated with the DCA to remove the displacement drift of the DCA, non-linear modelling of piezoelectric actuators is presented in this chapter.

There are many different methods for modeling piezoelectric actuators. However, universal approximators are available with proven ability in
system modeling; hence, the modeling technique is no longer such a critical issue. In this chapter, Nonlinear Auto-Regressive models with eXogenous inputs (NARX), which is commonly used for classical system identification, is utilized. In addition, to reduce complexity and increase the accuracy of the NARX model, appropriate inputs to the model must be selected.

To assess the significance of the possible inputs, a subtractive clustering derived neuro-fuzzy network is used which provides visibility of the internal operation and lets the researcher understand the role of each input in the model in comparison with other inputs. This method is very helpful in finding the most appropriate input arrangement. The first paper in this chapter, entitled “Fuzzy Modeling of a Piezoelectric Actuator”, is focused on this assessment.

In order to use the NARX model for position estimation, the accuracy of the estimated displacement is an important factor. One of the phenomena that reduce the accuracy of modelling is error accumulation. Because piezoelectric actuators are dynamic systems, the current value of their displacement is dependent on previous values of displacement. Therefore in dynamic models, in addition to piezoelectric-voltage-based signals, previously estimated values of displacement are also used as inputs to the model. Therefore, the current output error depends on previous output errors and hence the estimation error may accumulate. Error accumulation is a problem in a NARX model and in an extreme situation the model can become unstable. The second paper, entitled “A new hybrid method for sensorless control of piezoelectric actuators”, proposes a novel hybrid method to deal with the error accumulation phenomena. This is achieved through the use of a method that employs a velocity signal, which is related to the electric current passing through the piezoelectric actuator, to reduce
the effect of error accumulation on the output displacement of the model. This method is then theoretically and experimentally verified.
4.2 Fuzzy modeling of a piezoelectric actuator

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Fuzzy modeling of a piezoelectric actuator

Morteza Mohammadzaheri, Steven Grainger and Mohsen Bazghaleh

Abstract. In this research, a piezoelectric actuator was modeled using fuzzy subtractive clustering and neuro-fuzzy networks. In the literature, the use of various modeling techniques (excluding techniques used in this article) and different arrangements of inputs in black box modeling of piezoelectric actuators for the purpose of displacement prediction has been reported. Nowadays, universal approximators are available with proven ability in system modeling; hence, the modeling technique is no longer such a critical issue. Appropriate selection of the inputs to the model is, however, still an unsolved problem, with an absence of comparative studies. While the extremum values of input voltage and/or displacement in each cycle of operation have been used in black box modeling inspired by classical phenomenological methods, some researchers have ignored them. This article focuses on addressing this matter. Despite the fact that classical artificial neural networks, the most popular black box modeling tools, provide no visibility of the internal operation, neuro-fuzzy networks can be converted to fuzzy models. Fuzzy models comprise of fuzzy rules which are formed by a number of fuzzy or linguistic values, and this lets the researcher understand the role of each input in the model in comparison with other inputs, particularly, if fuzzy values (sets) have been selected through subtractive clustering. This unique advantage was employed in this research together with consideration of a few critical but subtle points in model verification which are usually overlooked in black box modeling of piezoelectric actuators.

Keywords: Piezoelectric actuators; fuzzy; ANFIS; input and output extrema; input arrangement; black box modeling.
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A$</td>
<td>a fuzzy set</td>
</tr>
<tr>
<td>$B$</td>
<td>a fuzzy set</td>
</tr>
<tr>
<td>$C$</td>
<td>centre of a cluster</td>
</tr>
<tr>
<td>$D$</td>
<td>density function</td>
</tr>
<tr>
<td>$f$</td>
<td>function</td>
</tr>
<tr>
<td>$m$</td>
<td>the number of rules of a FIS</td>
</tr>
<tr>
<td>$n$</td>
<td>the number of inputs to a FIS</td>
</tr>
<tr>
<td>$p, q, r$</td>
<td>parameters of FIS consequent s</td>
</tr>
<tr>
<td>$r_a$</td>
<td>range of influence</td>
</tr>
<tr>
<td>$r_i, r_j$</td>
<td>input and output order</td>
</tr>
<tr>
<td>$t$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$x$</td>
<td>a datum in clustering</td>
</tr>
<tr>
<td>$w$</td>
<td>the weight of a rule in a FIS</td>
</tr>
<tr>
<td>$y$</td>
<td>output (μm)</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>input variables in the Preisach model</td>
</tr>
<tr>
<td>$\mu(.)$</td>
<td>membership grade in FISs</td>
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<tr>
<td>$\mu(...)$</td>
<td>a weight function in (1)</td>
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### Greek Letters

<table>
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<td>a weight function in (1)</td>
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### Indices

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<td>$d$</td>
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<td>$s$</td>
<td>sampling</td>
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<td>$i$</td>
<td>counter</td>
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## 1 Introduction

Piezoelectric actuators are being increasingly studied and used in different areas of science and technology due to their nanometre displacement resolution, wide bandwidth, fast response and high stiffness (Zhang et al., 2010). As a result of this broad application, mathematical modeling of these devices has become an important task for engineers and scientists. This article concerns models that predict the displacement of a stack piezoelectric actuator based on its voltage.

Finite element techniques have been used to model piezoelectric actuators (Soderkvist, 1998, Han et al., 1999, Wang and Wereley, 1998); however, models derived through this method do not suit real time control (Boukari et al., 2011) which is a very important application for models of piezoelectric actuators (Xie et al., 2009, Zhang et al., 2007). The IEEE standard on
piezoelectricity (Meeker, 1987) led to some linear lumped-parameter models for piezoelectric actuators suitable for control purposes. These linear models proved inadequate in many instances due to the significant nonlinearities in piezoelectric actuator behavior (Song and Li, 1999). In the 1990’s, some physics-based nonlinear models were introduced but these had limitations such as inability to model minor hysteresis loops (Jung and Kim, 1994), vulnerability to measurement noise (Leigh and Zimmerman, 1991), and difficult parameter identification procedures (Rakotondrabe, 2011).

In 1995, the Preisach phenomenological model originally devised by the German physicist in 1935 to describe hysteresis phenomenon in magnetic systems (Preisach, 1935), was used to model piezoelectric actuators (Ge and Jouaneh, 1995) and is now widely accepted (Boukari et al., 2011, Liaw and Shirinzadeh, 2011, Zhang et al., 2009). The Prandtl-Ishlinskii model was derived from the Preisach model which fails to model asymmetric hysteresis loops (Zareinejad et al., 2010). Extended and modified versions of the Preisach model have also been published (Song and Li, 1999, Li and Tan, 2004, Dupre et al., 2001, Ge and Jouaneh, 1997, Han and Zhu, 2009, Yu et al., 2002).

In the Preisach model, numeric values of some parameters that cannot be determined merely based on a phenomenological description of the system are estimated using input-output (voltage-displacement) data. Due to this fact, some authors have categorized the Preisach model as a black box model (Makaveev et al., 2001, Sixdenier et al., 2008). This model needs a ‘whole domain’ solution, that is, model equations (i.e. a double integral) must be solved in an area including the very first moment. In classical black box modeling, this deficiency can be avoided and more computationally efficient models can be made. A variety of black box models have been
used for this purpose such as Multi-Layer Perceptrons (MLPs) (Zhang et al., 2010, Dong et al., 2008, Yang et al., 2008), Radial Basis Function Networks (RBFNs) (Dang and Tan, 2007), Recurrent Fuzzy Neural Networks (RFNNs) (Lin et al., 2006a), wavelet neural networks (Lin et al., 2006b), Nonlinear Auto-Regressive Moving Average models with eXogenous inputs (NARMAX) (Deng and Tan, 2009) and non-standard artificial neural networks (Hwang et al., 2001). With the availability of universal approximators (Ying, 1998a, Ying, 1998b, Chen and Chen, 1995a, Chen and Chen, 1995b, Chen et al., 1995c, Park and Sandberg, 1993), the modeling approach or the arrangement of inputs is more important than modeling techniques. In this article, fuzzy subtractive clustering and adaptive neuro fuzzy inference systems (ANFISs, also known as neuro-fuzzy networks) are employed to explore the most appropriate arrangement of inputs to a black box model of a piezoelectric stack actuator. Subtractive-clustering-based-neuro-fuzzy networks result in fuzzy models which can indicate the importance of each model input; this possibility is very helpful in finding the most appropriate input arrangement. Other universal approximators (e.g. MLPs and RBFNs) lack this critical advantage.

2 Background theory

2.1 Preisach Model

Let us consider a one-dimensional piezoelectric actuator with a varying input voltage. In Figure 1, vertical and horizontal axes of the (b) diagram, $\alpha$ and $\beta$, both present input (voltage, $u$ in this paper). According to the Preisach model, the displacement ($y$) at any point can be calculated as (Yang et al., 2008):
Figure 1: A varying input voltage \(u\) applied to the actuator (a), and its corresponding integration area for the Preisach model of (1), (b)

In the Preisach model, (1) will be converted to the following equations (Song and Li, 1999):

\[
y(t) = \iint_{\beta>\alpha} \mu(\alpha, \beta) d\alpha d\beta. \tag{1}
\]

\[
y(t) = \min_{u(\alpha)} \int_{S(0 \rightarrow \alpha_{\text{max}})} u(y) dy \int_{\beta} \mu(\alpha, \beta) d\beta (\text{ascending}), \tag{2}
\]

\[
y(t) = \max_{u(\alpha)} \int_{S(0 \rightarrow \alpha_{\text{max}})} u(y) dy \int_{\beta} \mu(\alpha, \beta) d\beta (\text{descending}). \tag{3}
\]

where \(S\) is integration area. (2) and (3) can be rewritten as (Zhang et al., 2009):

\[
y(t) = y_{\text{min}} + \int_{S(\alpha_{\text{max}} \rightarrow \alpha)} \mu(\alpha, \beta) d\alpha d\beta (\text{ascending}), \tag{4}
\]

\[
y(t) = y_{\text{max}} - \int_{S(\alpha_{\text{max}} \rightarrow \alpha)} \mu(\alpha, \beta) d\alpha d\beta (\text{descending}). \tag{5}
\]
where $y'_{\text{max}}$ is the last local maximum when descending and $y'_{\text{min}}$ is the last local minimum when ascending. (4) and (5) show that the current input (voltage), last extremum of the input ($u_{\text{ext}} = u_{\text{min}}$ or $u_{\text{max}}$) and its corresponding output ($y_{\text{ext}} = y'_{\text{min}}$ or $y'_{\text{max}}$) are used in estimating the current output ($y(t)$) according to the Preisach model. Hence, these extremum values can be used in black box modeling including neuro-fuzzy modeling.

### 2.2 Neuro-fuzzy Networks

Figure 2 shows the structure of a linear Sugeno type fuzzy inference system (FIS) with two inputs and two fuzzy sets (values) for each input. In this structure, the ‘antecedent’ of each fuzzy rule contains fuzzy sets and the ‘consequent’ of each rule is a first order polynomial. $x$ and $y$ are numeric inputs to the FIS. $X$ and $Y$ are input variables (e.g. length). $x$, a value of $X$, is the input to $A_1$ and $A_2$ and leads to two membership grades of $\mu_{A_1}(x)$ and $\mu_{A_2}(x)$, similarly, $y$, a value of $Y$, is the input to $B_1$ and $B_2$ and leads to two membership grades of $\mu_{B_1}(y)$ and $\mu_{B_2}(y)$. All the membership grades of a rule should pass AND function to result in the weight of that rule. For the FIS shown in Figure 2(a),

$$w_1 = \text{AND}(\mu_{A_1}(x), \mu_{B_1}(y)),$$

and

$$w_2 = \text{AND}(\mu_{A_2}(x), \mu_{B_2}(y)),$$

where $w_1$ and $w_2$ are the weight of rules shown in Figure 2(a). In Sugeno-type FISs, the consequent is totally independent of inputs. A consequent can
have the order of zero or one and generates the output of a rule. For a
system with $n$ inputs, a first order consequent has $n+1$ parameters. For the
FIS shown in Figure 2(a),

$$f_i = p_i x + q_i y + r_i,$$

(8)

where $p_i$, $q_i$, and $r_i$ are consequent parameters and $f_i$ is the rule output.
The output of the FIS is the weighted sum of the outputs of the rules.

$$f = \sum_{i=1}^{m} f_i w_j,$$

(9)

where $m$ is the number of rules and $f$ is the output of the FIS.

A sugeno type FIS as shown in Figure 2(a) can be converted to a neuro-
fuzzy network (or ANFIS) shown in Figure 2(b). In neuro fuzzy networks,
activation functions (i.e. membership functions) are adjusted during training
rather than weights. The first layer of nodes of a neuro-fuzzy network uses
membership functions of the corresponding FIS as activation functions. The
second layer nodes use AND function (product in this case) as the activation
function and the activation function of the third layer nodes are weighted
sum functions. In the nodes of the fourth layer, rule outputs are calculated
and the last layer includes a node with a sum activation function. The
training procedure involves gradient error back propagation to adjust the
coefficients of the activation functions of the second layer (i.e. fuzzy
membership functions) and least square of error to adjust the coefficients of
the activation functions of the fourth layer (i.e. FIS consequent parameters)
(Ghaffari et al., 2007, Jang et al., 2006).
2.3 Subtractive clustering

In fuzzy inference systems, if all the inputs have an identical number of fuzzy sets (values), the number of fuzzy rules in the model equals the number of fuzzy sets allocated to each input (e.g. 3) raised to the power of the number of inputs. Therefore, sometimes, too many rules are needed to cover all the input space. In order to reduce the number of fuzzy rules with
minimum loss of accuracy, ‘subtractive clustering’ is applied (Ghaffari et al., 2007, Jang et al., 2006, Mathworks, 2011).

In this method, for each set of input data \( (x_i, i = 1, \ldots, n) \) in an \( m \)-dimensional space, a density value is calculated:

\[
D_j = \sum_{j=1}^{n} \exp \left( -\frac{\left\| x_j - x_i \right\|^2}{\left( r_a/2 \right)^2} \right),
\]

where

\[
\left\| x_j - x_i \right\| = \sqrt{\sum_{k=1}^{n} (x^k_j - x^k_i)^2} \quad \text{(distance)},
\]

and \( r_a \) = Range of Influence (a positive number).

The point with the highest density is defined as the centre of the first cluster. The centre of the first cluster is named \( C_1 \), and its density is named \( D_{C_1} \). A cluster is a hyper sphere with a centre of \( C_i \) and radius of \( r_i \) in an \( m \)-dimensional space. Later, each cluster is used as the antecedent of a fuzzy rule. After the definition of the centre of the first cluster, the density of other points is redefined as

\[
D_i = D_i - D_{C_1} \exp \left( -\frac{\left\| x_i - x_{C_1} \right\|^2}{\left( r_b/2 \right)^2} \right),
\]

where \( r_b \) = Squash Factor.
If the redefined density of any point exceeds the ‘Accept Ratio’, it is defined as the centre of a cluster, and then, the density of other points are redefined again:

\[ D_i = D_i - \sum_{j=1}^{p} D_{c_j} \exp\left(-\frac{\|x_i - x_{c_j}\|^2}{(r_i/2)^2}\right), \]  

(13)

where \( p \) = the number of already defined clusters.

After any stage of redefinition, the density of the centres of previously defined clusters are re-calculated. If their density yields lower than ‘Reject Ratio’, those clusters are eliminated. This process continues till the clusters do not change between two sequential stages. The model derived from subtractive clustering is used as the initial neuro-fuzzy model for training.

3 Data gathering

The set up includes a NEC AE0505D44H40 stack piezoelectric actuator and a PHILTECH D20 optical sensor connected to a PC through dSPACE and voltage amplifiers. The system was excited by three triangle waves of voltage with maximum and minimum amplitude of ±20 V and slopes of ±80 V/s, ±800 V/s and ±8000 V/s, for a period of 2s each. Furthermore, it was excited by a repeating stair function, shown in Figure 3, and a chirp function, both in the range of ±20 V and for a time period of 2s. Data gathered through these excitations were used as the training data. The data obtained through the excitation of the set up with 20sin10 V was used as the validation data.
Modeling

4.1 Model structure

Nonlinear Auto-Regressive models with eXogenous inputs (NARX models) are commonly used for classical system identification (Nelles, 2001). According to the NARX structure, for a single input-single output system like a one-dimensional piezoelectric actuator (Nelles, 2001),

\[
y(t) = f \left( \begin{array}{c} u(t-t_d), u(t-t_d-T_s), \ldots, u(t-t_d-r_u T_s), \\ y(t-T_s), y(t-2T_s), \ldots, y(t-r_y T_s) \end{array} \right),
\]

where \( t_d \) is delay time, and \( r_u \) and \( r_y \) are input and output orders respectively. \( f \) is an approximated nonlinear function. The sampling time of 0.001 s, the delay time of 0.001 s and orders up to three were selected based
CHAPTER 4. NON-LINEAR MODELLING OF PIEZOELECTRIC ACTUATORS

on a previous successful black box modeling work on stack piezoelectric actuators (Zhang et al., 2009). However, inspired by the Preisach model (see (4) and (5)), the extrema of the system’s input and output have been also used as inputs to the model together with the classical inputs to NARX models. In this article, the involvement of extrema in black box modeling of a piezoelectric stack actuator is particularly addressed.

4.2 Different approaches to model validation

In system identification, there are two different approaches for validating models: one-step prediction and simulation. One-step prediction is the normal way of assessing black box models of piezoelectric actuators in the literature (Zhang et al., 2010, Song and Li, 1999, Li and Tan, 2004, Dong et al., 2008, Yang et al., 2008, Dang and Tan, 2007, Zhang et al., 2009). In this approach, all the inputs to the model are recalled from the memory, or their real values are assumed to be available:

\[
\hat{y}(t) = f \left( u(t-t_d), u(t-t_d-T_s), \ldots, u(t-t_d-r_s T_s), \right) \\
y(t-T_s), y(t-2T_s), \ldots, y(t-r_s T_s) \right), \quad (15)
\]

where the variable(s) with a hat represent estimated values. In one-step prediction, all real values of model inputs during an operation are given to the model, and the output of the model is compared to the recorded value of the system output.

However, if the model is used in predictive control (with a horizon of two or more) (Mohammadzaheri and Chen, 2010) sensor-less control or process simulation, the delayed outputs will not be available, so one-step prediction is not applicable; thus, previously estimated values of system output(s) are
needed to be used as model inputs. This is called model validation for simulation:

\[
\hat{y}(t) = f\left(u(t-t_d), u(t-t_d-T_s), \ldots, u(t-t_d-r_s T_s), \hat{y}(t-T_s), \hat{y}(t-2T_s), \ldots, \hat{y}(t-r_s T_s)\right).
\]  

(16)

As a result, the inevitable error of estimated outputs returns to the validation process and increases the resultant error repeatedly. This phenomenon is called ‘error accumulation’ (Mohammadzaheri and Chen, 2010, Mohammadzaheri et al., 2010).

Due to error accumulation, a high number of delayed outputs can decrease the accuracy in simulation and increase the accuracy in one-step prediction, especially in the case of deficiencies in the modeling structure or algorithm. This explains why the Preisach model, which is independent of delayed outputs, is used successfully in real-time control applications in spite of its deficiency. In summary, as a result of error accumulation, the decrease of the error in model validation for one step prediction does not necessarily lead to the decrease of the estimation error for simulation or real time application. This statement is confirmed by the results listed in Table 1. Only highly accurate models can both benefit from delayed outputs as their inputs and overcome the error accumulation for a significant period of time.
Table 1: Simulation (left hand side number) and one-step prediction errors (right hand side number) for a number of fuzzy models with different output orders ($r_y$). All models have been made by subtractive clustering and neuro-fuzzy modeling.

<table>
<thead>
<tr>
<th>Output order</th>
<th>Error without extrema ($\mu m$)</th>
<th>Error with $u_{ex}$ ($\mu m$)</th>
<th>Error with both extrema ($\mu m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_y=1$</td>
<td>2.2227/0.0837</td>
<td>0.2184/0.0222</td>
<td>0.2421/0.0290</td>
</tr>
<tr>
<td>$r_y=2$</td>
<td>8.0331/0.0339</td>
<td>0.1145/0.0185</td>
<td>0.1866/0.0292</td>
</tr>
<tr>
<td>$r_y=3$</td>
<td>1.2911/0.0362</td>
<td>0.1747/0.0194</td>
<td>0.1025/0.0259</td>
</tr>
</tbody>
</table>

4.3 Modeling results

In black box modeling of piezoelectric actuators, the extrema of input (voltage) (Deng and Tan, 2009, Zhang et al., 2009, Zhang and Tan, 2010, Kim et al., 2009) and the output (displacement) (Yang et al., 2008) have sometimes been used as the inputs to black box models, in some other works, these variables have not been utilized in modeling (Dong et al., 2008). In the literature there is no comparison to indicate the superiority of any of these approaches. In this section, fuzzy subtractive clustering (see Section 2.3) is employed to address this matter. In subtractive clustering, a higher number of fuzzy sets are allocated to the inputs which are more influential on forming clusters or operation areas (an operation area is a cluster or an antecedent of a fuzzy rule). If an input has only one fuzzy set, repeated in all rules, it will have no effect on the result, and that input can be overlooked in modeling (Jang et al., 2006). This is sometimes interpreted as: the inputs with more fuzzy sets have a more important role on the behavior of the system (Ahmadpour et al., 2009). This statement is not general, and the operation areas of rules need to be checked before finalizing the judgment about the importance of model inputs.

In this research, with a series of experimental data, a number of Sugeno-type fuzzy models were made through subtractive clustering. Membership
functions (fuzzy sets) are Gaussian and the consequent is a first order polynomial, AND method is product, and OR method is algebraic sum (Jang et al., 2006, Mathworks, 2011). The input arrangement of the fuzzy model is shown in (17):

\[
\hat{y}(t) = f\left(u(t - 0.001), u(t - 0.002), u(t - 0.003), u_{ext}, y(t - 0.001), y(t - 0.002), y(t - 0.003), y_{ext}\right).
\]

Range of Influence and Squash Factor were retained fixed at 1.25 and 0.5 in modeling; Accept and Reject Ratios were subject to change. Different pairs of Accept and Reject Ratios would result in different numbers of fuzzy sets allocated to each input. The results are listed in Table 2. Standard data preparation (Mohammadzaheri et al., 2009) was done prior to modeling.

According to Table 2, it appears that the extrema have a low number of fuzzy sets, and thus a low grade of importance in modeling, and they can be ignored. However, it is not correct. Figure 4 shows a fuzzy inference system of Table 2 with 10 rules. The first six inputs have similar fuzzy sets in the rules, differing from the last two inputs. It means, if a number of the first six inputs (with high numbers of fuzzy sets) are eliminated, they might be replaced by others, but if both extrema are neglected, their role cannot be fulfilled by the first six inputs. As an example, in the FIS shown in Figure 4, the operation areas (antecedents) of rules 2 and 10 will be almost the same without the extrema. As a result, involving the extrema (both or one of them) has a critical role in defining operation areas, and the extrema should not be overlooked. Results listed in Table 1 confirm this conclusion.
Table 2: Number of fuzzy rules resulting from a pair of Accept and Reject Ratios (in parentheses in the first column) and the number of fuzzy sets allocated to each model input

<table>
<thead>
<tr>
<th>No. of rules</th>
<th>$u(t-3t_s)$</th>
<th>$u(t-2t_s)$</th>
<th>$u(t-t_s)$</th>
<th>$y(t-3t_s)$</th>
<th>$y(t-2t_s)$</th>
<th>$y(t-t_s)$</th>
<th>$u_{ex}$</th>
<th>$y_{ex}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31(0.05,0.01)</td>
<td>20</td>
<td>20</td>
<td>22</td>
<td>22</td>
<td>24</td>
<td>20</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>24(0.2,0.05)</td>
<td>19</td>
<td>19</td>
<td>19</td>
<td>21</td>
<td>18</td>
<td>19</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>20(0.5,0.15)</td>
<td>15</td>
<td>16</td>
<td>16</td>
<td>18</td>
<td>16</td>
<td>17</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>13(0.8,0.3)</td>
<td>13</td>
<td>13</td>
<td>13</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>10(0.95,0.5)</td>
<td>10</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The results listed in Table 1 and shown in Figures 5 ((a) and (b)) have been gathered through subtractive clustering followed by neuro-fuzzy modeling. An effort has been made to achieve the best results with each arrangement of inputs; however, neuro-fuzzy networks are a subset of artificial neural networks (ANNs), and it is a difficult and unsolved problem to find the best ANN structure for each specific application; thus, a fairly large ANN is usually employed to deal with relatively complex approximation problems (Deng et al., 2008, Mohammadzaheri et al., 2012).

In this research, around 20 fuzzy rules were used for modeling; a higher number of rules did not lead to higher accuracy. The resolution of the measurement system (the lowest measurable value of the displacement) is 0.09 μm. Thus, errors below this value are not meaningful and can be assumed equal to 0.09 μm. Simulation errors, which are a few times higher than the resolution for a time period of 2s (2000 instants), correspond to highly accurate models.
Figure 4: A fuzzy model of a piezoelectric actuator with 10 rules
CHAPTER 4. NON-LINEAR MODELLING OF PIEZOELECTRIC ACTUATORS

Figure 5: Verification results in the form of hysteresis diagrams of 20 cycles of simulation (a) and one-step prediction (b), and in the form of time response of simulation (c) for a fuzzy model with orders of three for both inputs and outputs and with both the extrema and 20 fuzzy rules. The model has been made for the piezoelectric actuator detailed in Section 3.
5 Conclusion

This article addresses the use of subtractive-clustering-based fuzzy modeling for piezoelectric actuators, which gives an insight to the model and makes it possible to assess the significance of the inputs to models. In this research, a number of experiments were undertaken on a stack piezoelectric actuator. After preparation, these data were used in fuzzy subtractive clustering, then in neuro fuzzy modeling which resulted in a number of fuzzy models. The aforementioned modeling method is a black box method, not using the laws of nature. However, its outcomes, fuzzy models, mainly formed by linguistic or fuzzy values, convey valuable information about the model. This helps significantly in assessing how critical each model input is. In this research fuzzy modeling was followed by a thorough model validation, with data different from training data, both for one-step prediction and simulation. This research not only resulted in a number of accurate fuzzy models for piezoelectric actuators, but also provided a guideline for future modeling works especially for real time applications.

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4.3 A new hybrid method for sensorless control of piezoelectric actuators

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A new hybrid method for sensorless control of piezoelectric actuators

Mohsen Bazghaleh, Morteza Mohammadzaheri, Steven Grainger, Ben Cazzolato and Tien-Fu Lu

Abstract. This paper offers a new hybrid position estimation method for the sensorless control of piezoelectric actuators. Often mathematical models which map easy to measure signals to displacement are used in this role. The proposed method is aimed at enhancing the accuracy of commonly accepted black box models. Three easy to measure signals are available to be used to estimate displacement. One, the induced voltage, is not suitable for piezoelectric stacks. Two others are more generally usable: the piezoelectric voltage and the sensing voltage. This paper proposes, then theoretically and experimentally verifies a hybrid algorithm that uses the two latter signals to produce estimates of displacement with improved accuracy.

Keywords: Piezoelectric actuators; sensorless control; displacement estimation; black box modeling; fuzzy modeling.

1 Introduction

Sensorless control of the displacement of piezoelectric actuators has attracted much attention to avoid the expense and practical limits of precision displacement sensors (Aphale et al., 2007, Fairbairn et al., 2011, Fleming and Moheimani, 2006, Park and Moon, 2010, Ronkanen et al., 2011). The basis of sensorless control is to estimate displacement using easy
to measure electrical signals. In general, three signals may be used for this purpose: the voltage applied to the piezoelectric actuator or in short, the piezoelectric voltage (see Figure 1) (Aphale et al., 2007), the voltage across the sensing resistor, or at the output of the piezoelectric actuator, explained in Section 2.1 (the sensing voltage (see Figure 1)) (Fleming and Moheimani, 2006), and the induced voltage in electrodes or segments of the piezoelectric actuator which are not subject to the applied voltage (mostly used in piezoelectric tubes) (Kuiper and Schitter, 2010). The use of these signals, the sensing voltage, the induced voltage and the piezoelectric voltage is briefly explained in the following three paragraphs.

The sensing voltage signal can be used to obtain the electrical current passing the piezoelectric actuator (explained in Section 2.1), and the charge is found through integration. The charge that passes the piezoelectric actuator has a direct relation with its displacement in normal operational areas, so is a precise indicator of the displacement (Fleming and Moheimani, 2006, Comstock, 1981). However, in a digital environment, due to the problem of drift, after a while, the calculated charge is not equal to the real charge (Bazghaleh et al., 2010).

The induced voltage signal has been suggested as an indicator of displacement using an arrangement of capacitors (Kuiper and Schitter, 2010), also linear mathematical models have been offered to map the induced voltage to the piezoelectric voltage (Moheimani and Yong, 2008, Yong and Moheimani, 2009); however, to date, no direct mapping between displacement and the induced voltage has been reported in sensorless control of piezoelectric actuators.
CHAPTER 4. NON-LINEAR MODELLING OF PIEZOELECTRIC ACTUATORS

The piezoelectric voltage, on the other hand, has been adequately mapped onto displacement through a variety of dynamic mathematical models (Park and Moon, 2010, Li and Tan, 2004, Mohammadzaheri et al., 2012b, Schitter and Stemmer, 2004, Song and Li, 1999, Yang et al., 2008, Yu et al., 2002, Zhang and Tan, 2010, Zhang et al., 2009, Zhang et al., 2010). Analytical models have failed to model this complex relationship, and instead, dynamic phenomenological (e.g. the Presiach or the Rayleigh models) (Park and Moon, 2010, Yu et al., 2002) and classical black box models (e.g. artificial neural networks or neurofuzzy networks) (Yang et al., 2008, Mohammadzaheri et al., 2012c) have been employed to perform this task. It has been shown that static models (those not considering a history of the system input/output) are not adequate tools to model piezoelectric actuators. Alternatively, dynamic models have shown promising outcomes (Mohammadzaheri et al., 2012b).

Piezoelectric actuators are dynamic systems; that is, the current value of their displacement is dependent on previous values of displacement. Therefore, in highly accurate dynamic models, in addition to piezoelectric-voltage-based signals (the history and/or derivatives and/or extrema of the piezoelectric voltage), previously estimated values of displacement are also used as inputs to the model (Mohammadzaheri et al., 2012b). This however, gives rise to the error accumulation phenomenon (explained in Section 2.2) which is a major issue in modeling of dynamic systems in general (Mohammadzaheri et al., 2012b). Many researchers have contributed to sensorless control of piezoelectric actuators through developing new mathematical models to map the piezoelectric voltage onto displacement more effectively. This paper introduces a new approach to decrease the effect of error accumulation in displacement estimation of piezoelectric actuators through an algorithm that also incorporates the sensing voltage.
2 Existing popular approaches towards sensorless control of piezoelectric actuators

Two of the most popular displacement estimation circuits are shown in Figures 1 and 3: $V_i(t)$ is input voltage to the piezoelectric actuator (coming from a voltage amplifier), $V_s(t)$ and $R_s$ are the sensing voltage and the sensing resistance respectively. $i_p(t)$ is the electrical current passing the piezoelectric actuator; $V_p(t)$ and $q_p(t)$ are the piezoelectric actuator voltage and charge respectively, and $d$ is the displacement of the piezoelectric actuator.

2.1 Displacement estimation using the sensing voltage

There is a linear relation between the charge and the displacement of the piezoelectric actuator (Fleming and Moheimani, 2005, Newcomb and Flinn, 1982). At higher frequencies, in this case more than 100 Hz, the phase lag between charge and displacement starts to increase which makes it undesirable for higher frequency tracking applications. In this paper, to avoid phase lag the excitation signal is assumed to be less than 100 Hz. Therefore:

$$d(t) = Kq_p(t)$$  \hspace{1cm} (1)

According to Figure 1, displacement can be estimated as below

$$\hat{d}(t) = K \int_0^t i_p(t) \, dt$$  \hspace{1cm} (2)
or

\[ \dot{d}(t) = \frac{K}{R_s} \int_0^{t_f} V_s(t) \, dt \]  

Figure 1: A schematic of a digital circuit that outputs displacement with the input of the sensing voltage

However, in the circuit shown in Figure 1, the analog to digital converter (ADC) is not ideal and its offset voltage together with the dielectric leakage of the piezoelectric actuator introduce a bias voltage term, \( V_{\text{bias}} \), and in practice, the estimated displacement is

\[ \dot{d}(t) = \frac{K}{R_s} \int_0^t (V_s(t) + V_{\text{bias}}) \, dt, \]  

where \( t_f \) is the final time of operation. The discrepancy between (3) and (4) leads to an estimation error, namely ‘drift’. In the discrete domain, \( d(k) \) represents the current displacement and \( d(k-i) \) represents the value of displacement at \( i \) instants ago; the time interval between two consecutive instants is the sampling time of the model, \( T_s \).
If \( n_r = \frac{L_f}{t_s} \), then

\[
\hat{d}(t) = \frac{K}{R_k} \sum_{k=1}^{n_r+1} (V_s(k) + V_{bias}).
\]  

If the number of terms of discrete sum function \( n_r \) is large, then the resultant error or drift will be considerable even with a small \( V_{bias} \). Drift compensation was not addressed in this research. Figure 2 shows the drift which increases as the operations progresses.

![Figure 2: Actual (-) and estimated (--) piezoelectric actuator displacement using the circuit shown in Figure 1 with a sampling time of 0.001 s](image-url)
2.2 Displacement estimation using the piezoelectric voltage

A variety of dynamic models have been used to map the piezoelectric voltage to displacement. In terms of the essence of inputs to the model, dynamic models are divided into two groups: in some phenomenological models (i.e. different forms of Preisach models), the previous values of displacement are not used as the inputs to the model, although these values definitely influence the current value of displacement. This can be interpreted as the previous values of displacement have been assumed to be indirectly represented by the piezoelectric-voltage-based signals (the history and/or derivatives and/or extrema of the piezoelectric voltage). Preisach models, originally developed by a physicist in the area of magnetism (Preisach, 1935), need a whole-domain solution from the very first instant of operation to estimate the current displacement, moreover phenomenological models focus on hysteresis modeling (e.g. (Yu et al., 2002)), whereas hysteresis is not the main issue in cases such as tube piezoelectric actuators.

The second group of dynamic models in the area are classical black-box models such as universal approximators with mathematically proven capability in system identification (Mohammadzaheri, 2012a). These techniques offer modeling accuracy and computational efficiency (Mohammadzaheri, 2012a). Sensorless control of piezoelectric actuators using highly accurate black box models, shown in Figure 3, is the main focus of this article.
As previously mentioned, in order to achieve higher accuracy, the previously estimated values of displacement are used as inputs to this model. As a result, during sensorless operation, the inevitable error in the estimated displacements feeds back to the model and influences the next estimated displacement.

Consider a model of $f$: $\hat{d}(k) = f\left(\mathbf{V}_{ps}, d(k-1)\right)$,

$$\text{where } \mathbf{V}_{ps} \text{ is a vector of the present and a number of previous piezoelectric-voltage values. Variables with a hat represent estimated values. The estimation error } e \text{, is given by}$$

$$e(k) = d(k) - \hat{d}(k).$$

According to these definitions, during operation:

$$e(2) = d(2) - f\left(\mathbf{V}_{ps}, d(1)\right) \text{ or } d(2) = f\left(\mathbf{V}_{ps}, d(1)\right) + e(2),$$
So

\[ e(3) = d(3) - f(V_{ps}, f(V_{ps}, d(1)) + e(2)) \], \quad (9) \]

and

\[ e(4) = d(4) - f(V_{ps}, f(V_{ps}, d(1)) + e(2)) + e(3) \], \quad (10) \]

and

\[ e(k) = d(k) - f(V_{ps}, f(V_{ps}, \ldots f(V_{ps}, f(V_{ps}, d(1)) + e(2)) + e(3)) + e(4) + \ldots + e(k-1)). \] \quad (11) \]

Thus, the resultant error is likely to increase as the operation progresses, that is, larger \( k \) very often leads to a larger \( e(k) \). This phenomenon is called error accumulation. In other words, the estimation error at each instant affects all future estimated values of displacement. This article addresses this issue and offers an original approach to reduce its effect on displacement estimation of piezoelectric actuators.

### 3 Proposed hybrid model

In this section, a method is offered to estimate the previous values of the displacement in such a way as to reduce estimation errors. Then these new
estimated values are used as inputs to the black box model, such that the influence of error accumulation will decrease.

If \( v \) represents the velocity of the piezoelectric actuator at the point where displacement is measured, for one instant before the current time, as a result of an approximate discrete derivation:

\[
\frac{d(k-1) - d(k-2)}{T_s} \approx v(k-1) \text{ or }
\]

\[
d(k-1) \approx d(k-2) + T_s v(k-1) \quad (12)
\]

For two instants before the current time:

\[
d(k-2) \approx d(k-3) + T_s v(k-2) \quad (13)
\]

(12) and (13) can be combined:

\[
d(k-1) \approx d(k-3) + T_s \left[ v(k-1) + v(k-2) \right]. \quad (14)
\]

Similarly,

\[
d(k-1) \approx d(k-4) + T_s \left[ v(k-1) + v(k-2) + v(k-3) \right], \quad (15)
\]

or in general, for \( N \) instants earlier:

\[
d(k-1) \approx d(k-N) + T_s \left[ \sum_{j=1}^{N-1} v(k-j) \right] \quad (16)
\]

where \( N \) defines the number of previous displacements used in estimation.
The velocity can be estimated by taking the temporal derivative of (1) in the case of the validity of (1):

\[ v = K_i p \]  
(17)

Hence,

\[ d(k-1) = d(k-N) + KT_s \left[ \sum_{j=1}^{N-1} i_p(k-j) \right] \]  
(18)

During estimation, (18) will be used as

\[ \hat{d}(k-1) = \hat{d}(k-N) + KT_s \left[ \sum_{j=1}^{N-1} i_p(k-j) \right] \]  
(19)

or

\[ \hat{d}(k-1) = \hat{d}(k-N) + \frac{KT_s}{R_s} \left[ \sum_{j=1}^{N-1} \left( V_s(k-j) + V_{bias} \right) \right] \]  
(20)

\( V_{bias} \) is very small and its effect is negligible if \( N \) is a small number (see Section 2.1), so it is approximately assumed that:

\[ \hat{d}(k-1) = \hat{d}(k-N) + \frac{KT_s}{R_s} \left[ \sum_{j=1}^{N-1} V_s(k-j) \right] \]  
(21)

Eq. (21) can be adjusted so as to offer equivalents for \( \hat{d}(k-2) \) or \( \hat{d}(k-3) \) or any other displacement-based inputs to the black box model, using earlier estimated values of displacements. Eq. (21) and their aforementioned equivalents are the heart of the proposed idea of this paper.
Displacement estimation with a black box model of (6), with or without use of the proposed method can be compared. Subscripts b and h, respectively, represent use of the black box model solely and use of the proposed hybrid method. The initial value of displacement \( \hat{d}(1) \) is available (usually \( \hat{d}(1) = 0 \) in practice). For \( N = 3 \) and \( k = 4 \), without the proposed method, From (10),

\[
\hat{d}_b(4) = f(V_{ps}, f(V_{ps}, f(V_{ps}, f(V_{ps}, d(1)) + e_b(2)) + e_b(3))). \tag{22}
\]

Two different estimation errors are involved in the argument of (22). With the proposed method,

\[
\hat{d}_h(4) = f(V_{ps}, d(1) + Kr_s \left[ \sum_{j=0}^{3} V_s(4 - j) \right]), \tag{23}
\]

No estimation error is involved in the argument of (23), if (21) is valid.

Similarly, for \( k = 7 \), from (11) and (6), in the case of sole usage of a black box model,

\[
\hat{d}_b(7) = f(V_{ps}, \ldots, f(V_{ps}, f(V_{ps}, f(V_{ps}, d(1)) + e_b(2)) + e_b(3)) + e_b(4) + \ldots) + e_b(6). \tag{24}
\]

Five estimation errors are involved in the calculation of the estimated displacement.
However, in the case of the proposed hybrid model:

\[
\hat{d}_h(7) = f \left[ V_p, \left( f \left( d(1) + K_t \sum_{j=1}^{4} V_s(4-j) \right) \right) + e_h(4) + K_t \sum_{j=1}^{7} V_s(j-7) \right]
\]

(25)

only one estimation error is involved in the calculation of the estimated displacement. Similarly, it can be concluded for the first order model presented in (6), with \( N = 3 \), \( \hat{d}_h(10) \), \( \hat{d}_h(13) \), \( \hat{d}_h(16) \) and \( \hat{d}_h(k) \) have only two, three, four and \((k-4)/3\) estimation errors involved in their estimation calculations, respectively. In general, \((k-N-1)/N\) estimation errors are involved in calculating \( \hat{d}_h(k) \) for model (6). This simply means the inevitable estimation error at each instant does not affect all subsequent estimated values of displacement; thus, the influence of error accumulation decreases significantly.
Figure 4: A schematic of the proposed method for the model introduced in (6), $Z^{-1}$ represents a unit delay

However, this conclusion is based on the assumption of the validity of (1) which is true at excitation frequencies below 100 Hz, and is also based on the validity of (21) which is not completely true. Large values of $N$ (introduced in (16)) may lead to a considerable error due to conversion errors (see Section 2.2). In summary, as $N$ increases, error accumulation decreases and the aforementioned error increases. Therefore, the value of $N$ is a trade-off.

4 Data gathering and black box modeling

The set up includes a NEC/TOKIN AE0505D44H40 stack piezoelectric actuator and a PHILTECH D20 optical sensor connected to a PC through a DS1104 dSPACE and a voltage amplifier. The system was excited by three triangle waves with maximum and minimum amplitude of $\pm 20$ V and
frequencies of 1 Hz, 10 Hz and 100 Hz, for a period of 2 s each. All experiments were performed under no external forces, so the models are valid under this condition (such as with Atomic Force Microscopy). In scanning devices, the piezoelectric actuators follow a raster pattern (Schitter et al., 2007, Abramovitch et al., 2007, Moheimani, 2008); triangle waves of excitation are employed to generate this pattern of motion; that is, in one direction (let us say \( x \)), the tube should track a triangular waveform, and in the other direction (let us say \( y \)), the tube should track a very slowly increasing ramp.

In this research, a Sugeno-type fuzzy model was fitted to the aforementioned data:

\[
\hat{d}(k) = f(V_p(k-1), V_p(k-2), V_p(k-3), V_{\text{ext}}, d(k-1)),
\]

where \( V_{\text{ext}} \) is the last extremum value of the piezoelectric voltage. Antecedent parameters were tuned via the steepest descent method and consequent parameters were tuned using least square of error. The fuzzy model comprises 25 fuzzy rules, initially selected through subtractive clustering. The fuzzy modeling process and its performance have been detailed in (Mohammadzaheri et al., 2012c).

5 Experimental results

The achieved fuzzy model has been tested operating alone (with \( \hat{d}(k-1) \) arising directly from \( \hat{d}(k) \)) and within the circuit shown in Figure 4 for one second (1000 instants). A variety of triangle waves, the most prevalent waveform in piezoelectric actuation, with the domain of ±20 V and a range of frequencies have been used in the tests. Results (Mean of Absolute Error,
MAE) are presented in Table 1 and depicted in Figures 5 and 6. The improvement in accuracy ranges from 27% to 47% for Table 1. It can be seen there is an optimal value for $N$. This confirms the trade-off mentioned in Section 3. It is expected the optimal $N$ is dependent upon the noise properties of the sensing voltage used to determine velocity and is the subject of future research. The results indicate the capability of the method to estimate displacement at the given range of frequency. Due to the RC circuit between the sensing resistor and the piezoelectric actuator, at low frequencies, the voltage drop across the sensing resistor decreases significantly. This causes a significant reduction in the signal-to-noise ratio (SNR) of the sensing voltage, $V_s(t)$, introduced in Section 2, which means this signal cannot be measured accurately. Therefore, this method is not suitable to estimate displacement when the applied voltage is low frequency or a DC signal.

Table 1: Mean of absolute values of estimation error for one second (1000 instants) in micro meters. The range of voltage is [-20, +20] V and the range of displacement is approximately [-5, +5] micro meters.

<table>
<thead>
<tr>
<th></th>
<th>Fuzzy</th>
<th>Hybrid $N=2$</th>
<th>Hybrid $N=3$</th>
<th>Hybrid $N=4$</th>
<th>Hybrid $N=5$</th>
<th>Hybrid $N=6$</th>
<th>Hybrid $N=7$</th>
<th>Hybrid $N=8$</th>
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<td>0.3170</td>
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<td>0.3900</td>
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<tr>
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<td>0.5213</td>
<td>0.5332</td>
<td>0.4349</td>
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<td>1.0656</td>
<td>1.0219</td>
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<td>1.1741</td>
<td>1.1046</td>
<td>0.8524</td>
<td>1.7515</td>
</tr>
</tbody>
</table>
Figure 5: The accuracy of solely the fuzzy model and the hybrid algorithm (with N=6) to predict displacement generated by a triangle excitation voltage wave at the frequency of 40 Hz and the domain of 20V after 0.9 seconds of operation.
6 Conclusion

In this paper, a new method has been introduced for sensorless control of piezoelectric stack actuators that increases the estimation accuracy of black box models of these actuators through reducing the effect of error accumulation. This is achieved through the use of a method that employs two electrical signals (amongst three known signals for this purpose which one of them is not applicable on piezoelectric stack actuators). The method employs a black box model which maps the piezoelectric voltage onto displacement together with a complementary algorithm that uses the sensing voltage.
References


5 Model-based Drift Correction of the Digital Charge Amplifier

5.1 Introduction

While the DCA, proposed in Chapter 3, significantly reduced hysteresis and improved the linearity of piezoelectric actuators, it suffers from drift which reduces the positioning accuracy over a considerable period of time.

This chapter proposes a model-based drift correction technique for removal of the drift and to improve the tracking performance of the DCA. It is shown how the model, developed in Chapter 4, and which crucially does not suffer from drift, can be integrated with a charge amplifier in order to remove the charge amplifier’s inherent drift, while the charge amplifier itself tackles the issue of hysteresis. In this respect, data fusion is utilized to integrate the reliability and short term accuracy of the DCA, with the long term accuracy of the non-linear model to obtain the benefit of both techniques.

Experimental results clearly show the elimination of drift and improvements in tracking performance.
5.2 A digital charge amplifier for hysteresis elimination in piezoelectric actuators

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Contributions to the Paper | Developed the model, performed experimental work, analyzed data and wrote the manuscript.
Signature | Date 11/9/13

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Contributions to the Paper | Supervised research, reviewed manuscript.
Signature | Date 11/09/13
A digital charge amplifier for hysteresis elimination in piezoelectric actuators

Mohsen Bazghaleh, Steven Grainger, Morteza Mohammadzaheri, Ben Cazzolato and Tien-Fu Lu

Abstract. Piezoelectric actuators are commonly used for nanopositioning due to their high resolution, low power consumption and wide operating frequency but they suffer hysteresis which affects linearity. In this paper, a novel digital charge amplifier is presented. Results show that hysteresis is reduced by 91% compared with a voltage amplifier but over long operational periods the digital charge amplifier approach suffers displacement drift. A non-linear ARX model with long-term accuracy is used with a data fusion algorithm to remove the drift. Experimental results are presented.

Keywords: Piezoelectric actuators; digital implementation; hysteresis; charge control; data fusion; drift; non-linear ARX model.

1 Introduction

The most commonly used nanopositioner is the piezoelectric actuator. Aside from being compact in size, they are capable of nanometre resolution in displacement, have high stiffness, provide excellent operating bandwidth and high force output (Devasia et al., 2007). Consequently they have been widely used in many applications ranging from scanning tunneling microscopes (STM) (Wiesendanger, 1994) to vibration cancellation in disk drives (Ma and Ang, 2000). Typically piezoelectric actuators are driven by
CHAPTER 5. MODEL-BASED DRIFT CORRECTION OF THE DIGITAL CHARGE AMPLIFIER

voltage, however voltage driven methods suffer from hysteresis and creep which are nonlinear in nature and reduce the positioning accuracy (Leang and Devasia, 2002).

Creep is the result of the remnant polarization which continues to change after the applied signal reaches its final value and typically is an issue at low frequencies (IEEE Standard on Piezoelectricity, 1987). Hysteresis is due to the polarization of microscopic particles (Damjanovic, 2005) and depends on both the currently applied voltage as well as that previously applied (Kuhnen and Janocha, 1998). Physically it means that, for similar voltage input, the piezoelectric actuator may have different displacement values. The relationship between applied voltage and displacement also changes subject to the amplitude and the frequency of the applied voltage, thus it is both non-linear and dynamic in nature (Adriaens et al., 2000).

A variety of approaches have been used to tackle the hysteresis of piezoelectric actuators. Feed-forward control uses an inverse-model, such as the Preisach model (Ge and Jouaneh, 1995) or Maxwell resistive model (Goldfarb and Celanovic, 1997b), to compensate for hysteresis. Because of uncertainties in the piezo parameters, the feedforward inverse-model technique can suffer from lack of robustness. Another approach is displacement feedback control (Shieh et al., 2004) but this is limited by sensor-induced noise, high cost, additional complexity and limited bandwidth (Fleming and Leang, 2008).

Charge regulator and capacitor insertion techniques have also been used (Minase et al., 2010). The capacitor insertion method (Kaizuka and Siu, 1988) involves using a capacitor in series with the piezoelectric actuator and, while it is effective in dealing with hysteresis, significantly reduces the
operating range because of the voltage drop across the capacitor. Charge regulation was first introduced by Comstock (1981) who showed that by regulating the charge across a piezoelectric actuator the hysteresis is reduced significantly. However a charge amplifier approach has historically been complicated and expensive to implement due to the complexity of the analog circuitry required to meet stringent performance targets (Fleming and Moheimani, 2005). Additionally, in the generic charge amplifier, the load capacitor is charged up because of the uncontrolled nature of the output voltage and once the output voltage reaches the power supply rails, the charge amplifier saturates. To avoid this undesirable effect additional methods have been used which increases the implementation complexity (Fleming and Moheimani, 2004). Comstock (1981) used an initialization circuit to short circuit the sensing capacitor periodically; however, it causes undesirable disturbances at high frequencies. Fleming and Moheimani (2004) used two additional amplifiers to synthesize charge operation at low frequencies.

This paper presents a novel digital charge amplifier with model-based drift compensation for the elimination of hysteresis. In Section 2 the digital charge amplifier is described. Piezoelectric actuator modeling and the model-based drift correction are discussed in Section 3. The experimental setup and results are in Section 4. Conclusions are provided in Section 5.

2 A digital charge amplifier

Figure 1 shows the digital charge amplifier that forms the basis of this work. It consists of a voltage amplifier, digital to analog converter (DAC), analog to digital converter (ADC) and a digital signal processor (DSP). A sensing
resistor is placed in series with the piezoelectric stack actuator and a protection circuit protects the ADC from high voltage.

integrate the piezoelectric actuator current.

\[ q_p = \int I_s(t) dt. \]  \hspace{1cm} (1)

Because of the protection circuit resistor \( R_p \) and the ADC input resistance\( R_{\text{inputADC}} \), the piezoelectric actuator current is given by

\[ I_s(t) = \frac{V_s(t)}{R_s \|(R_p + R_{\text{inputADC}})} \]  \hspace{1cm} (2)

where \( V_s \) is the sensing voltage across the sensing resistor.

Substituting (2) into (1) gives
\[ q_p = \int \frac{V_s(t)}{R_T \|(R_p + R_{\text{inputADC}})} \, dt. \]  
\hspace{1cm} (3)

The protection resistor and input resistance are in series and together they are in parallel with sensing resistor \( R_s \), so the total resistance is

\[ R_T = R_s \|(R_p + R_{\text{inputADC}}). \] \hspace{1cm} (4)

Substituting (4) in (3) gives

\[ q_p = \frac{1}{R_T} \int V_s(t) \, dt. \] \hspace{1cm} (5)

Therefore the charge across the piezoelectric actuator is equal to the integral of the voltage across the sensing resistor divided by the total resistance. A high gain feedback loop is used to equate the desired charge with the actual charge.

The transfer function can now be derived. By considering a capacitor \( C_L \) as a model of the piezoelectric actuator (Fleming and Moheimani, 2005, Huang \textit{et al.}, 2010), the transfer function is:

\[ H(s) = \frac{V_s(s)}{V_o(s)} = \frac{R_T C_L s}{R_T C_L s + 1} \] \hspace{1cm} (6)

By taking the Z transform, the discrete transfer function between the output of the voltage amplifier \( V_o(z) \) and sensing voltage \( V_s(z) \) will be

\[ H(z) = \frac{V_s(z)}{V_o(z)} = \frac{z-1}{z - e^{-\frac{R_T}{R_T C_L}}} \] \hspace{1cm} (7)
where $T_s$ is the sampling time. Moreover, the transfer function of the digital charge drive algorithm is

$$V_o(z) = k(q_{\text{desired}} - \frac{T}{(z-1)R_T}V_1(z)) \quad (8)$$

where $k$ is the closed loop gain and $q_{\text{desired}}(z)$ is the desired charge. Substituting (8) in (7) results in

$$\frac{z - e^{-\frac{T}{R_Tc_L}}}{z - 1}V_1(z) = k(q_{\text{desired}}(z) - \frac{T_s}{(z-1)R_T}V_1(z)). \quad (9)$$

By considering $V_1(z) = \frac{q_{\text{actual}}(z)R_T(z-1)}{T_s}$, where $q_{\text{actual}}(z)$ is the actual charge across the piezo, the discrete transfer function from the input (desired charge) to the output (actual charge) is given by

$$\frac{q_{\text{actual}}(z)}{q_{\text{desired}}(z)} = \frac{T_k}{z - e^{-\frac{T}{R_Tc_L}}} + \frac{T_k}{R_T} \quad (10)$$

For a typical piezoelectric stack actuator with a capacitance of 3.4 $\mu$F using the digital charge amplifier with a 100 $\Omega$ sensing resistor and sampling frequency 40 kHz, the frequency response has zero dB gain with approximately zero phase-shift up to 10 kHz. This is the theoretical bandwidth. In Section 4 the practical bandwidth will be shown.

The principal difficulty in using this technique is drift of the displacement output. In the digital charge amplifier, because the ADC is not ideal, it
suffers from current leakage. This can lead to a bias voltage $V_{\text{bias}}$ at the input and is the main reason for the drift in the measured charge which is given by

$$q_{\text{measured}} = \frac{1}{R_c} \int (V_o(t) + V_{\text{bias}}) dt.$$  \hfill (11)

Due to the integration of the voltage bias, the measured charge will drift resulting in miscalculation of the actual charge across the piezoelectric actuator. The output voltage $V_o(t)$, which is equal to $k(q_{\text{desired}} - q_{\text{measured}})$, will also drift until it reaches the power supply rails.

Some drift removal methods have been proposed including the use of a low-pass filter (LPF) bias estimator, the integrator reset method and a modified integrator. These have been discussed in detail in previous work (Bazghaleh et al., 2010). The LPF bias estimator is easy to implement but does not work at frequencies lower than its cut-off frequency. The modified integrator has less drift compared to a LPF bias estimator but it has some signal distortion at high frequencies and the integrator reset is only useful if it is known that the signal crosses zero regularly. None of these methods have been shown to be successful in removing drift over a wide range of frequencies. In the next section a model-based drift correction method is described to improve the performance of the DCA.

### 3 Model-based drift correction

This section describes how the model-based drift correction operates. The block diagram is shown in Figure 2 and comprises an integrator unit, a nonlinear autoregressive exogenous model (NARX) and a data fusion block.
With reference to Figure 1, $I_s$ is calculated by dividing $V_s$ by $R_s$ and $V_p$ is calculated by subtracting $V_s$ from $V_o$.

![Figure 2: Model-based drift correction block diagram](image)

The integrator unit is used to integrate current passing through the piezo to calculate the charge. As described before, using the digital charge amplifier alone can provide an accurate estimate of charge but it will drift over time due to small errors in the voltage measurement and hence calculated current. As the new charge is calculated iteratively using the previous charge, the charge errors are accumulated and the error grows over time.

Data fusion is used to combine the reliability and short term accuracy of the digital charge amplifier with the long term accuracy of the model to remove the drift on the DCA and improve the accuracy of positioning.

### 3.1 Nonlinear autoregressive exogenous model (NARX)

The charge on the piezo actuator has a non-linear relationship with the applied piezo voltage and to model this, a black-box NARX model was
used. A NARX model determines the current value of a time series (in this case the charge output of the model) as a non-linear function of both past values of the time series and current and past inputs. The non-linear function is determined through training the black-box model. In this instance the model charge output is determined from the previous charge output of the model and present and past piezo voltages \( V_p \).

The model was trained by exciting the piezo with three triangle waves with slopes of \( \pm 80 \, \text{V/s} \), \( \pm 800 \, \text{V/s} \) and \( \pm 8000 \, \text{V/s} \) with maximum and minimum amplitude of \( \pm 20 \, \text{V} \) for a duration of 2s each. Furthermore, the piezo was excited by a \( \pm 20 \, \text{V} \) chirp function with a frequency range of 1 Hz to 100 Hz for a time period of 2 s. Data gathered through these excitations was used as the training data.

The voltage to displacement relationship of piezoelectric actuators exhibits rate-dependant behavior. This makes modeling difficult and therefore the NARX model is not accurate at all rates as shown in the first column of Figure 9. Later it will be described how this model can be used to remove the drift as the model’s important feature is that it is drift free.

### 3.2 Data fusion

The data fusion comprises two weighting coefficients \( w_a \) and \( w_b \) for the digital charge amplifier and the NARX model respectively. The sum of the weighting coefficients is unity. If \( w_a \) is one and \( w_b \) is zero, the system is a pure integrator and if \( w_a \) is zero and \( w_b \) is one, the system only relies on the model. Choosing \( w_a \) between zero and one can combine the benefits of both techniques. As illustrated in Figure 2, the optimal output charge is
\[ q_{\text{optimal}}[k] = w_a q_{\text{DCA}}[k] + w_b q_{\text{model}}[k] \] (12)

and the output of the integrator is given by

\[ q_{\text{DCA}}[k] = q_{\text{optimal}}[k-1] + T_s (I_s[k] + e_{ls}[k]), \] (13)

where \( T_s \) is the sampling time, \( I_s \) is the piezoelectric current and \( e_{ls} \) is the current error. Considering \( q_{\text{actual}} \) as the actual charge on the piezo and \( e_m \) as the error at the output of the NARX model, the charge in the model is estimated as

\[ q_{\text{model}}[k] = q_{\text{actual}}[k] + e_m[k]. \] (14)

Substituting (13) and (14) into (12) gives

\[ q_{\text{optimal}}[k] = w_a q_{\text{optimal}}[k-1] + w_a T_s I_s[k] + w_a T_s e_{ls}[k] + w_b q_{\text{actual}}[k] + w_b e_m[k]. \] (15)

Considering \( e_{q,\text{opt}} \) as the error in the optimal output charge,

\[ q_{\text{optimal}}[k] = q_{\text{actual}}[k] + e_{q,\text{opt}}[k]. \] (16)

Substituting (16) in (15) gives

\[ q_{\text{actual}}[k] + e_{q,\text{opt}}[k] = w_a (q_{\text{actual}}[k-1] + e_{q,\text{opt}}[k-1]) + w_a T_s I_s[k] + w_a T_s e_{ls}[k] + w_b q_{\text{actual}}[k] + w_b e_m[k] \] (17)

Considering \( q_{\text{actual}}[k] = q_{\text{actual}}[k-1] + T_s I_s[k] \),
\[ q_{\text{actual}}[k] + e_{q,\text{opt}}[k] = q_{\text{actual}}[k] + w_a e_{q,\text{opt}}[k-1] + w_T e_{k}[k] + w_e e_m[k] \]  

(18)

By cancelling \(q_{\text{actual}}[k]\) and simplifying (18) the following expression for the error in the optimal charge output is given by

\[ e_{q,\text{opt}}[k] = w_a e_{q,\text{opt}}[k-1] + w_T e_{k}[k] + w_e e_m[k]. \]  

(19)

Taking the z-transform of both sides of (19) and assuming zero-state conditions

\[ e_{q,\text{opt}}[z](1 - w_a z^{-1}) = w_T e_{k}[z] + (1 - w_a) e_m[z]. \]  

(20)

Rearranging (20) gives

\[ e_{q,\text{opt}}[z] = w_T e_{k}[z] / (1 - w_a z^{-1}) + (1 - w_a) e_m[z] / (1 - w_a z^{-1}). \]  

(21)

Taking the inverse z-transform of (21) gives

\[ e_{q,\text{opt}}[k] = (1 - w_a) \left( \sum_{n=0}^{k-1} w_a^n e_m[k-n] \right) \]

\[ + w_T \left( \sum_{n=0}^{k-1} w_a^n e_{k}[k-n] \right). \]  

(22)

It can be seen in the second term of (22) that as long as \(w_a\) is less than unity the effects of previous current errors diminishes over time. Simplifying this equation, the error on the optimal output will be
\[ e_{q,\text{opt}}[k] = \sum_{n=0}^{k-1} (w^n_T w_k e_{k-n}) + (1-w_k)e_m[k-n]) \]  

(23)

Because of the central limit theorem, the input current error \( e_{in} \) is considered to have a normal distribution with mean \( \mu_1 \) and standard deviation \( \sigma_1 \) and the model error \( e_m \) is assumed to have a normal distribution with mean \( \mu_2 \) and standard deviation \( \sigma_2 \) (Rice, 1995). The sum of two normally distributed signals is also normally distributed. Therefore (23) will be

\[ e_{q,\text{opt}}[k] = \sum_{n=0}^{k-1} (w^n e_{f(k-n)}) \]  

(24)

where \( f(k-n) \) has a normal distribution with mean

\[ \mu_f = T_s w_n \mu_t + (1-w_n)\mu_2 \]  

(25)

and variance

\[ \sigma_f^2 = T_s^2 w_n^2 \sigma_1^2 + (1-w_n)^2 \sigma_2^2. \]  

(26)

Eqn. (24) has a normal distribution with mean and variance

\[ \mu_{e,\text{opt}} = \mu_t \sum_{n=0}^{k-1} w^n \]  

(27)

\[ \sigma_{e,\text{opt}}^2 = \sigma_1^2 \sum_{n=0}^{k-1} (w^n)^2 \]  

(28)
If the system is a pure integrator, \( w_a \) is equal to one and (27) and (28) will diverge, the error will be accumulated and the output will drift. If \( w_a \) is between zero and one, (27) and (28) will converge and therefore there will not be drift of the output and

\[
\mu_{e,\text{opt}} = \frac{(T_w \mu_t + (1 - w_a) \mu_2)}{1 - w_a}
\]

(29)

\[
\sigma^2_{e,\text{opt}} = \frac{(T^2 w_a^2 \sigma_1^2 + (1 - w_a)^2 \sigma_2^2)}{1 - w_a^2},
\]

(30)

The mean square error (MSE) on the optimal output is

\[
MSE_{e,\text{opt}} = \mu^2_{e,\text{opt}} + \sigma^2_{e,\text{opt}}.
\]

(31)

The optimal value \( w_a \) is achieved when the MSE is minimized. In Section 4, the experimental results will show this method eliminates drift and reduces the error.

4 Experimental results and discussion

4.1 Experimental setup

The proposed technique was validated experimentally by using a piezoelectric stack actuator AE0505D44H40 from NEC. The displacement of the piezoelectric stack actuator was measured using a strain gauge from Vishay Electronics (EA-06-125TG-350), which is only used to evaluate the performance of the DCA. All digital algorithms, estimation and control functions were written in Matlab/Simulink then compiled for a dSPACE DS1104 development platform. Matlab/Simulink was also used for actuator characterization (Figure 3).
4.2 DCA linearity results

Figure 4 illustrates the improvement in linearity offered by the new digital charge amplifier compared to a standard voltage amplifier. Figure 4a shows the hysteresis loop when the piezo is driven by a voltage amplifier and Figure 4b shows the results when the piezo is driven by the DCA. It can be seen that the hysteresis loop is significantly reduced in the DCA. For a displacement range of 11.54 μm, the digital charge amplifier has maximum hysteresis of 144 nm while it is 1598 nm for the voltage amplifier for a driving frequency of 10 Hz. Therefore, the digital charge amplifier has reduced the hysteresis by 91%.
Figure 4: Experimental hysteresis loop of a piezoelectric stack actuator AE0505D44H40 subjected to a 10 Hz sine wave driven by a) a voltage amplifier and b) DCA

Displacement range = 11.54 µm

Maximum Hyst = 1598 nm

Displacement range = 11.54 µm
4.3 Drift and model-based drift correction results

Figure 5 shows experimentally the drift effect on the displacement of the piezoelectric actuator when it is driven by the digital charge amplifier with no drift removal and when the model-based drift correction method is used. It clearly shows no drift on displacement when it is driven using the model-based drift correction method.

![Figure 5: Experimental results for displacement of a piezoelectric actuator driven by the DCA, showing desired displacement (Black) versus the displacement when no drift removal technique (Blue) is used and when the model-based drift correction method (Red) is used.](image)

In this experimental setup, \( \mu_t = -0.00041 \) A, \( \sigma_t = 8.023 \times 10^{-5} \), which are the mean and standard deviation of input current noise respectively, and the model error has mean and standard deviation, \( \mu_2 = 1 \times 10^{-5} \) C and \( \sigma_2 = 4 \times 10^{-6} \) respectively. These values were obtained experimentally through statistical analysis of the ADC input and model output when driven
by a 10 Hz triangle waveform. By setting sampling time $T_s = 0.0001$ s, then (31) can be written as

$$MSE_{	ext{opt}}(w_a) = \frac{(-4.1 \times 10^{-8} w_a + 1 \times 10^{-5} (1 - w_a))^2}{(1 - w_a)^2} + \frac{(1.68 \times 10^{-15} w_a^2 + 1 \times 10^{-5} (1 - w_a)^2)}{1 - w_a^2}.$$ \hspace{1cm} (32)

It can be seen in (32) that the MSE is only dependent on $w_a$. Figure 6 shows MSE plotted against values of $w_a$.

![Figure 6: Mean square error plot for different $w_a$](image)

The MSE has a minimum value $4.76 \times 10^{-13}$ C$^2$ when $w_a$ is 0.9460. This value is chosen for $w_a$ to minimize the MSE. Compared to the model alone which has a MSE $1.16 \times 10^{-10}$ C$^2$, the model based drift correction method
has reduced the MSE by a factor 243 while it has removed the drift on the digital charge amplifier completely.

4.4 Frequency response of the output displacement to input charge

Figure 7 illustrates experimentally measured displacement versus charge at different frequencies. This shows that at higher frequencies the area within the loop between charge and displacement increases. This loop is not hysteresis but rather a linear phase shift. Figure 8 illustrates the frequency response of displacement output to charge input for the stack piezo. It can be seen that for frequencies higher than 100 Hz the phase lag starts to increase, which makes it undesirable for higher frequency tracking applications. As described by Goldfarb and Celannovic (1997a, 1997b) the relation between the charge and the displacement of the piezo introduces a phase lag as shown in Figure 8, and arises from the nature of the piezoelectric actuator itself.
Figure 7: Measured displacement versus charge for a sinusoidal input of 1 Hz, 20 Hz, 100 Hz and 500 Hz
Figure 8: Measured frequency response of the output displacement to input charge
4.5 SNR

Table 1: Signal to noise ratio (SNR) of the sensing voltage for $R_s$ equal to 10 Ω, 200 Ω and 400 Ω and at frequencies 0.01 Hz, 0.1 Hz, 1 Hz and 10 Hz.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>$R_s$=10 Ω</th>
<th>$R_s$=200 Ω</th>
<th>$R_s$=400 Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01 Hz</td>
<td>-20.91</td>
<td>-4.97</td>
<td>0.15</td>
</tr>
<tr>
<td>0.1 Hz</td>
<td>-18.94</td>
<td>14.34</td>
<td>19.97</td>
</tr>
<tr>
<td>1 Hz</td>
<td>3.92</td>
<td>34.27</td>
<td>39.81</td>
</tr>
<tr>
<td>10 Hz</td>
<td>24.65</td>
<td>40.98</td>
<td>41.29</td>
</tr>
</tbody>
</table>

Because of the RC circuit between the sensing resistor and the piezo, at low frequencies the voltage drop across the sensing resistor decreases significantly. This causes a reduction in the signal-to-noise ratio (SNR) which is shown in Table 1. Therefore the accuracy of measured charge decreases with decreasing frequency, which causes an increase in the error of the output displacement. Increasing the sensing resistance can increase the voltage drop and therefore improve the SNR. However, increasing the sensing resistor can cause the sensing voltage to reach the maximum ADC input voltage at lower frequencies and therefore limits the bandwidth of the DCA.

Choosing the appropriate sensing resistor is an important task when using the DCA. The sensing resistor should not be so small that the noise on the measured charge increases significantly and also it should not be so large that $V_s$ becomes more than the input ADC limitation at the required bandwidth.

The value of the sensing resistor should be set for each application. To select the best value for $R_s$, the maximum displacement at the maximum
frequency should be considered. Then $R_s$ should be calculated such that the sensing voltage will not be greater than the maximum ADC voltage. Using the highest possible sensing resistor increases the SNR at low frequencies.

### 4.6 Voltage drop

The voltage drop across any sensing element limits the piezoelectric actuator voltage range. Table 2 compares the maximum voltage drop for two hysteresis linearization techniques from the literature that have been evaluated experimentally (a grounded charge amplifier (Fleming and Moheimani, 2005, Minase et al., 2010), capacitor insertion (Kaizuka and Siu, 1988, Minase et al., 2010)) and the DCA at different frequencies. To make the data comparable, the displacement range is set to be ±5 µm. It can be seen that at lower frequencies the DCA voltage drop is significantly smaller than the voltage drop of the grounded charge amplifier or capacitor insertion method, thus maximizing the displacement that can be achieved for a given supply voltage.

**Table 2: Comparison of voltage drop and percentage of voltage drop using three alternative methods at four different frequencies**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Grounded charge amplifier ($C_s = 30 \mu F$)</th>
<th>Capacitor insertion ($C_s = 1 \mu F$)</th>
<th>DCA ($R_s = 55 \Omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 Hz</td>
<td>2.72(12%)</td>
<td>51.42(72%)</td>
<td>0.01(0.05%)</td>
</tr>
<tr>
<td>1 Hz</td>
<td>2.41(11%)</td>
<td>52.38(72%)</td>
<td>0.02(0.1%)</td>
</tr>
<tr>
<td>10 Hz</td>
<td>2.48(11%)</td>
<td>53.28(72%)</td>
<td>0.25(1.2%)</td>
</tr>
<tr>
<td>75 Hz</td>
<td>2.32(11%)</td>
<td>53.53(72%)</td>
<td>1.93(8.8%)</td>
</tr>
</tbody>
</table>
4.7 Tracking performance

In Figure 9 the desired displacement is compared against the actual displacement at different frequencies using the model-alone (when $w_a$ is zero and $w_b$ is one) and model-based drift correction.

<table>
<thead>
<tr>
<th>Freq</th>
<th>Model-alone</th>
<th>Model-based drift correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1Hz</td>
<td><img src="image1" alt="1Hz Model-alone" /></td>
<td><img src="image2" alt="1Hz Model-based drift correction" /></td>
</tr>
<tr>
<td>10Hz</td>
<td><img src="image3" alt="10Hz Model-alone" /></td>
<td><img src="image4" alt="10Hz Model-based drift correction" /></td>
</tr>
<tr>
<td>100Hz</td>
<td><img src="image5" alt="100Hz Model-alone" /></td>
<td><img src="image6" alt="100Hz Model-based drift correction" /></td>
</tr>
</tbody>
</table>

Figure 9: Experimental results of actual and desired charge in 1 Hz, 10 Hz and 100 Hz by using the Model-based drift correction method and the model-alone

It can be seen in Figure 9 that for 1 Hz and 100 Hz the model-alone is not as accurate because the NARX model is tuned at 10 Hz and is unable to track...
the rate dependent dynamics of the piezoelectric actuator. As the weighting of the digital charge amplifier is close to one (Section 4.3) the model errors do not significantly impact the tracking performance of the model-based drift correction results.

Figure 10 illustrates the output of the system in three different scenarios: a) when the system only relies on the DCA ($w_a$ is one and $w_b$ is zero) b) when the system only relies on the model ($w_a$ is zero and $w_b$ is one) c) model-based drift correction (when appropriate $w_a$ and $w_b$ are chosen). It clearly can be seen that model-based drift correction removes the drift on the DCA while its accuracy is much better than the model-alone.
Figure 10: Response to a decaying 5 Hz triangular input signal. The black square marker is desired displacement, the blue line is the displacement when the system only relies on the model ($w_a$ is zero and $w_b$ is one), the green line is when the system only relies on the DCA ($w_a$ is one and $w_b$ is zero) and the red line is the displacement with model-based drift correction (when appropriate $w_a$ and $w_b$ are chosen).

5 Conclusion

An innovative digital charge amplifier has been introduced to reduce the non-linear behavior of a piezoelectric actuator. For the experiments conducted, it was found that the charge amplifier reduced the hysteresis by 91% at 10 Hz. Compared to a grounded charge amplifier, the DCA has less voltage drop, does not need gain tuning and is far more cost effective. Moreover, a model-based drift correction technique has been proposed to remove the drift and improve the tracking performance of the digital charge
amplifier. It uses data fusion to integrate the reliability and short term accuracy of a digital charge amplifier, with the long term accuracy of the NARX model, to realise the benefit of both techniques. This easily implemented, digital charge drive approach overcomes many of the limitations of previous charge drive solutions, opening up new applications where the performance of a charge drive is beneficial such as scanning probe microscopy (Fleming and Leang, 2008). Future work will focus on improving the model and investigating methods to increase the bandwidth, especially at low frequencies.

References


6 Bandwidth Extension of the Digital Charge Amplifier

6.1 Introduction

As is mentioned in Chapter 3, the DCA’s lowest frequency of operation is limited due to the reduction of signal-to-noise-ratio of the sensing voltage at low frequencies.

In order to extend the DCA’s operational bandwidth, a novel hybrid digital method is proposed. As explained in Chapter 5, the relationship between applied voltage and displacement shows rate-dependant behaviour. Thus designing a model that can work over a wide range of operations is a complicated task. In this chapter, a non-linear model was designed and trained to estimate displacement based on the piezoelectric voltage only at low frequencies. The model and charge-based displacement estimators were used together through a complementary filter, to increase the bandwidth of displacement estimation and control. Moreover, the proposed method is designed to be capable of driving grounded-loads such as piezoelectric tube actuators.
Experimental results clearly show that both the tracking performance of the new method is improved and it can also operate over a wider frequency bandwidth.
6.2 A novel digital charge-based displacement estimator for sensorless control of a grounded-load piezoelectric tube actuator

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# Statement of Authorship

<table>
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## Author Contributions

By signing the Statement of Authorship, each author certifies that their stated contribution to the publication is accurate and that permission is granted for the publication to be included in the candidate's thesis.

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<tr>
<th>Name of Principal Author (Candidate)</th>
<th>Mr. Mohsen Bazghaleh</th>
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<tbody>
<tr>
<td>Contribution to the Paper</td>
<td>Developed the model, developed theory, performed experimental work, analyzed data and Wrote the manuscript.</td>
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<th>Dr. Steven Grainger</th>
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<tr>
<td>Contribution to the Paper</td>
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</tbody>
</table>
A novel digital charge-based displacement estimator for sensorless control of a grounded-load piezoelectric tube actuator

Mohsen Bazghaleh, Steven Grainger, Morteza Mohammadzaheri, Ben Cazzolato and Tien-Fu Lu

Abstract. Piezoelectric tube actuators are widely used in nanosensing applications, especially in scanning probe microscopes to manipulate matter at nanometre scale. Accurate displacement control of these actuators is critical, and in order to avoid the expense and practical limits of highly accurate displacement sensors, sensorless control has recently attracted much attention. As the electrical charge on these actuators is an accurate indicator of their displacement exhibiting almost no hysteresis over a wide range of frequencies, it suggests that charge measurement can replace displacement sensors. However, charge-based methods suffer from poor low frequency response and voltage drop across the sensing capacitor. In this paper, a displacement estimator is presented that complements a digitally implemented charge amplifier with an artificial neural network (ANN) designed and trained to estimate the piezoelectric tube’s displacement using the piezoelectric voltage at low frequencies of excitation where the charge methods fail. A complementary filter combines the grounded-load digital charge amplifier (GDCDE) and the ANN to estimate displacement over a wide bandwidth and to overcome drift. The discrepancy between the desired and estimated displacement is fed back to the piezoelectric actuator using proportional control. Experimental results highlight the effectiveness of the proposed design.

Keywords: Piezoelectric tube actuators; sensorless control; displacement estimation; artificial neural network; complementary filter.
1 Introduction

Scanning Probe Microscopes (SPMs) enable the manipulation of matter at nanometre scale (Moheimani, 2008). In traditional SPMs, a tripod positioner with three stack piezoelectric actuators are employed for movement in the x, y and z directions. Binnig and Smith (1986) used a piezoelectric tube scanner for the first time in 1986. Compared to tripod positioners, piezoelectric tube scanners offer better accuracy, higher bandwidth and an easier manufacturing process (Devasia et al., 2007) while their smaller size simplifies vibration isolation (Chen, 1992). Piezoelectric tubes are the foremost actuators in atomic force microscopy (AFM) (Abramovitch et al., 2007, Kuiper and Schitter, 2010) and are likely to remain the most widely used positioning actuators in other micro-scale and nano-scale positioning tasks for several years (Moheimani, 2008); such as displacement of fiber optics (Leung et al., 2008), ultrasonic applications (Hui et al., 2010) and in ink jet printers (Chen and Tsao, 1977).

In order to avoid the use of expensive highly accurate displacement sensors, sensorless control has recently attracted much attention. The voltage across the piezoelectric tube (piezoelectric voltage) can represent its displacement. However, the relationship between actuator displacement and the piezoelectric voltage suffers from hysteresis and creep which are nonlinear in nature and reduce the positioning accuracy (Leang and Devasia, 2002).

Creep results from remnant polarization which causes the displacement of the piezoelectric actuator to change once the applied voltage is held fixed, and typically is an issue at low frequencies (Minase et al., 2010a). Hysteresis happens due to the polarization of microscopic particles. In practice, it means that, for similar values of the piezoelectric voltage, the
piezoelectric actuator has a variety of displacement values, and this cannot be described with linear models.

Charge regulation and capacitor insertion techniques have been used for linearization (Minase et al., 2010b). Charge regulation relies on the fact that there is almost no hysteresis or other nonlinearity between the charge across the piezo and the displacement of the piezo, so the electrical charge on the piezoelectric can represent displacement more effectively than the applied piezoelectric voltage. Charge regulation was first introduced by Comstock (1981) but has historically been complicated and expensive to implement due to the complexity of the analog circuitry required to meet stringent performance targets (Fleming and Moheimani, 2005).

This paper presents a novel sensorless approach that combines charge-based displacement estimation with mapping of piezoelectric voltage to displacement using an artificial neural network to increase the bandwidth of the developed system and to remove drift. In Section 2 the design principle is described. Section 3 explains the experimental set up. The closed loop control system is analyzed in Section 4. Section 5 addresses ANN model design for low frequency operation. Experimental results are presented in Section 6 and the conclusions are drawn in Section 7.

## 2 Design principles

Over a wide range of frequencies, the electrical charge \( q_p \) on a piezoelectric actuator is proportional to its displacement \( d \) (Fleming and Moheimani, 2005, Newcomb and Flinn, 1982); and may be expressed as

\[
d(t) = Kq_p(t).
\]
Figure 1 shows the basis of the proposed grounded-load digital charge-based displacement estimator (GDCDE) which aims to estimate displacement using $q_p$. In GDCDE, a sensing resistor ($R_s$) is placed in series with the piezoelectric tube actuator and an isolation amplifier is used to measure the voltage, $V_s$, across this resistor.

The integral of electrical current equals charge, thus displacement can be estimated by

$$\hat{d}(t) = K q_p(t) = K \int_0^t i_p(t) dt = \frac{K}{R_s} \int_0^t V_s(t) dt,$$

where $i_p$ is the electric current passing through the piezoelectric actuator, and $t_f$ is the elapsed time.

Due to the RC circuit formed by the sensing resistor and the piezoelectric actuator which may be approximated as a capacitor, at low frequencies the sensing voltage is very small which causes a reduction in its signal to noise ratio (SNR). Also, the analog to digital converter (ADC) is not ideal and suffers from offset voltage and drift. These issues together with dielectric leakage of the piezoelectric actuator causes a bias voltage, $V_{bias}$ and in practice, the estimated displacement is

$$\hat{d}(t) = \frac{K}{R_s} \int_0^t (V_s(t) + V_{bias}) dt.$$

In this paper, at low frequencies, an artificial neural network (ANN) has been designed and trained to map the voltage across the piezoelectric actuator, $V_p$, to displacement. A complementary filter merges these two
displacement estimation methods to increase the bandwidth of the hybrid system. The complementary filter comprises a low pass filter \( W_{\text{ANN}} \) on the output of the ANN and a high pass filter \( W_{\text{GDCDE}} \) on the output of the GDCDE. The high pass and low pass filters used are given by

\[
W_{\text{GDCDE}}(s) = \frac{s}{s + \omega_c}, \tag{4}
\]

\[
W_{\text{ANN}}(s) = \frac{\omega_c}{s + \omega_c}, \tag{5}
\]

and

\[
W_{\text{ANN}}(j\omega) + W_{\text{GDCDE}}(j\omega) = 1 \tag{6}
\]

Where \( \omega_c \) is the cut-off frequency, \( \pi \) rad/s (0.5 Hz). \( \omega_c \) was determined experimentally and is related to the decreasing SNR with frequency. During operation, the complementary filter produces the estimated displacement:

\[
\hat{d} = W_{\text{GDCDE}}\hat{d}_{\text{GDCDE}} + W_{\text{ANN}}\hat{d}_{\text{ANN}} \tag{7}
\]
As shown in Figure 1, in the discrete domain, the charge of the $k^{th}$ sample is estimated as

$$q_p(k) = \frac{T_s}{z-1} i_p(k) = \frac{T_s z^{-1}}{1 - z^{-1}} i_p(k). \tag{8}$$

where $T_s$ is the sampling time of the system. As a result,

$$q_p(k) = T_i i_p(k-1) + q_p(k-1). \tag{9}$$

In order to improve displacement estimation and remove drift Eq. (9) is replaced by (Bazghaleh et al., 2011)

$$q_p(k) = T_i i_p(k-1) + \frac{\hat{d}(k-1)}{K}. \tag{10}$$

This uses the output of the complementary filter in discrete integration instead of the output of the GDCDE (Figure 2).

![Figure 2: The proposed displacement estimation system](image)
3 Experimental setup

The proposed technique was validated experimentally by using a piezoelectric tube actuator PT130.24 from PI. The tube has one grounded inner electrode and four equally distributed outer electrodes (Figure 3).

PI-D-510 capacitive sensors together with an E-852.10 signal conditioner and a cubical end-effector were used to measure actual displacement. During the experiment, the capacitive sensor is held relative to the tube using an aluminum cylinder (Figure 4).
CHAPTER 6. A NOVEL DIGITAL CHARGE-BASED DISPLACEMENT ESTIMATOR FOR SENSORLESS CONTROL OF A GROUNDED-LOAD PIEZOELECTRIC TUBE ACTUATOR

In scanning devices, the end-effector typically follows a raster pattern (Moheimani, 2008, Abramovitch et al., 2007, Schitter et al., 2007); that is, in one axis (say $x$), the tube should track a triangular waveform, and in the other (say $y$), the tube should track a very slowly increasing ramp, e.g. quasi-static with respect to $x$. For test purposes, the $y$ electrode is earthed, replaced by a DC signal or open circuited (Moheimani, 2008, Bhikkaji and Moheimani, 2009, Bhikkaji et al., 2007a, Bhikkaji et al., 2007b). In this research, both $y^+$ and $y^-$ electrodes were earthed and the displacement controller drove one electrode ($x^+$) directly and an inverted signal drove $x^-$ to increase the displacement range (Fleming and Leang, 2008). All algorithms were developed in MATLAB/Simulink then compiled for a dSPACE DS1104 development platform.

Figure 5 shows the control system loop closed on estimated displacement. It consists of a displacement estimation unit, a voltage amplifier, digital to analog converter (DAC) and an analog to digital converter (ADC). The displacement estimation unit is as described before and shown in Figure 2.
The high gain \((K_c = 230 \text{ V}/\mu\text{m})\) feedback loop is used to equalize the desired displacement \(d_r\) with the estimated displacement \(\hat{d}\).

4 Control system analysis with the GDCDE

From Eq. (1) and as the electrical charge is equal to the integral of the electric current:

\[
\hat{d}_{\text{GDCDE}} = K \frac{V_s}{R_s}
\]

or

\[
V_s = \frac{\hat{d}_{\text{GDCDE}} R_s}{K}. \tag{12}
\]

By considering a capacitor \(C_L\) as an approximate model of the piezoelectric actuator (Fleming and Moheimani, 2005, Huang et al., 2010), the relation
between the input voltage, $V_i$, and the sensing voltage of the $x+$ electrode (as shown in Figure 1), $V_s$, is

$$
\frac{V_i}{V_s} = \frac{R_i C_L s + 1}{R C_L s}.
$$

(13)

Combining Eqs. (12) and (13) results in the following expression

$$
V_i = \frac{R C_L s + 1}{R C_L s} \frac{\dot{d}_{GDCDE}}{K} = \frac{R C_L s + 1}{K C_L} \dot{d}_{GDCDE}.
$$

(14)

In the control circuit, shown in Figure 5 with a gain of $K_c$, the input voltage is

$$
V_i = K_c (d_i - \dot{d}).
$$

(15)

If the GDCDE alone is used, then the input voltage is

$$
V_i = K_c (d_i - \dot{d}_{GDCDE}).
$$

(16)

Equating Eq. (14) with Eq. (16),

$$
V_i = \frac{R C_L s + 1}{K C_L} \dot{d}_{GDCDE} = K_c (d_i - \dot{d}_{GDCDE}).
$$

(17)

Therefore,

$$
\left( \frac{R C_L s + 1}{K C_L} + K_c \right) \dot{d}_{GDCDE} = K_c d_i.
$$

(18)

or,
\[ \frac{\dot{d}_{\text{GCDE}}}{d_c} = \frac{K_c}{R C_s s + 1 + K_c} = \frac{K_c K C_{L}}{R C_s s + 1 + K_c K C_{L}}. \]  

(19)

As a result, the break (critical) frequency of the displacement estimation loop is

\[ f_{\text{critical}} = \frac{1 + K_c K C_{L}}{R C_{L}} \]  

(20)

The piezoelectric tube PT130.24 has an approximate capacitance of 8.5 nF, and the resistance of the sensing resistor is 50 kΩ. As a result, with \( K = K_c = 0 \), \( f_{\text{critical}} = 374.5 \) Hz; in other words, \( f_{\text{critical}} > 374.5 \) Hz (2352 rad/s). This shows a high gain is a satisfactory controller over a wide range of frequencies. The analysis is valid while Eqs. (1) and (11) remain valid.

5 ANN model

The piezoelectric tube was excited by triangular waves with magnitudes of 20 V, 40 V and 60 V at frequencies of 0.1 Hz and 0.5 Hz for 10 s each. The training frequencies were selected to be below those at which the charge displacement estimation operates effectively but above DC. The data gathered with the excitation magnitude of 40 V at 0.5 Hz were used for validation and the rest for ANN modeling. Additional frequencies, as listed in Figure 7, were later used to test the complete control system.
5.1 Model structure

The structure of the model, mapping the piezoelectric voltage represented by \( V_p \), to displacement, is Nonlinear Auto-Regressive with eXogenous inputs or NARX (Nelles, 2001); that is

\[
d(t) = F \left( V_p(t - t_d), V_p(t - t_d - T_s), \ldots, V_p(t - t_d - r_v T_s), \right),
\]

(21)

Where \( t \) is time, \( t_d \) is the delay time, \( T_s \) is the sampling time, \( r_v \) and \( r_d \) are the piezoelectric voltage and displacement orders respectively, and \( F \) is an artificial neural network. In the discrete domain, the model will be

\[
d(k) = F \left( V_p(k - r_{de}), V_p(k - r_{de} - 1), \ldots, V_p(k - r_{de} - r_v), \right),
\]

(22)

where \( r_{de} = \frac{t_d}{T_s} \) and \( k = \frac{t}{T_s} \). The value of \( k \) (index) is higher than the maximum of \( (r_{de} + r_v) \) and \( r_d \), namely \( r_{max} \).

In the modeling process the values of \( t_d \), \( T_s \), \( r_v \) and \( r_d \) are estimated based on the available knowledge of the system dynamics.

A semi-linear ANN, with both linear and nonlinear activation functions in the hidden layer, especially suitting systems with slight nonlinearities (Mohammadzaheri et al., 2009, Mohammadzaheri et al., 2012a) was employed in this research. Figure 6 shows a typical semi-linear ANN where \( T \) and \( W \) vectors/matrices are the weights of connections of the ANN and \( b \) represents the bias.
\[ d(k) = R \times \left( \sum_{i=0}^{r+q} W_{Li} V_p(k-i) + \sum_{i=1}^{q_b} T_{Li} d(k-i) + b_L \right) + \sum_{i=0}^{r+b} W_{N1} V_p(k-i) + \sum_{i=1}^{q_b} T_{Ni} d(k-i) + b_N + b. \]  

Figure 6: A semi-linear ANN (Mohammadzaheri et al., 2012a)

5.2 Data preparation

In input-output-data-based modeling, if some variables have values with higher magnitudes than others, they are likely to influence the modeling process more significantly (Ghaffari et al., 2007). Therefore, the experimental data are first normalized in black box modeling.
After normalization, the data, which originally have two columns in this case (the piezoelectric voltage and displacement), are rearranged for the purpose of neural network training. The prepared data forms a matrix with \( r_d + r_v + 1 \) input columns and one output column as shown in Eq. (25) where \( n \) is the number of data elements in each column of raw data.

\[
\begin{bmatrix}
V_p(k_{\min} - r_{de}) & V_p(k_{\min} - r_{de} - 1) & \cdots & V_p(k_{\min} - r_{de} - r_v) \\
V_p(k_{\min} - r_{de} + 1) & V_p(k_{\min} - r_{de}) & \cdots & V_p(k_{\min} - r_{de} - r_v + 1) \\
\vdots & \vdots & \ddots & \vdots \\
V_p(n - r_{de}) & V_p(n - r_{de} - 1) & \cdots & V_p(n - r_{de} - r_v)
\end{bmatrix}
\]

\[
\begin{bmatrix}
d(k_{\min} - 1) & d(k_{\min} - 2) & \cdots & d(k_{\min} - r_d) \\
d(k_{\min}) & d(k_{\min} - 1) & \cdots & d(k_{\min} - r_d + 1) \\
\vdots & \vdots & \ddots & \vdots \\
d(n - 1) & d(n - 2) & \cdots & d(n - r_d)
\end{bmatrix}
\]

(25)

where \( k_{\min} = \max((r_{de} + r_v), r_d) + 1 \).

The Nguyen-Widrow algorithm was used for weight initialisation (Nguyen and Widrow, 1990) and Levenberg-Marquardt-Batch-Error-Back-Propagation was employed to train the ANN (Mohammadzaheri et al., 2009).

5.3 Validation

There are two different approaches to validate models: one-step-prediction and simulation. In one-step-prediction, all the inputs to the model are
recalled from memory, or their real values are assumed to be available. For the dynamic model presented in Eq. (21), the one-step-prediction output of the model is

\[
\hat{d}(t) = F\left(V_p(t-t_a), V_p(t-t_a-T_s), \ldots, V_p(t-t_a-r_v T_s), d(t-T_s), d(t-2T_s), \ldots, d(t-r_a T_s)\right),
\]

(26)

where the variable(s) with a hat represent estimated values.

However, in simulation of dynamic models, delayed model outputs are used as model inputs after the very first simulation instants and Eq. (21) changes to Eq. (22):

\[
\hat{d}(t) = F\left(V_p(t-t_a), V_p(t-t_a-T_s), \ldots, V_p(t-t_a-r_v T_s), \hat{d}(t-T_s), \hat{d}(t-2T_s), \ldots, \hat{d}(t-r_a T_s)\right),
\]

(27)

Therefore, the inevitable error of estimated outputs returns to the validation process and increases the resultant error due to ‘error accumulation’ (Nelles, 2001, Mohammadzaheri et al., 2012b, Bazghaleh et al., 2013). In this research, \( t_a = T_s = 0.001 \) s, and \( r_v = r_a = 7 \) were selected empirically for the neural network model.

## 6 Experimental results

The voltage drop across any sensing element limits the piezoelectric actuator voltage range. Table 1 compares the maximum voltage drop for two hysteresis linearization techniques from the literature that have been evaluated experimentally (a grounded charge amplifier (Fleming and Moheimani, 2005), capacitor insertion (Kaizuka and Siu, 1988)) and the
GDCDE at different frequencies. To make the data comparable, the displacement range is set to be ±6 µm. It can be seen that at lower frequencies the GDCDE voltage drop is significantly smaller than the voltage drop of the grounded charge amplifier or capacitor insertion method, thus maximizing the displacement that can be achieved for a given supply voltage.

Table 1: Comparison of the maximum voltage drop and its proportion to the input (driver) voltage expressed as a percentage using the three different methods at the frequencies 5, 10, 20 and 50 Hz.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Grounded-load charge amplifier ((C_s = 78 \text{ nF}))</th>
<th>Capacitor insertion ((C_{\text{series}} = 10 \text{ nF}))</th>
<th>GDCDE ((R_s = 20 \text{ KΩ}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Hz</td>
<td>26 (30%)</td>
<td>82.5 (57%)</td>
<td>0.4147 (0.69%)</td>
</tr>
<tr>
<td>10 Hz</td>
<td>14.9 (19%)</td>
<td>82.5 (57%)</td>
<td>0.8294 (1.36%)</td>
</tr>
<tr>
<td>20 Hz</td>
<td>10.4 (14%)</td>
<td>82.5 (57%)</td>
<td>1.6588 (2.69%)</td>
</tr>
<tr>
<td>50 Hz</td>
<td>8.83 (12.8%)</td>
<td>82.5 (57%)</td>
<td>4.1469 (6.01%)</td>
</tr>
</tbody>
</table>

The value \(C_{\text{series}}\) for capacitor insertion was chosen from (Minase et al., 2010b) and value \(C_s\) for grounded-load charge amplifier was chosen from (Fleming and Moheimani, 2005).

Table 2 compares the lowest operational frequency for each method where displacement is linear with the applied signal. It indicates that this method can extend low frequency operation.

Table 2: Lowest frequency of linear operation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest frequency</td>
<td>2 Hz</td>
<td>1 Hz</td>
<td>0.8 Hz</td>
<td>1 Hz</td>
<td>0.04 Hz</td>
</tr>
</tbody>
</table>

Figure 7 shows the control performance of the proposed GDCDE, the designed ANN and their combination through the complementary filter. The
mean of absolute control error defined as the ratio of the mean of absolute
displacement error to the range of displacement varies from 0.5% to 2%.
Figure 8 shows the desired displacement versus the actual displacement and
the piezoelectric voltage (control input) respectively and Figure 9 illustrates
the open-loop frequency response of the piezoelectric tube actuator.

<table>
<thead>
<tr>
<th>Freq (Hz)</th>
<th>ANN Model</th>
<th>GDCDE</th>
<th>Complementary filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
<tr>
<td>0.07</td>
<td><img src="image4.png" alt="Graph" /></td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
<tr>
<td>0.1</td>
<td><img src="image7.png" alt="Graph" /></td>
<td><img src="image8.png" alt="Graph" /></td>
<td><img src="image9.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
Figure 7: Experimental results for tracking at different frequencies using different displacement estimators. The dot-dashed line (-.-) is desired displacement, the solid line (-) is the actual displacement and the dashed line (--) is the open-loop response.
Figure 8: a) Displacement versus open-loop applied voltage at a sinusoidal frequency of 5 Hz. b) Control performance for tracking a sinusoidal reference displacement at 5 Hz.
This research considered the design of a digital sensorless control system for the displacement of piezoelectric tube actuators. As the electrical charge is an accurate estimator of displacement over a wide range of excitation frequencies, the sensing voltage (Figure 1) was employed to estimate charge. A grounded-load digital charge-based displacement estimator was developed to replace displacement sensors on this basis. However, charge-based displacement estimation methods fail to perform satisfactorily at low frequencies. A semi-linear neural network was designed and trained to estimate displacement based on the piezoelectric voltage at low frequencies.
(<0.5 Hz) (Figure 1). The ANN and charge-based displacement estimators were used together through a complementary filter to increase the bandwidth of displacement estimation and control. Using a proportional controller, promising experimental results were achieved.

References


CHAPTER 6. A NOVEL DIGITAL CHARGE-BASED DISPLACEMENT ESTIMATOR FOR SENSORLESS CONTROL OF A GROUNDED-LOAD PIEZOELECTRIC TUBE ACTUATOR
7 Conclusions and Recommendations for Future Work

The aim of this research was to investigate the efficacy of a synergistic approach to the creation of hybrid digital algorithms which tackle challenges arising in the control of non-linear devices such as piezoelectric actuators. The following section draws conclusions from the research presented in this thesis, while recommended future work is presented in Section 7.2.

7.1 Conclusions

Piezoelectric actuators are the most commonly used nanopositioning actuators. However, they suffer from performance issues arising from their fundamental non-linearities.

The literature review, conducted as part of this thesis, indicates that charge drive is a method which can significantly reduce the nonlinearities of the piezoelectric actuator. Therefore, in this thesis charge drive is used as a promising starting point.
In the first step, a comprehensive analysis was carried out to establish the performance limitations of the charge drive approach and revealed that the charge methods adopted to date were implemented using analog circuitry and as a result have historically been complicated and expensive to implement due to the complexity of the analog circuitry.

In order to overcome these performance limitations this thesis focused on using digital technologies as a platform. This is central to enabling a synergistic approach that combines multiple conceptual technologies as required (e.g. Artificial Neural Networks (ANNs), fuzzy logic and complementary filters) in an integrative fashion.

A novel digital charge amplifier (DCA) was derived for a piezoelectric actuator that overcomes some inherent limitations found in analog charge amplifiers developed in previous research such as poor low frequency performance and significant voltage drop. While the DCA significantly reduced the hysteresis and improved the linearity of piezoelectric actuators the DCA suffers from its own limitations but the synergistic approach opens up the possibility of integration with other displacement control methods to improve the overall performance of the resulting displacement controller. The computational performance of modern digital devices ensures the viability of this.

To address the issue of drift in the DCA a model-based drift correction technique was proposed. It has been shown how a drift free model can be integrated with the DCA to remove the DCA’s inherent drift while the DCA itself tackles the issue of hysteresis. To maximize model accuracy, a novel hybrid method was proposed which incorporates a velocity signal into the model to reduce the effect of error accumulation.
Furthermore to extend the DCA operational bandwidth, a technique is proposed that integrates a non-linear model designed and trained to estimate displacement based on the piezoelectric voltage only at low frequencies. The model and charge-based displacement estimators were effectively utilized together through a complementary filter. In addition, the proposed method is designed to be capable of driving grounded-loads such as piezoelectric tube actuators.

This research has satisfied the aim of investigating the efficacy of a synergistic approach to the creation of solutions that allow new levels of performance to be achieved.

### 7.2 Recommendations for future work

It is recommended that further research be undertaken in the following areas:

- Electrically, a piezoelectric actuator behaves as a non-linear capacitor which is the main reason for nonlinearities. The piezo current, which can be estimated using the sensing resistor, and piezo voltage may be combined to estimate the real-time value of the non-linear capacitor. This value, along with the piezo voltage, can be used to estimate the piezo charge which could be used to implement a new type of charge drive. Moreover, an estimate of the real-time value of the non-linear capacitor may lead to a more accurate controller which can be tuned in real-time. This technique may take advantage of synergistic approaches for better accuracy (e.g. using modeling and complementary filter).
A controller should be designed for a multi-axis nanopositioner, e.g., piezoelectric tube actuators. The most significant difficulty is the existence of cross-coupling between the x, y and z axes. Due to the complexity of this cross-coupling, a comprehensive controller that can compensate the effect of cross-coupling has not been carried out. The proposed synergistic approach should be investigated in addressing this issue to obtain improved performance.

Recently the piezoelectric strain-induced voltage has been used to estimate the tube deflection. There is not any research that uses strain-induced charge rather than strain-induced voltage to estimate the deflection of the tube. In addition, there is a potential of integrating the strain-induced charge with the proposed methods in order to increase the accuracy of the displacement controller.
Appendix A. Experimental Setup

In this thesis two separate setups were used; one for driving a piezoelectric stack actuator and the other for driving a piezoelectric tube actuator both of which are described in the following sections.

A.1 Driving a piezoelectric stack actuator

A.1.1 General description

Figure A.1 shows the schematic of the experimental setup for driving a piezoelectric stack actuator. It is driven by a voltage amplifier through a sensing circuit. The sensing circuit, Figure A.2, consists of a sensing resistor which is in series with the piezo and a protection circuit to protect the dSPACE from high voltage. The sensing circuit provides the sensing voltage to the dSPACE in order to estimate the charge across the piezo, as described in Chapter 3. The displacement of the piezoelectric actuator is provided by a strain gauge, attached to the piezoelectric actuator. The strain gauge is in a full bridge configuration in order to compensate for temperature. The strain gauge is connected to a strain gauge signal conditioner. The dSPACE controller board, installed in a PC, provides the control signal to the voltage amplifier, and samples the output of the sensing circuit and the strain gauge signal conditioner.
A.1.2 Sensing circuit

Figure A.2 shows the circuit diagram of the sensing circuit. The sensing resistor, $R_{\text{sensing}}$, is in series with the piezoelectric stack actuator in order to provide the sensing voltage to the dSPACE. The resistor $R_p$ and the diodes $D_1$ and $D_2$ are used to protect the dSPACE from high voltage.
A.1.3 The piezoelectric stack actuator

In the first experimental setup a piezoelectric stack actuator, AE0505D44H40, from NEC is used. The specifications are:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer:</td>
<td>NEC</td>
</tr>
<tr>
<td>Model number:</td>
<td>AE0505D44H40</td>
</tr>
<tr>
<td>Dimensions:</td>
<td>5 x 5 x 40 mm</td>
</tr>
<tr>
<td>Maximum driving voltage</td>
<td>150 VDC</td>
</tr>
<tr>
<td>Recommended driving voltage</td>
<td>100 VDC</td>
</tr>
<tr>
<td>Displacement @ Maximum driving voltage</td>
<td>42.0±6.6 µm</td>
</tr>
<tr>
<td>Displacement @ Recommended driving</td>
<td>28.0±6.6 µm</td>
</tr>
<tr>
<td>voltage</td>
<td></td>
</tr>
<tr>
<td>Generated force (Compression):</td>
<td>850 N</td>
</tr>
<tr>
<td>Resonance frequency</td>
<td>34 kHz</td>
</tr>
<tr>
<td>Capacitance</td>
<td>3.4 µm</td>
</tr>
<tr>
<td>Insulation resistance</td>
<td>5 MΩ</td>
</tr>
</tbody>
</table>

A.1.4 The strain gauge

The strain gauge EA-06-125TG-350 from the Vishay Measurement Group, Inc, is used to measure the displacement of the stack actuator. It is glued to the top and bottom of the stack actuator and, in order to compensate for the temperature variation, a full bridge configuration is used. A strain gauge signal conditioner drives the strain gauge sensor and provides the displacement signal to the dSPACE. The specifications are:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer:</td>
<td>Vishay Micro-Measurements Division, Inc</td>
</tr>
<tr>
<td>Model number:</td>
<td>EA-06-125TG-350</td>
</tr>
<tr>
<td>Maximum strain:</td>
<td>3% gauge length - 0.1 mm</td>
</tr>
<tr>
<td>Resistance@ 24ºC</td>
<td>350.0 Ω ± 0.2%</td>
</tr>
</tbody>
</table>
APPENDIX A. EXPERIMENTAL SETUP

Strain gauge amplifier:

Manufacturer: Electronics Workshop, School of Mech. Eng., Univ. of Adelaide

Bridge excitation voltage: +5 VDC
Bridge type: Full Wheatstone bridge
Variable gain stage: 5000 V/V
Fixed gain Stage: 100 V/V
Offset voltage: ±12 VDC

A.1.5 Piezo driver

A power amplifier E-865.10 from Physik Instrumente (PI) is used to drive the piezoelectric stack actuator. It provides voltage ranging from -20 V to 120 V, with a maximum power of 30 W and a gain of 10. It has one BNC type input connector and one LEMO type output connector.

Manufacturer: PI
Model number: E-865.10 - LVPZ Power Amplifier Module
Output voltage range: -20 to +120 V
Max. average output current: 0.22 A
Max. output current: 2.0 A
Max. average output power: 30 W
Voltage gain: 10

A.2 Driving the piezoelectric tube actuator

A.2.1 General description

The experimental setup used to drive the piezoelectric tube actuator is shown in Figure A.3. The two opposite electrodes of the tube are each driven by an individual voltage amplifier through a sensing circuit, which
provides the sensing voltages to dSPACE to calculate the charge, described in Chapter 6. The displacement is measured using a capacitive sensor and a signal conditioner which provides the displacement signal to the dSPACE.

Figure A.3: The schematic of the experimental setup for driving a piezoelectric tube actuator

A.2.2 Test rig of the tube actuator

Figure A.4a shows the test rig for the tube scanner. The tube scanner is mounted into an aluminium base to secure one end of the tube and to hold the tube vertically. The piezoelectric tube scanner deflects the end-effector in the x, y and z directions. Deflections change the capacitance between the end-effector and the capacitive sensor that allowing the movement of the end-effector to be measured.

Figure A.4b shows the cylindrical cover which is mounted on the aluminium cylinder to protect the tube actuator and to keep the capacitive sensors parallel with the end-effector.
APPENDIX A. EXPERIMENTAL SETUP

Figure A.4: The piezoelectric tube actuator setup a) with no cover b) covered by the cylinder holding the capacitive displacement sensors

A.2.3 Sensing circuit

Figure A.5 shows the circuit diagram of the sensing circuit for driving the tube actuator. The sensing resistor, $R_{\text{sensing}}$, is in series with the piezoelectric tube actuator. The Isolation amplifier is used to provide the sensing voltage to the dSPACE while protecting the dSPACE from high voltage.
A.2.4 The piezoelectric tube actuator

The piezoelectric tube actuator is a PT130.24 from PI. The tube has one inner electrode and four equally distributed outer electrodes (Figure A.6).

![Schematic of piezoelectric tube scanner PT130.24]

**Figure A.6: Schematic of piezoelectric tube scanner PT130.24**

The specifications are as follows:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer:</td>
<td>PI</td>
</tr>
<tr>
<td>Model number:</td>
<td>PT130.24</td>
</tr>
<tr>
<td>Dimensions (L x OD x ID*):</td>
<td>30 x 10.0 x 9.0 mm</td>
</tr>
<tr>
<td>Number of outer electrodes/ inner electrode</td>
<td>4/1</td>
</tr>
<tr>
<td>Maximum operating voltage</td>
<td>200 V</td>
</tr>
<tr>
<td>Axial contraction @ Maximum operation voltage</td>
<td>9 µm</td>
</tr>
<tr>
<td>Radial contraction @ Maximum operation voltage</td>
<td>3 µm</td>
</tr>
<tr>
<td>XY deflection @ Maximum operation voltage</td>
<td>10 µm</td>
</tr>
<tr>
<td>Capacitance</td>
<td>4 x 8.5 nF</td>
</tr>
</tbody>
</table>

*L (length), OD (outer diameter), ID (inner diameter)
A.2.5 The capacitive sensor

The capacitive sensor is from PI, model number D-510.050. It allows for non-contact measurement of the end-effector displacement. The signal conditioner E-852.10 from PI is used to interface the capacitive sensor in order to provide an appropriate displacement signal to dSPACE. The specifications are:

**Capacitive sensor:**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer:</td>
<td>PI</td>
</tr>
<tr>
<td>Model number:</td>
<td>D-510.050</td>
</tr>
<tr>
<td>Nominal measurement range:</td>
<td>50 µm</td>
</tr>
<tr>
<td>Min. gap</td>
<td>25 µm</td>
</tr>
<tr>
<td>Max. gap</td>
<td>375 µm</td>
</tr>
<tr>
<td>Static resolution</td>
<td>&lt;0.001 % of measurement range</td>
</tr>
<tr>
<td>Dynamic resolution</td>
<td>&lt;0.002 % of measurement range</td>
</tr>
<tr>
<td>Linearity</td>
<td>&lt;0.1 % of nominal measurement range</td>
</tr>
</tbody>
</table>

**Capacitive sensor signal conditioner:**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer:</td>
<td>PI</td>
</tr>
<tr>
<td>Model number:</td>
<td>E-852.10</td>
</tr>
<tr>
<td>Number of channel:</td>
<td>1</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>0.3 / 3 / 10 kHz</td>
</tr>
<tr>
<td>Output voltage</td>
<td>-10 to +10 V / -5 to +5 V / 0 to +10 V</td>
</tr>
</tbody>
</table>

A.2.6 Piezo driver

A power amplifier is used to drive the piezoelectric tube actuator. It provides a voltage range of ±180V with a gain of 50.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer:</td>
<td>Elec. Workshop, School of Mech. Eng., Univ. Adelaide</td>
</tr>
<tr>
<td>Output voltage range:</td>
<td>-180 to +180V</td>
</tr>
</tbody>
</table>
APPENDIX A. EXPERIMENTAL SETUP

Bandwidth: DC to 10 kHz
Input voltage range: -5 to +5V
Voltage gain: 50

A.2.7 dSPACE controller board

dSPACE DS1104 is an Input/output controller board. It is used to deliver the appropriate signal to the piezo driver and record the outputs from the sensing circuits and strain gauge signal conditioner or capacitive sensor signal conditioner.

This board has eight analog/digital converters (ADCs) and eight digital/analog converters (DACs). The voltage range of all ADCs and DACs is ±10 V.
Appendix B. Relevant Conference Papers

B.1 An Innovative digital charge amplifier to reduce hysteresis in piezoelectric actuators

Australasian Conference on Robotics and Automation (ACRA), Brisbane, Australia, 2010.
An innovative digital charge amplifier to reduce hysteresis in piezoelectric actuators
Mohsen Bazghaleh, Steven Grainger, Ben Cazzolato, Tien-fu Lu
School of Mechanical Engineering, The University of Adelaide, SA 5005, AUSTRALIA
Mohsen.bazghaleh@adelaide.edu.au

Abstract
Smart actuators are the key components in a variety of nanopositioning applications, such as scanning probe microscopes and atomic force microscopes. Piezoelectric actuators are the most common actuators among a variety of smart actuators due to their high resolution, low power consumption and wide operating frequency but they suffer hysteresis which affects linearity. In this paper, an innovative digital charge amplifier is presented to reduce hysteresis in stack piezoelectric actuators. Experimental results are presented.

1 Introduction
Piezoelectric actuators have been used in many applications such as atomic force microscopy (Binnig, Quate et al. 1986), inkjet printers (Maeda 1983), fuel injectors in diesel engines (MacLachlan, Elvin et al. 2004), loudspeakers (Kompanek 1965) and many other applications.

The piezoelectric actuator is a type of nanopositioning actuator for which a mechanical displacement results when the electric field across it is changed. Compared to other nanopositioning actuators, typically piezoelectric actuators have high resolution, high force, wide operating frequency and low power consumption.

Creep and hysteresis, which are non-linear characteristics of piezoelectric materials, can reduce the accuracy of a piezoelectric actuator for positioning. When a sudden voltage is applied to a piezoelectric actuator, the length will respond quickly then change slowly due to the creep effect. This is the result of the polarisation of the piezoelectric actuator which continues to change after the applied voltage reaches its final voltage. Commonly the creep effect is an issue at low frequencies (IEEE Standard on Piezoelectricity, 1988).

When the input voltage is gradually increased, the displacement differs from when the voltage is decreased for the same applied voltage which is resulting in hysteresis.

Many techniques have been implemented to reduce the hysteresis such as: model-based control (Goldfarb and Celanovic 1997), displacement feedback control (Fanson and Caughey 1990) and charge control techniques (Fleming and Moheimani 2005).

The Preisach model (Ge and Jouaneh 1995) and the Maxwell resistive model (Goldfarb and Celanovic 1997) are two important mathematical modelling methods. The inverse of these models have been used to remove hysteresis using feedforward elements. There are many drawbacks associated with model-based control techniques. These models are static in nature, as the drive frequency increases the error will also increase. Also they need pre-processing to calculate the appropriate model for each different piezoelectric actuator.

One of the easiest ways to reduce hysteresis is to use the capacitor insertion method (Kaizuka and Siu 1988) which involves using a capacitor in series with the piezoelectric actuator. However this method reduces the operating range of a piezoelectric actuator because of the voltage drop across the capacitor.

Using a charge regulator is another. Comstock (1981) showed that by regulating the charge across the piezoelectric actuator, hysteresis will reduce significantly. The main problem with this approach is that because the sensing capacitor, piezo and opamp are not ideal, the voltage across the sensing capacitor will contain an offset voltage and this can cause drift on the piezoelectric actuator until the output voltage saturates.
APPENDIX B. RELEVANT CONFERENCE PAPERS

Comstok (1981) used an initialization circuit to remove the drift, however, it causes undesirable disturbances at high frequencies. Fleming and Moheimani (2004) proposed an extra voltage feedback loop to improve the low frequency response of their charge amplifier but this extra voltage feedback reduces the bandwidth.

In this paper, a new digital charge amplifier is presented. It significantly reduces the nonlinear behaviour of piezoelectric actuators such as hysteresis. It is more cost effective than analog charge amplifiers and this new design addresses the limited operating frequency of existing piezoelectric actuators. In Section 2 the analog charge amplifier is described. The design and analysis of a digital charge amplifier are presented in Section 3. In Section 4, three different methods to remove the drift are described followed by a conclusion in Section 5.

2 Background

Figure 1 shows a circuit for a simple analog charge amplifier. The feedback loop is used to equalize the reference voltage, \( V_{\text{ref}} \), with the voltage across the sensing capacitor. The sensing capacitor is used to integrate the current, and the voltage across it is proportional to charge. \( R_1 \) is used to model the opamp input terminal leakage and \( R_2 \) is due to the leakage of the sensing capacitor. If the opamp, sensing capacitor and the stack piezo were ideal, \( R_1 \) and \( R_2 \) will be removed. Therefore, the transfer function will be

\[
\frac{q_L}{V_{\text{ref}}} = C_S
\]

where \( C_S \) is the capacitance of the sensing capacitor. Equation (1) represents an ideal charge amplifier but in practice the main issue is the existence of the two resistors. By considering these resistors the transfer function between stack charge and input voltage will be

\[
\frac{q_L}{V_{\text{ref}}} = C_S \frac{s}{s + \frac{1}{R_1 C_{\text{piezo}}}}
\]

Equation (2) is a high pass filter with cut off frequency:

\[
\omega_c = \frac{1}{R_1 C_{\text{piezo}}}
\]

At frequencies below \( \omega_c \) the load impedance is much higher than \( R_1 \). Therefore, in contrast with infinite DC impedance, a DC current passes through \( R_1 \) which applied DC compliance voltage \( V_0 = R_1 i_{dc} \) across the piezo (Fleming and Moheimani 2005).

As an example, by using a stack piezo AE050D44H40 from NEC with a capacitance of 3.4 \( \mu \)F, and to limit the DC current to 10 mA, for the 40V maximum DC offset voltage across the piezo, a parallel resistor 4 \( k\Omega \) is needed. As an illustration, figure 2 shows the frequency response of this analog charge amplifier. It can be seen that at frequencies of less than 24 Hz the phase lead exceeds 26 degree which causes distortion for tracking applications.
APPENDIX B. RELEVANT CONFERENCE PAPERS

3 Design and analysis

Figure 3 shows the digital charge amplifier that forms the basis of this work. It consists of an analog power amplifier, DAC, ADC and a DSP. A shunt resistor is placed in series with the stack piezoelectric actuator and a protection circuit protects the DSP from high voltage.

In general, this circuit measures the charge across the piezoelectric actuator, and by using a closed-loop control system it tries to equalize the desired input charge signal with the actual charge, \( Q_{\text{piezo}} \). To measure the charge, the system integrates the current which passes the piezoelectric actuator and is given by

\[
q_{\text{piezo}} = \int i(t) \, dt \tag{4}
\]

Because of protection circuit resistor \( R_p \) and the DSP input impedance \( R_{\text{inputDSP}} \), the piezoelectric actuator current is given by:

\[
i(t) = \frac{V_s(t)}{R_{\text{shunt}} \| (R_p + R_{\text{inputDSP}})} \tag{5}
\]

Substituting Equation (5) into (4) gives the piezo charge:

\[
q_{\text{piezo}} = \int \frac{V_s(t)}{R_{\text{shunt}} \| (R_p + R_{\text{inputDSP}})} \, dt \tag{6}
\]

The protection resistor and input impedance are in series and together they are in parallel with shunt resistor \( R_{\text{shunt}} \), so the total resistance is:

\[
R_{\text{total}} = R_{\text{shunt}} \| (R_p + R_{\text{inputDSP}}) \tag{7}
\]

Substituting Equation (7) in (6):

\[
q_{\text{piezo}} = \frac{1}{R_{\text{total}}} \int V_s(t) \, dt \tag{8}
\]

Therefore the charge across the piezoelectric actuator is equal to the integral of the voltage across the shunt resistor divided by the total resistance.
Figures 4 and 5 illustrate the improvement in linearity offered by the new digital charge amplifier compared to a standard voltage amplifier. The displacement of the stack piezoelectric actuator was measured using a strain gauge. For the same displacement range of 11.54 μm, the digital charge amplifier has maximum hysteresis of 144 nm while it is 1598 nm for the voltage amplifier. Therefore, the digital charge amplifier has reduced the hysteresis by 91%.

It can be shown that the discrete transfer function from the input (desired charge) to the output (actual charge) is given by \( G(z) = \frac{z-1}{z-1} = 1 \).

In other words, unity gain (up to the Nyquist limit of the A/D rate and loop rate of the controller).

The most significant difficulty in using this technique is drift.

4 Drift removal

Because the analog to digital converter is not ideal, it suffers from current leakage. This can cause a bias voltage \( V_{bias} \) in the input. This voltage is the main reason for the drift in charge which is given by:

\[
q_{Piezo} = \frac{1}{R_{Total}} \int (V_s(t) + V_{bias}) dt
\]

This voltage bias can cause miscalculation of the actual charge across the piezoelectric actuator, thus the voltage applied to the piezoelectric actuator will drift and finally saturate.

In the following section, several methods to remove the drift are described.

4.1 Integrator reset

Comstock (1981) uses an analog initialization circuit to reset the circuit to avoid drift. A switch is used to short out the sensing capacitor and sets the voltage across it to zero, thus restarting the circuit periodically.

A similar idea is used in the digital implementation in this paper but here it does not happen periodically. Figure 6 shows the block diagram of this method. Once the voltage across the piezoelectric actuator is equal to zero, the integrator will be restarted. This process is implemented within the DSP so no additional hardware is required.

The result is presented in figure 7. As you can see, this technique distorts the signal when the charge crosses zero.
4.2 LPF bias estimator

The easiest way to calculate the bias is using a low pass filter and then removing the bias from the original signal. Figure 8 shows the block diagram of a LPF bias estimator.

The response of a pure integrator and bias estimator to a sine wave of frequency 100 rad/s and a DC bias of 100 μC is shown in Figure 9. It can be seen that the pure integrator has more drift when compared to the bias estimator. After 4 seconds, the pure integrator has 1524 μC error while the bias estimator has 195 μC. This method has reduced the drift by 87%.

4.3 Modified Integrator

A common solution to remove the drift in the output of an integrator is to replace the integrator with a first order low pass filter. However using a LPF can produce errors in the magnitude and phase, especially at frequencies lower than the cut off frequency. To solve this problem a modified integrator can be used (Hu and Wu 1998).
Figure 10 shows the modified integrator. At high frequencies the feedback loop gain is zero. Therefore, the transfer function behaves like a low pass filter. At low frequencies the feedback loop acts to remove the DC drift (Hu and Wu 1998).

If the output signal does not exceed the limitation level in the saturation block, then the transfer function will be a pure integrator. If the input signal reaches the limiting level, the output signal will be

\[ \frac{1}{s + w_c} \int \frac{w_c}{s + w_c} - z = y \tag{10} \]

This modified integrator can solve the problem of a pure integrator but the difficulty with this method is to know the limitation of the applied charge.

Figure 11 demonstrates the output of pure integrator and modified integrator shown in Figure 10. It is clear that the pure integrator suffers from drift while the modified integrator has removed the drift.

Although the modified integrator has less drift compared to other drift removal techniques, it has some signal distortion at high frequency. The LPF bias estimator has more drift and it has a simpler implementation. In dynamic applications, if it is known that the signal crosses zero in each period the best option is Integrator reset as it can remove drift completely and distortion is minimal.

5 Conclusion
An innovative digital charge amplifier has been introduced to reduce the non-linear behaviour of a piezoelectric actuator. It reduced the hysteresis by 91% at 10 Hz. Techniques have also been developed to compensate for the drift that plagues charge amplifiers. Compared to analog techniques the digital method shows higher performance and is likely to be far more cost effective.

6 Acknowledgment
The authors would like to thank Jayesh L. Minase for his support and fruitful discussions.

References
B.2 Model-based drift correction of a digital charge amplifier

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Model-based drift correction of a digital charge amplifier
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Abstract – Piezoelectric actuators are the most common among a variety of smart actuators due to their high resolution, low power consumption and wide operating frequency but they suffer hysteresis which affects linearity. In this paper a novel digital charge amplifier is presented which reduces hysteresis and linearizes the piezoelectric actuator. A data fusion algorithm uses a non-linear ARX model to remove drift. Experimental results are presented.

Keywords – Modeling, piezoelectric actuator, data fusion.

1 INTRODUCTION
In the piezoelectric actuator a mechanical displacement results when the electric field across it is changed. Piezoelectric actuators have been used in many applications such as atomic force microscopy [1], inkjet printers [2], fuel injectors in diesel engines [3], loudspeakers [4] and many other applications. For nanpositioning, piezoelectric actuators have high resolution, high force, wide operating frequency and low power consumption in comparison with other nanopositioning actuators.

When the input voltage is gradually increased, the displacement differs from when the voltage is decreased for the same applied voltage resulting in hysteresis. Many techniques have been implemented to reduce hysteresis such as: model-based control [5], displacement feedback control [6] and charge control techniques [7].

The Preisach model [8] and the Maxwell resistive model [9] are two important mathematical modeling methods. The inverse of these models have been used to remove hysteresis using feedforward elements. There are many drawbacks associated with model-based control techniques. These models are typically static in nature and as the drive frequency increases the error will also increase. Also they commonly need pre-processing to calculate the appropriate model for each different piezoelectric actuator.

One of the easiest ways to reduce hysteresis is to use the capacitor insertion method [10] which involves using a capacitor in series with the piezoelectric actuator. However this method reduces the operating range of a piezoelectric actuator because of the voltage drop across the capacitor.

Using a charge regulator is another. Comstock [11] showed that by regulating the charge across the piezoelectric actuator, hysteresis will reduce significantly. The main problem with this approach is that because the sensing capacitor, piezo and opamp are not ideal, the voltage across the sensing capacitor will contain an offset voltage and this can cause drift on the piezoelectric actuator until the output voltage saturates.

Comstock [11] used an initialization circuit to remove the drift, however, it causes undesirable disturbances at high frequencies. Fleming and Moheimani [12] proposed an extra voltage feedback loop to improve the low frequency response of their charge amplifier but this extra voltage feedback reduces the bandwidth.

A digital charge amplifier [13] has been shown to significantly reduce the nonlinear behavior of piezoelectric actuators due to hysteresis. It is more cost effective than analog charge amplifiers but drift is a major drawback.

In this paper, a technique employing data fusion has been presented to remove the drift and improve the performance of the digital charge amplifier.

In Section 2 the digital charge amplifier is described. Model-based drift correction is presented in Section 3. In Section 4, the experimental result will be described followed by a conclusion in Section 5.

2 DIGITAL CHARGE AMPLIFIER
Figure 1 shows the digital charge amplifier that forms the basis of this work. It consists of an analog voltage amplifier, DAC, ADC and a DSP. A sensing resistor is placed in series with the stack piezoelectric actuator and a protection circuit protects the DSP from high voltage.

In general, this circuit measures the charge across the piezoelectric actuator, and by using a closed-loop control system it tries to equalize the desired input charge signal with the actual charge. To measure the charge, the system integrates the current which passes the piezoelectric actuator which gives an estimate of the charge in the piezo, given by

\[ q_{\text{piezo}} = \int i(t) \, dt \quad (1) \]

Because of protection circuit resistor \( R_p \) and the DSP input impedance \( R_h \), the piezoelectric actuator current is given by

\[ i(t) = \frac{V_S(t)}{R_{\text{sensing}}} \left( \frac{1}{(R_p + R_h)} \right), \quad (2) \]

Substituting Equation (2) into (1) gives the piezo charge

\[ q_{\text{Piezo}} = \int \frac{V_S(t)}{R_{\text{sensing}}} \left( \frac{1}{(R_p + R_h)} \right) \, dt \quad (3) \]

The protection resistor and input impedance are in series and together they are in parallel with sensing resistor \( R_{\text{sensing}} \), so the total resistance is:

\[ R_T = R_{\text{sensing}} \left( \frac{1}{(R_p + R_h)} \right) \quad (4) \]
Substituting Equation (4) in (3) gives the charge in the piezo,

\[ q_{\text{Piezo}} = \frac{I}{R_T} \int V_S(t) dt. \]  

Therefore the charge across the piezoelectric actuator is equal to the integral of the voltage across the sensing resistor divided by the total resistance.

Figures 2 and 3 illustrate the improvement in linearity offered by the new digital charge amplifier compared to a standard voltage amplifier. The displacement of the stack piezoelectric actuator was measured using a strain gauge. For the same displacement range of 11.54 µm, the digital charge amplifier has maximum hysteresis of 144 nm while it is 1598 nm for the voltage amplifier. The digital charge amplifier has reduced the hysteresis by 91%.

In the digital charge amplifier, because the analog to digital converter is not ideal, it suffers from current leakage. This
can cause a bias voltage $V_{bias}$ at the input. This voltage is the main reason for the drift in charge which is given by

$$q_{\text{Piezo}} = \frac{I}{R} \left( V_s(t) + V_{bias} \right) dt.$$  \hspace{1cm} (6)

This voltage bias can cause miscalculation of the actual charge across the piezoelectric actuator, thus the voltage applied to the piezoelectric actuator will drift and finally saturate the amplifier.

In [13], three techniques (reset integrator, low pass filter (LPF) and modified integrator) have been proposed to remove the drift in the digital charge amplifier.

In the reset integrator technique, once the voltage across the piezoelectric actuator is equal to zero, the integrator will be restarted. By restarting the integrator, the accumulated errors in the integrator output will be removed; therefore the drift in the output will be removed. When the integrator is restarted it means that the measured charge is changed instantaneously from a non-zero value to zero. This sudden change is equivalent to a scaled step signal and, as the step signal has high frequency components, introduces distortion.

With the LPF technique, a low pass filter is used to measure the bias which can subsequently be subtracted from the original signal. The drawback of this technique is that the input signal should be greater than the cut off frequency of the low pass filter if distortion is to be avoided.

To solve this problem a modified integrator can be used as shown in Figure 4 [14]. At high frequencies the feedback loop gain is zero and the transfer function behaves like a low pass filter. At low frequencies the feedback loop acts to remove the DC drift.

![Figure 4. Block diagram of a modified integrator](image)

If the output signal does not exceed the limits in the saturation block the transfer function will be that of a pure integrator. If the input signal reaches the limiting level, the output signal will be

$$\frac{l}{s + w_c} x + \frac{w_c}{s + w_c} z = y.$$ \hspace{1cm} (7)

This modified integrator thus solves the problem of a pure integrator but the difficulty with this method is to know the limits of the applied charge.

Although the modified integrator has less drift compared to other drift removal techniques, it has some signal distortion at high frequency. The LPF bias estimator has more drift than the other techniques and it has a simpler implementation. In dynamic applications, if it is known that the voltage across piezoelectric actuator crosses zero in each period the best option is integrator reset as it can remove drift completely and distortion is minimal.

All of the techniques have some limitations. In the next section a model-based drift correction technique is proposed which can remove drift with better performance than the others.

## 3 MODEL-BASED DRIFT CORRECTION

The charge across the piezoelectric actuator can be estimated by integrating the current passing through it. This technique can provide an accurate estimate of charge but it will drift over time due to small errors in the current measurement. This is because the new charge is calculated iteratively using the previous charge, the charge errors are accumulated and the error grows over time.

The charge can also be estimated by a nonlinear autoregressive exogenous model (NARX), shown in Figure 5. In this model the input is the voltage across the piezoelectric actuator and the output is charge. The output charge is noisy but it does not drift.

Data fusion is used to combine the reliability and short term accuracy of the digital charge amplifier with the long term accuracy of the NARX model to get the benefit of both techniques.

### 3.1 Modelling charge

The charge across the piezoelectric actuator has a nonlinear relation with the voltage across it. A nonlinear autoregressive exogenous model (NARX) is used to calculate the output charge. A wavelet network has been used to model the nonlinearity of piezoelectric actuator. The piezoelectric actuator is considered as a black box model.

For a system with a single input of $u$ and a single output of $y$ like a one-dimensional piezoelectric actuator, the governing equations are

$$y(t) = f \left( u(t - t_d), u(t - t_d - t_s), ..., y(t - t_s), ..., y(t - 2t_s), ..., y(t - r_s t_s) \right),$$ \hspace{1cm} (8)

where $t_s$ is sampling time, $t_d$ is delay time, and $r_u$ and $r_y$ are input and output orders respectively. The variable $f$ can be approximated by any nonlinear function. In this work, $f$ is a wavelet neural network, and $t_s = 0.001s$, $t_d = 0.001s$, $r_u = 2$ and $r_y = 4$. The input of NARX model, in this research, is voltage across the piezoelectric actuator, and its output is the charge across the piezoelectric actuator. The accuracy of this model and its output are influenced by the noise; however, drift has no effect on this model.
3.2 Data fusion

Figure 5 shows the block diagram of the model-based drift correction. $w_a$ and $w_b$ are weighting coefficients for the digital charge amplifier and piezoelectric model. The sum of $w_a$ and $w_b$ is 1.

If $w_a$ is one and $w_b$ is zero, the system is a pure integrator and if $w_a$ is zero and $w_b$ is one, the system only relies on model. Choosing $w_a$ between zero and one can mix the benefit of both techniques. The output optimal charge is

$$q_{\text{optimal}}[k] = w_a \cdot q_{\text{DCA}}[k] + w_b \cdot q_{\text{model}}[k].$$  \hfill (9)

The output of the integrator is given by

$$q_{\text{DCA}}[k] = q_{\text{optimal}}[k-1] + T_s \cdot (I_s[k] + e_{I_s}[k]),$$  \hfill (10)

where $T_s$ is the sampling time, $I_s$ is piezoelectric current and $e_{I_s}$ is the input noise. Considering $e_m$ as an error on the output of the NARX model

$$q_{\text{model}}[k] = q_{\text{real}}[k] + e_m[k].$$  \hfill (11)

Substituting Equations (10) and (11) in Equation (9) gives

$$q_{\text{optimal}}[k] = w_a \cdot q_{\text{optimal}}[k-1] + w_a \cdot T_s \cdot I_s[k] + \ldots + T_s \cdot e_{I_s}[k] + w_b \cdot q_{\text{real}}[k] + w_b \cdot e_m[k].$$  \hfill (12)

Simplifying this equation, the final error on the optimal output will be:

$$e_{q_{\text{opt}}}[k] = (1 - w_a) \cdot \sum_{n=0}^{k-1} w_a^n \cdot e_m[k] + \ldots + T_s \cdot \sum_{n=0}^{k+1} w_a^n \cdot e_{I_s}[k].$$  \hfill (13)

In this equation $e_{q_{\text{opt}}}[k]$ is optimal output error which is equal to two geometric series. If the system is a pure integrator, $w_a$ is equal to one, the geometric series will diverge, the error will be accumulated and the output will drift. If $w_a$ is between zero and one the geometric series will converge and therefore there will not be drift of the output.

Because the error in the digital charge amplifier is small compared to the model error in the short term, increasing $w_a$ can increase the accuracy, on the other hand increasing $w_a$ will increase the convergence time. In summary, there is a trade-off between accuracy and convergence time.

4 EXPERIMENTAL RESULTS

The proposed technique is validated experimentally by using a stack piezoelectric actuator AE0505D44H40 from NEC. To evaluate the displacement, optical sensor D20 from PHILTECH is used.

Figure 6 compares desired charge with actual charge at different frequencies using different techniques. It can be seen that the reset integrator has poor performance, especially at low frequencies, with some discontinuities in the output signal. The modified integrator output has distortion which is significant particularly at low frequencies. The reset integrator and modified integrator have removed drift but at the cost of some distortion in the output signal.

It can be seen that the model based drift correction has better performance at all frequencies.
5 CONCLUSION

In this paper, a digital charge amplifier is presented which linearizes the output of the piezoelectric actuator. A model-based drift correction technique has been proposed to remove the drift and improve the tracking performance of the digital charge amplifier. It uses data fusion centre to integrate the reliability and short term accuracy of a digital charge amplifier with the long term accuracy of the NARX model to get the benefit of both techniques. The experimental results show the improvement in the tracking performance compared to other techniques.

REFERENCES


B.3 Using frequency-weighted data fusion to improve the performance of a digital charge amplifier

Using Frequency-weighted Data Fusion to Improve the Performance of a Digital Charge Amplifier

M. Bazghaleh, S. Grainger, B. Cazzolato and T. Lu

Abstract—Piezoelectric actuators are the most common among a variety of smart actuators due to their high resolution, low power consumption and wide operating frequency but they suffer hysteresis which affects linearity. In this paper a novel digital charge amplifier is presented which reduces hysteresis and linearizes the piezoelectric actuator. A frequency-weighted data fusion algorithm uses a non-linear ARX model to remove drift and increase the bandwidth of digital charge amplifier. Experimental results are presented.

I. INTRODUCTION

In the piezoelectric actuator a mechanical displacement results when the electric field across it is changed. Piezoelectric actuators have been used in many applications such as atomic force microscopy [1], inkjet printers [2], fuel injectors in diesel engines [3], loudspeakers [4] and many other applications. For nanopositioning, piezoelectric actuators have high resolution, high force, wide operating frequency and low power consumption in comparison with other nanopositioning actuators.

When the input voltage is gradually increased, the displacement differs from when the voltage is decreased for the same applied voltage resulting in hysteresis. Many techniques have been implemented to reduce hysteresis such as: model-based control [5], displacement feedback control [6] and charge control techniques [7].

The Preisach model [8] and the Maxwell resistive model [9] are two important mathematical modeling methods. The inverse of these models have been used to remove hysteresis using feedforward elements. There are many drawbacks associated with model-based control techniques. These models are typically static in nature and as the drive frequency increases the error will also increase. Also they commonly need pre-processing to calculate the appropriate model for each different piezoelectric actuator.

One of the easiest ways to reduce hysteresis is to use the capacitor insertion method [10] which involves using a capacitor in series with the piezoelectric actuator. However this method reduces the operating range of a piezoelectric actuator because of the voltage drop across the capacitor.

Using a charge regulator is another. Comstock [11] showed that by regulating the charge across the piezoelectric actuator, hysteresis will reduce significantly. The main problem with this approach is that because the sensing capacitor, piezo and opamp are not ideal, the voltage across the sensing capacitor will contain an offset voltage and this can cause drift on the piezoelectric actuator until the output voltage saturates.

Comstock [11] used an initialization circuit to remove the drift, however, it causes undesirable disturbances at high frequencies. Fleming and Moheimani [12] proposed an extra voltage feedback loop to improve the low frequency response of their charge amplifier but this extra voltage feedback reduces the bandwidth.

A digital charge amplifier [13] has been shown to significantly reduce the nonlinear behavior of piezoelectric actuators due to hysteresis. Because a low cost microcontroller can be used to implement the digital charge amplifier, it is more cost effective than analog charge amplifiers but drift is a major drawback.

In this paper, a technique employing data fusion has been presented to remove the drift and improve the performance of the digital charge amplifier.

In Section II the digital charge amplifier is described. A frequency-weighted data fusion algorithm is presented in Section III. In Section IV, the experimental results will be described followed by a conclusion in Section V.

II. DIGITAL CHARGE AMPLIFIER

Fig. 1 shows the digital charge amplifier that forms the basis of this work. It consists of an analog voltage amplifier, DAC, ADC and a DSP. A sensing resistor is placed in series with the stack piezoelectric actuator and a protection circuit protects the DSP from high voltage.

This circuit measures the charge across the piezoelectric actuator, and by using a closed-loop control system it tries to equalize the desired input charge signal with the actual charge $q_{\text{Piezo}}$. To measure the charge, the system integrates the current which passes the piezoelectric actuator and is given by

$$q_{\text{Piezo}} = \int i(t) dt$$

Because of protection circuit resistor $R_p$ and the DSP input impedance $R_{il}$, the piezoelectric actuator current is given by

$$i(t) = \frac{v(t)}{R_p + R_{il}}$$

where $v(t)$ is the voltage across the sensing capacitor.
Substituting Equation (2) into (1) gives the piezo charge

\[ q_{\text{Piezo}} = \int \frac{V_S(t)}{R_{\text{Sensing}} || (R_P + R_{II})} dt \]  
\[ (3) \]

The protection resistor and input impedance are in series and together they are in parallel with sensing resistor \( R_{\text{Sensing}} \), so the total resistance is

\[ R_T = R_{\text{Sensing}} || (R_P + R_{II}) \]  
\[ (4) \]

Substituting Equation (4) in (3)

\[ q_{\text{Piezo}} = \frac{1}{R_T} \int V_S(t) dt \]  
\[ (5) \]

Therefore the charge across the piezoelectric actuator is equal to the integral of the voltage across the sensing resistor divided by the total resistance.

Fig. 2 and Fig. 3 illustrate the improvement in linearity offered by the digital charge amplifier compared to a standard voltage amplifier. The displacement of the stack piezoelectric actuator was measured using a strain gauge. For the same displacement range of 11.54 µm, the digital charge amplifier has maximum hysteresis of 144 nm while it is 1598 nm for the voltage amplifier. The digital charge amplifier has reduced the hysteresis by 91%.
Fig. 4 shows the experimental result of the drift effect on the displacement of the piezoelectric actuator when it is driven by the digital charge amplifier. In the digital charge amplifier, because the analog to digital converter is not ideal, it suffers from current leakage. This can cause a bias voltage $V_{bias}$ at the input. This voltage is the main reason for the drift in charge which is given by

$$ q_{Piezo} = \frac{1}{R_T} \int (V_S(t) + V_{bias}) dt $$

This voltage bias can cause miscalculation of the actual charge across the piezoelectric actuator, thus the voltage applied to the piezoelectric actuator will drift and finally saturate the amplifier.

When the integrator is restarted it means that the measured charge is changed instantaneously from a non-zero value to zero. This sudden change is equivalent to a scaled step signal and, as the step signal has high frequency components, introduces distortion.

With the LPF technique, a low pass filter is used to measure the bias which can subsequently be subtracted from the original signal. The drawback of this technique is that the input signal should be greater than the cut off frequency of the low pass filter if distortion is to be avoided.

To solve this problem a modified integrator can be used as shown in Fig. 5 [14]. At high frequencies the feedback loop gain is zero and the transfer function behaves like a low pass filter. At low frequencies the feedback loop acts to remove the DC drift.

$$ Y(s) = \frac{X(s)}{1 + \frac{w_c}{s + w_c}} $$

This modified integrator thus solves the problem of a pure integrator but the difficulty with this method is to know the limits of the applied charge.

Although the modified integrator has less drift compared to other drift removal techniques, it has some signal distortion at high frequency. The LPF bias estimator has more drift than the other techniques and it has a simpler implementation. In dynamic applications, if it is known that the voltage across piezoelectric actuator crosses zero in each period the best option is integrator reset as it can remove drift completely and distortion is minimal.

All of the techniques have some limitations. In the next section a weighted-function data fusion technique is proposed which can remove drift with better performance than the others.

### III. FREQUENCY-WEIGHTED FILTERS

The charge across the piezoelectric actuator can be estimated by integrating the current passing through it. This technique can provide an accurate estimate of charge but it will drift over time due to small errors in the current measurement. This is because the new charge is calculated...
iteratively using the previous charge, the charge errors are accumulated and the error grows over time. Also at low frequencies, less than 1 hertz, the piezoelectric impedance is much higher than the impedance of the sensing resistor. Therefore the sensing voltage $V_s$ is below the noise floor and it is impossible for the DCA to accurately measure the charge. Hence a DCA is not suitable for low frequencies.

The hysteresis of a piezoelectric actuator has rate-dependent characteristics. It means that by increasing the frequency of the input voltage, the width of the hysteresis curve increases and it rotates clockwise. The rate dependent characteristic of a piezoelectric actuator makes modeling difficult. Therefore rate-independent hysteresis is easier to model [15]. In this paper, to reduce the complexity, an autoregressive exogenous model (NARX) has been used to estimate the charge using a rate-independent model. It has better performance at low frequency than the DCA output does not suffer drift.

Frequency-weighted filters are used to combine the benefits of the rate-independent model with the digital charge amplifier. At low frequency, the output charge relies more on the model while at high frequency it uses the reliability and short term accuracy of the digital charge amplifier with the long term accuracy of the NARX model to get the benefit of both techniques.

A. Modelling charge

The charge across the piezoelectric actuator has a nonlinear relation with the voltage across it. A nonlinear autoregressive exogenous model (NARX) is used to calculate the output charge.

For a system with a single input of $u$ and a single output of $y$ like a one-dimensional piezoelectric actuator, the governing equations are

$$y(t) = f(u(t-t_d), u(t-t_d-\delta), \ldots, u(t-t_d-r_u t_s), y(t-t_y), \ldots, y(t-2 t_s), \ldots, y(t-r_y t_s))$$

where $t_s$ is the sampling time, $t_d$ is the delay time, $r_u$ and $r_y$ are input and output orders respectively. The variable $f$ can be approximated by any nonlinear function. In this work, $t_s=0.001\text{s}$, $t_d=0.001\text{s}$, $r_u=2$ and $r_y=4$. The input of the NARX model is the voltage across the piezoelectric actuator, and its output is the charge across the piezoelectric actuator. The accuracy of this model and its output are influenced by noise; however, drift has no effect.

B. Data Fusion

Fig. 6 shows the block diagram of the frequency weighted data fusion. The $W_{DCA}$ and $W_m$ are the digital charge amplifier and NARX modeling weighting filters. In each frequency, the summations of the weighting filters is one

$$W_{DCA}(j\omega) + W_m(j\omega) = 1.$$  (9)

Because the NARX model in rate-independent, at high frequency it is not accurate. If $W_{DCA}$ is one and $W_m$ is zero, the system is a pure integrator and if $W_{DCA}$ is zero and $W_m$ is one, the system only relies on model. Choosing $W_{DCA}$ between zero and one can mix the benefit of both techniques. The transfer functions of weighting filters are
It can be seen that at high frequencies, \( W_{DCA} \) is 0.99 and \( W_m \) is 0.01.

The output optimal charge is

\[
q_{\text{optimal}}[k] = W_{DCA} q_{\text{optimal}}[k-1] + \ldots + W_{DCA} T_s I_2[k] + w_{DCA} T_s e_{\text{ls}}[k] + \ldots + \sum_{n=0}^{k-1} w_a^{k+n} e_m[k] + \ldots
\]

Simplifying this equation, the final error on the optimal output will be:

\[
e_{q_{\text{opt}}}[k] = (1 - w_a) (\sum_{n=0}^{k-1} w_a^{k+n} e_m[k] + \ldots) + T_s (\sum_{n=0}^{k-1} w_a^{k+n} e_{\text{ls}}[k])
\]

In this equation, \( e_{q_{\text{opt}}}[k] \) is optimal output error which is equal to two geometric series. If the system is a pure integrator, \( w_{DCA} \) is equal to one, the geometric series will diverge, the error will be accumulated and the output will drift. If \( w_{DCA} \) is between zero and one the geometric series will converge and therefore there will not be drift of the output.

Because the error in the digital charge amplifier is small compared to the model error in the short term, increasing \( w_{DCA} \) can increase the accuracy, on the other hand increasing \( w_{DCA} \) will increase the convergence time. In summary, there is a trade-off between accuracy and convergence time.
APPENDIX B. RELEVANT CONFERENCE PAPERS

IV. EXPERIMENTAL RESULTS

The proposed technique is validated experimentally by using a stack piezoelectric actuator AE0505D44H40 from NEC. To evaluate the displacement, an optical sensor, D20 from PHILTECH, is used.

Fig. 7 compares desired position with actual position at different frequencies using the different techniques. It can be seen that the NARX model has better performance at low frequencies while the DCA is better at high frequencies. The frequency-weighted data fusion technique has a reasonable performance at both high frequency and low frequency while removing the drift.

V. CONCLUSION

In this paper, a digital charge amplifier is presented which linearizes the output of the piezoelectric actuator. A frequency-weighted data fusion technique has been proposed to remove the drift and improve the tracking performance of the digital charge amplifier. At low frequency it uses the NARX model and at high frequency it uses frequency-weighted filters to integrate the reliability and short term accuracy of a digital charge amplifier with the long term accuracy of the NARX model to get the benefit of both techniques. The experimental results show the improvement in the tracking performance compared to other techniques.

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